Assignment #12-key.

1(a) The Welsh speaker loves Mrs Jones.

There exists an x such that x speaks Welsh, and there's only one such x, and x loves Mrs Jones.

There exists an x such that Fx and $\forall y(Fy \supset y = x)$, and Lxm.

$\exists x((Fx \wedge \forall y(Fy \supset y = x)) \wedge Lxm)$

(b) Angharad loves the girl who loves Bryn.

There exists an x such that x is a girl and x loves Bryn, and there's only one such x, and Angharad loves x.

There exists an x such that Gx and Lxb, and $\forall y((Gy \wedge Lyb) \supset y = x)$, and Lax.

$\exists x(((Gx \wedge Lxb) \wedge \forall y((Gy \wedge Lyb) \supset y = x)) \wedge Lax)$

(c) The girl other than the girl who loves Bryn is Angharad.

There exists an x such that x is a girl and x loves Bryn, and there's only one such x, and the girl other than x is Angharad.

(i) There exists an x such that x is a girl and x loves Bryn, and there's only one such x:

$\exists x((Gx \wedge Lxb) \wedge \forall y((Gy \wedge Lyb) \supset y = x))$

(ii) The girl other than x is Angharad:

There exists a z such that z is a girl and z is not x, and there's only one such z, and z is Angharad.

$\exists z((Gz \wedge \neg z = x) \wedge \forall w(((Gw \wedge \neg w = x) \supset w = z) \wedge z = a))$

Combining (i) and (ii):

$\exists x(((Gx \wedge Lxb) \wedge \forall y((Gy \wedge Lyb) \supset y = x)) \wedge \exists z(((Gz \wedge \neg z = x) \wedge \forall w((Gw \wedge \neg w = x) \supset w = z)) \wedge z = a))$

(d) The shortest Welsh speaker loves the tallest Welsh speaker.

There exists an x such that x speaks Welsh and x is a shortest Welsh speaker, and there's only one such x, and x loves the tallest Welsh speaker.

(i) x is a shortest Welsh speaker:

For all w, if w speaks Welsh and w is not x, then w is taller than x.

$\forall w((Fw \wedge \neg w = x) \supset Mwx)$

(ii) there's only one such x:

$\forall y((Fy \wedge \forall w((Fw \wedge \neg w = y) \supset Mwy)) \supset y = x)$

(iii) x loves the tallest Welsh speaker:

There's a v such that v speaks Welsh, and v is a tallest Welsh speaker, and there's only one such v, and x loves v.

$\exists v(((Fv \wedge \forall w((Fw \wedge \neg w = v) \supset Mvw)) \wedge \forall z((Fz \wedge \forall w((Fw \wedge \neg w = z) \supset Mzw)) \supset z = v)) \wedge Lxv)$

Combining (i), (ii) and (iii):

$\exists x(((Fx \wedge \forall w((Fw \wedge \neg w = x) \supset Mwx)) \wedge \forall y((Fy \wedge \forall w((Fw \wedge \neg w = y) \supset Mwy)) \supset y = x))$
$\wedge \exists v(((Fv \wedge \forall w((Fw \wedge \neg w = v) \supset Mvw)) \wedge \forall z((Fz \wedge \forall w((Fw \wedge \neg w = z) \supset Mzw)) \supset z = v)) \wedge Lxv))$

2a)

1.	$m = n$	
2.	Fn	
3.	$\forall x(Fx \supset Gx)$	
4.	$\neg Gm$	
5.	$(Fn \supset Gn)$ ✓	($\forall'$) on 3, n/x
6.	$\neg Fn$ Gn	(g) on 5
7.	$*$ Gm	(L) on 1, 6
	$*$	

2b)

1.	∀x∃yRyx		
2.	¬∃xRxx ✓		
3.	¬∀x∃y(¬x = y ∧ Ryx) ✓		
4.	∀x¬Rxx		(¬∃) on 2
5.	∃x¬∃y(¬x = y ∧ Ryx) ✓		(¬∀) on 3
6.	¬∃y(¬a = y ∧ Rya) ✓		(∃) on 5, a/x
7.	∀y¬(¬a = y ∧ Rya)		(¬∃) on 6
8.	∃yRya ✓		(∀′) on 1, a/x
9.	¬Raa		(∀′) on 4, a/x
10.	Rba		(∃) on 8, b/y
11.	¬(¬a = b ∧ Rba) ✓		(∀′) on 7, b/y
12.	¬¬a = b ✓	¬Rba	(f) on 11
13.	a = b	*	(a) on 12
14.	Raa		(L) on 10, 13
	*		

3)

1.	∃x∀y((Fy ≡ y = x) ∧ Gx) ✓			
2.	¬∃x((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx) ✓			
3.	∀x¬((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx)			(¬∃) on 2
4.	∀y((Fy ≡ y = a) ∧ Ga)			(∃) on 1, a/x
5.	¬((Fa ∧ ∀y(Fy ⊃ y = a)) ∧ Ga) ✓			(∀′) on 3, a/x
6.	((Fa ≡ a = a) ∧ Ga) ✓			(∀′) on 4, a/y
7.	(Fa ≡ a = a) ✓			
	Ga			(b) on 6
8.	Fa		¬Fa	
	a = a		¬a = a	(h) on 7
			*	
9.	¬(Fa ∧ ∀y(Fy ⊃ y = a)) ✓	¬Ga		(f) on 5
		*		
10.	¬Fa ¬∀y(Fy ⊃ y = a) ✓			(f) on 9
11.	∃y¬(Fy ⊃ y = a) ✓			(¬∀) on 10
12.	¬(Fb ⊃ b = a) ✓			(∃) on 1, b/y
	Fb			
13.	¬b = a			(d) on 12
14.	((Fb ≡ b = a) ∧ Ga) ✓			(∀′) on 4, b/y
	(Fb ≡ b = a) ✓			
15.	Ga			(b) on 14
16.	Fb	¬Fb		
	b = a	¬b = a		(h) on 14
	*	*		