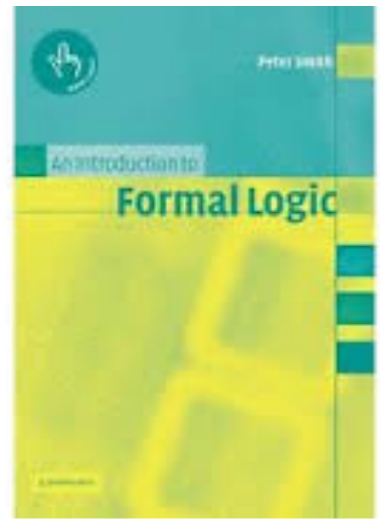
PL-UY 2004 Symbolic Logic Spring 2016 T/Th 8:30-10:20 RH425

Prof: Jonathan Bain Office: LC124 Hours: Weds 1-2 faculty.poly.edu/~jbain/logic

I. Description

This is an introduction to the methods and applications of 1st-order symbolic logic, including both propositional logic (**PL**) and relational predicate (or "quantifier") logic (**QL**). The course covers methods of testing arguments for deductive validity and deductive invalidity, as well as methods for identifying tautologies, contradictions and logical equivalences. In addition, the tree method will be used to demonstrate *soundness* and *completeness* of **PL** and **QL**.



II. Required Text:

Smith, P. (2003) *An Introduction to Formal Logic*, Cambridge Univ. Press.

III. Requirements

1. There will be a **homework assignment** handed out each week. The assignment for any given week will be due at the beginning of class on the Thursday of the following week (with exceptions noted on the schedule). ***Late assignments will not be accepted.*** Your final assignment grade will be calculated from the highest 10 of your 12 individual assignment grades.

2. There will be two **exams** and a **final**. The final will be minimally cumulative (i.e., it will emphasize the units not covered by the preceding exams, but may include a few questions from previous material). Makeup exams can only be given in very extenuating circumstances and only for legitimate reasons. Holiday scheduling is not a legitimate reason. Please schedule and manage your time effectively.

IV. Grade Distribution. Exams: 25% each. Assignments: 20%. Final: 30%

01. Introduction (Chaps 1-6)

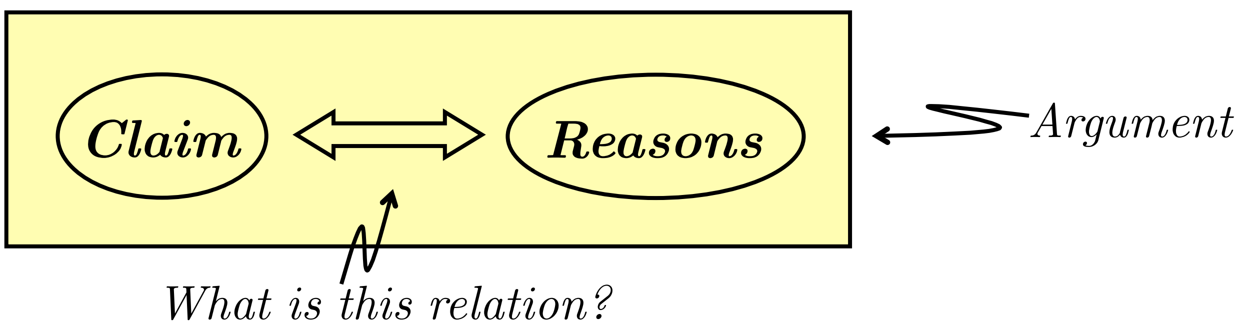
Purpose of course: Intro to methods of *symbolic logic*.

Logic = a subfield of *critical thinking*.

- In general, we want to be well-informed.
- To be well-informed requires being able to critically analyze the claims that people make.

Question: When should we believe a *claim* that someone makes?

One answer: When she give us *reasons* to believe the claim.



- *Critical thinking* is the analysis of this relation -- the analysis of *arguments*.
- *Logic* is the analysis of a particular type of argument called a *deductive argument*.

I. Arguments

An *argument* is a set of *declarative sentences* consisting of two parts:

(1) The *conclusion* = a sentence representing the claim being argued for.

(2) The *premises* = sentences that represent the reasons given for the claim.

> - *An argument is not a quarrel.*
> - *An argument is not just a claim.*

II. Two Types of Arguments

1. An *inductive argument* is an argument in which the premises provide *partial support* for the conclusion.

> *Ex.* (1) 80% of observed smokers got lung cancer.
> *So:* (2) Smoking causes lung cancer.

- The truth of the premise does *not* guarantee the truth of the conclusion.
- A variety of criteria can be suggested to strengthen the partial support that the premise provides the conclusion.
- Scientific reasoning is based primarily on inductive arguments.
- This course is *not* about inductive arguments!

2. A *valid-deductive argument* is an argument in which, *if* the premises are true, the conclusion *must* be true.

Ex1. (1) All humans are mortal.
(2) The French are humans.
So: (3) The French are mortal.

Ex2. (1) All animals with wings can fly.
(2) Pigs are animals with wings.
So: (3) Pigs can fly.

General Form. (1) All A's are B's.
(2) All C's are A's.
So: (3) All C's are B's.

Importance of General Form (or "pattern"):
If you can show that a general form of argument is valid, then all instances of it are valid.

Essence of this course:
We will develop techniques to analyze general forms of arguments to see if they are valid.

Def. 1. An *argument* is *valid* just when it has a valid form.

Def. 2. An *argument form* is *valid* just when there are no instances of that form in which all the premises are true and the conclusion is false.

Def. 3. An *argument form* is *invalid* just when it has an instance in which all premises are true and the conclusion is false. This instance, if it exists, is called a *counterexample* to the argument form.

Ex3.	(1)	All cows are reptiles.	**F**
	(2)	All reptiles have three eyes.	**F**
So:	(3)	All cows have three eyes.	**F**

Ex3′.	(1)	All cows are reptiles.	**F**
	(2)	All reptiles have udders.	**F**
So:	(3)	All cows have udders.	**T**

Ex4.	(1)	If Pippin is a cat, then Pippin is a mammal.	**T**
	(2)	Pippin is a mammal.	**T**
So:	(3)	Pippin is a cat.	**F**

INVALID!

Symbolic Logic is the study of general forms of deductive arguments by means of symbolic languages.

Two Types of Formal Language in this class:

(1) Propositional Logic (**PL**): Chapters 7-20.

(2) Predicate ("Quantifier") Logic (**QL**): Chapters 21-30

- These are ***languages*** (just like English, Spanish, C++, Prolog, Russian, Hindi, any branch of mathematics, *etc.*). They have their own *alphabet*, *grammar*, and *semantics*.
- They are ***very simple*** languages.
- Their advantage:
  - (i) *Rhetorically*: They are ***so simple*** that they can be used, together with a few formal rules, to prove validity or invalidity for most types of argument forms in any given "ordinary" language (English, Spanish, Russian, Hindi, *etc.*).
  - (ii) *Mathematically*: They are so simple that they can be used, together with a few formal rules, to provide a basis for most if not all branches of mathematics!

Chapter 7: Three Propositional Connectives

Goals:

- To construct a formal language (**PL**) based on the propositional connectives "and", "or", "not".
- To use **PL** to analyze deductive arguments for validity.

Idea: Use "and", "or", "not" to connect together simple *declarative sentences* into compound sentences.

Def. 1. A ***declarative sentence*** is a sentence that is either true or false, but not both at the same time.

Ex:	Pippin is a cat.	(declarative)
	Is Pippin a cat?	(inquisitive)
	Catch Pippin now!	(imperative)

*We will restrict **PL** to declarative sentences only! ("propositions")*

English Arguments: Examples

Ex1.	(1)	*Either* we are out of petrol *or* the carburetor is blocked.
	(2)	We are not out of petrol.
So:	(3)	The carburetor is blocked.

Ex2.	(1)	*Either* Jill is at the library *or* she is at the coffee bar.
	(2)	Jill isn't at the library.
So:	(3)	Jill is at the coffee bar.

General Form.	(1)	*Either A or B.*
	(2)	*Not A.*
So:	(3)	*B.*

VALID

Ex3. (1) It's not the case that Tony supports the policy and George does too.

(2) Tony supports the policy.

So: (3) George does not.

VALID

General Form. (1) Not (A and B).

(2) A.

So: (3) Not B.

Ex4. (1) It is not the case that Alice is clever and Bob is, too.

So: (2) Alice is not clever.

INVALID

Counterexample:
Suppose Alice is a brainiac and Bob is a dunce.

General Form. (1) Not (A and B).

So: (2) Not A.

General Semantic Rules for "and", "or", "not" in English:

1. *The **conjunction** of two propositions A and B is true just when A and B are true, and false otherwise.*

But Beware!

Exceptions in English to this rule:

Ex1. (1) There is not a word of truth to claims that Jill loves Jack.
(2) Jack loves Jill.

- Suppose both (1) and (2) are true separately.
- But the conjunction of (1) and (2) seems to deny that Jack loves Jill.

Ex2. (1) Eve became pregnant and she married Adam.
(2) Eve married Adam and she became pregnant.

- (1) and (2) seem to be making different claims.
- Example of "and" conveying *temporal succession*, and not simply *conjunction*.

2. *The* ***exclusive disjunction*** *of two propositions A and B is true just when exactly one of A and B is true, and false otherwise.*

The ***inclusive disjunction*** *of two propositions A and B is true just when at least on of A and B is true, and false otherwise.*

Excluive disjunction:

A or B and not both. "You can have soup or salad."

Inclusive disjunction:

A or B, perhaps both. "This bridge is open to trucks or cars."

- Need to specify which meaning the "or" in our formal language will have.

3. *The* ***negation*** *of a proposition A is true just when A is false, and false otherwise.*

Ex.	Joe is married.
Negation:	Joe is not married.
or:	It is not the case that Joe is married.

The Scope of a connective in English:

Ex. Either Jack took Jill to the party or he took Jo and he had some fun.

Meaning? Did he have fun with Jill or only with Jo?

How can this be specified?

Task: Remove all these ambiguities in meaning by constructing a simple formal language that is free of them.

Design Brief for Propositional Logic (PL)

(1) Need symbols for "and", "or", "not".

- Let "$\wedge$" represent bare conjunction (nothing else!).
- Let "$\vee$" represent inclusive disjunction (nothing else!).
- Let "$\neg$" represent bare negation ("It is not the case that...").

- New symbols in a foreign language alphabet!
- Not just abbreviations of English connectives!

(2) Need symbols to designate scope.

What does $A \vee B \vee C$ mean?

Analogy: What does $1 + 2 \times 3$ mean?

$(1 + 2) \times 3 = 9$

$1 + (2 \times 3) = 7$

- Adopt parentheses in **PL**, too.
- Every occurrence of "$\wedge$" and "$\vee$" must come with a pair of parentheses "(", ")".
- No need to do this for "$\neg$".

(3) Need a way to represent the simple declarative sentences that the connectives connect.

- Call them "atomic sentences".
- Use the following letters and primes:

P, Q, R, S, P', Q', ... , P'', Q'', ..., P''', Q''', ...

Example:

Translation Key:	P	*represents*	Jack loves Jill.
	Q	*represents*	Jill loves Jack.
	R	*represents*	Jo loves Jill.
	S	*represents*	Jack is wise.

What do the following sentences in **PL** mean in English?

(a) $\neg \mathsf{P}$

(b) $(\mathsf{S} \wedge \mathsf{P})$

(c) $\neg (\mathsf{P} \vee \mathsf{Q})$

(d) $(\neg \mathsf{P} \wedge \neg \mathsf{Q})$