Assignment #13-key.

1. (a) f(x) — Not a *wff*. This is a *term*.
   (b) Gf(x) = m — Not a *wff*. Gf(x) is not a term.
   (c) Mh(f(x), y)f(h(m, y)) — This is an *atomic wff*.
   (d) (Gh(x, f(n)) ⊃ ∀x∃z((Mxy ∧ Gf(z)) ≡ f(x))) — Not a *wff*. The second expression in the biconditional, f(x), is a term, not a *wff*.

2.

(a) No prime number larger than 2 is even.
For all x, if x is a prime number larger than 2, then x is not even.
For all x, if Px ∧ Lmx, then ¬Fx.
∀x((Px ∧ Lmx) ⊃ ¬Fx)

(b) The sum of two even numbers is always even.
For all x and all y, if x is even and y is even, then the sum of x and y is even.
For all x and all y, if Fx ∧ Fy, then Fh(x, y).
∀x∀y((Fx ∧ Fy) ⊃ Fh(x, y))

(c) Every even number greater than 2 is the sum of two prime numbers.
For all x, if x is even and x is greater than 2, then there is a y and there is a z, such that y is a prime and z is a prime, and x is the sum of y and z.
For all x, if Fx ∧ Lmx, then there is a y and there is a z, such that Py ∧ Pz, and x = h(y, z).
∀x((Fx ∧ Lmx) ⊃ ∃y∃z((Py ∧ Pz) ∧ x = h(y, z)))

3.

(a) Gf(m) ("The successor of 2 is odd.") This is an atomic *wwf*. It is true just when the object named by f(m) has the property (i.e., is in the extenssion of the property) named by G. The object named by f(m) is 3, and 3 has the property of being odd. So Gf(m) is true in *q* ($Q0^f$).

(b) ∃zGf(z) ("There are integers whose successors are odd.") This is an existential *wff*. It is true in *q* just when there is a z-variant of *q* that makes Gf(z) true. Let *q′* be a z-variant of *q* that maps z to 4. Then the object named by f(z) in *q′* is 5, and 5 has the property named by G in *q′*. So *q′* makes Gf(z) true ($Q0^f$). So ∃zGf(z) is true in *q* ($Q7^f$).

(c) ∀z(Gz ⊃ f(z) = m) ("Every odd integer has 2 as its successor.") This is a universal *wff*. It is true in *q* just when all z-variants of *q* make (Gz ⊃ f(z) = m) true. Let *q′* be a z-variant of *q* that maps z to 5. Then the object named by f(z) in *q′* is 6, which is not in the relation = to the object named by m (i.e., 6 is not equal to 2). So *q′* makes f(z) = m false ($Q0^f$). Since *q′* also makes Gz true ($Q0^f$), *q′* makes (Gz ⊃ f(z) = m) false ($Q4^f$). So not all z-variants of *q* make (Gz ⊃ f(z) = m) true. So ∀z(Gz ⊃ f(z) = m) is false in *q* ($Q6^f$).