

**Assignment #13. Due Thurs 5/5.**

1. Suppose  $G$  is a one-place predicate,  $M$  is a two-place predicate,  $f$  is a one-place function, and  $h$  is a two-place function. Which of the following are *wffs* of  $QL^f$ ? For those that are *wffs*, identify what type of *wff* it is.

- (a)  $f(x)$
- (b)  $Gf(x) = m$
- (c)  $Mh(f(x), y)f(h(m, y))$
- (d)  $(Gh(x, f(n)) \supset \forall x \exists z ((Mxy \wedge Gf(z)) \equiv f(x)))$

2. Use the translation key below to translate the following claims in English into  $QL^f$ .

Domain: {integers}

$m \Rightarrow 2$	$h \Rightarrow$ 2-place <i>sum</i> function
$F \Rightarrow$ __ is even	$f \Rightarrow$ 1-place <i>successor</i> function
$G \Rightarrow$ __ is odd	
$P \Rightarrow$ __ is prime	
$L \Rightarrow$ __ is less than __	

- (a) No prime number larger than 2 is even.
- (b) The sum of two even numbers is always even.
- (c) Every even number greater than 2 is the sum of two prime numbers.

3. Let  $q$  be a  $q$ -valuation for the vocabulary  $V = \{m, F, G, P, L, h, f\}$  of symbols that appear in the translation key in #2. Let the domain of  $q$  be  $D = \{\text{integers}\}$ . Let  $q$  assign  $m$  to 2, and let  $q$  assign to each predicate letter and each function letter, their intended extension in  $D$  (so, for example,  $q$  maps  $F$  to  $\{2, 4, 6, \dots\}$ , and it maps  $h$  to  $\{\langle 0, 0, 0 \rangle, \langle 0, 1, 1 \rangle, \langle 0, 2, 2 \rangle, \dots, \langle 1, 0, 1 \rangle, \langle 1, 1, 2 \rangle, \langle 1, 2, 3 \rangle, \dots\}$ ). Determine the truth values in  $q$  of the following *wffs* that have  $V$  as their vocabulary. Explain your answer by referring to the appropriate semantic rules of  $QL^f$ .

- (a)  $Gf(m)$  ("The successor of 2 is odd.")
- (b)  $\exists z Gf(z)$  ("There are integers whose successors are odd.")
- (c)  $\forall z (Gz \supset f(z) = m)$  ("Every odd integer has 2 as its successor.")