

**Assignment #10-key**

- 1a)
1.  $\forall x(Cx \supset (Fx/Nx))$
  2.  $\forall x(Fx \supset Bx)$
  3.  $\neg \forall x(Cx \supset Bx) \checkmark$
  4.  $\neg \neg \forall x(Cx \supset \neg Nx) \checkmark$
  5.  $\forall x(Cx \supset \neg Nx)$  (a) on 4
  6.  $\exists x \neg(Cx \supset Bx) \checkmark$  ( $\neg \forall$ ) on 3
  7.  $\neg(Ca \supset Ba) \checkmark$  ( $\exists$ ) on 6
  8.  $Ca$  (d) on 7
  9.  $\neg Ba$
  10.  $(Ca \supset (Fa \vee Na)) \checkmark$  ( $\forall'$ ) on 1
  11.  $\neg Ca \quad (Fa \vee Na) \checkmark$  (g) on 10
  - \*
  - $Fa \quad Na$  (e) on 11
  12.  $(Fa \supset Ba) \checkmark \quad (Ca \supset \neg Na) \checkmark$  ( $\forall'$ ) on 1, 5
  - $\neg Fa \quad Ba \quad \neg Ca \quad \neg Na$  (g) on 13
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- 1b)
1.  $\forall x(Fx \supset (Bx \equiv \neg Tx))$
  2.  $\neg \forall x(Fx \supset (Bx/Cx)) \checkmark$
  3.  $\neg \exists x(Tx \wedge \neg(Dx \supset Cx)) \checkmark$
  4.  $\neg \exists x(Fx \wedge \neg(Cx \vee Dx)) \checkmark$
  5.  $\exists x \neg(Fx \supset (Bx \vee Cx)) \checkmark$  ( $\neg \forall$ ) on 2.
  6.  $\forall x \neg(Tx \wedge \neg(Dx \supset Cx))$  ( $\neg \exists$ ) on 3.
  7.  $\forall x \neg(Fx \wedge \neg(Cx \vee Dx))$  ( $\neg \exists$ ) on 4.
  8.  $\neg(Fa \supset (Ba \vee Ca)) \checkmark$  ( $\forall'$ ) on 7, a/x.
  9.  $Fa$  (d) on 8.
  10.  $\neg(Ba \vee Ca) \checkmark$
  11.  $\neg Ba$  (c) on 10.
  12.  $\neg Ca$
  13.  $(Fa \supset (Ba \equiv \neg Ta)) \checkmark$  ( $\forall'$ ) on 1, a/x.
  14.  $\neg Fa \quad (Ba \equiv \neg Ta) \checkmark$  (g) on 13.
  - \*
  - $Ba \quad \neg Ba$  (h) on 14.
  15.  $\neg Ta \quad \neg \neg Ta \checkmark$
  - \*  $Ta$  (a) on 16.
  16.  $\neg(Ta \wedge \neg(Da \supset Ca)) \checkmark$  ( $\forall'$ ) on 6, a/x.
  - $\neg Ta \quad \neg \neg(Da \supset Ca) \checkmark$  (f) on 18.
  17.  $\neg \neg(Da \supset Ca) \checkmark$  (a) on 19.
  - \*  $(Da \supset Ca) \checkmark$
  - $\neg Da \quad Ca$  (g) on 20.
  18.  $\neg(Fa \wedge \neg(Ca \vee Da)) \checkmark$  ( $\forall'$ ) on 7, a/x.
  - \*  $\neg \neg(Ca \vee Da) \checkmark$  (f) on 22.
  19.  $\neg \neg(Ca \vee Da) \checkmark$  (a) on 23.
  - \*  $(Ca \vee Da) \checkmark$
  - $Ca \quad Da$  (e) on 24.
  - \* \*

- 1c)
1.  $\forall x((Fx \vee Gx) \supset Hx)$
  2.  $(\forall xHx \supset \forall xSx) \checkmark$
  3.  $\neg(\forall xFx \supset \forall xSx) \checkmark$
  4.  $\forall xFx$  (d) on 3.
  5.  $\neg\forall xSx \checkmark$
  6.  $\exists x\neg Sx \checkmark$  ( $\neg\forall$ ) on 5.
  7.  $\neg Sa$  ( $\exists$ ) on 6, a/x.
  8.  $\neg\forall xHx \checkmark$   $\forall xSx$  (g) on 2.
  9.  $\exists x\neg Hx \checkmark$   $Sa$  ( $\neg\forall$ ) on 8, ( $\forall'$ ) on 8, a/x.
  10.  $\neg Hb$  \* ( $\exists$ ) on 9, b/x.
  11.  $((Fb \vee Ga) \supset Hb) \checkmark$  ( $\forall'$ ) on 1, b/x.
  12.  $\neg(Fb \vee Gb) \checkmark$   $Hb$  (g) on 11.
  13.  $\neg Fb$  \* (c) on 12.
  14.  $\neg Gb$
  15.  $Fb$  ( $\forall'$ ) on 4, b/x.
- \*

- 2a)
1.  $\neg((\exists xFx \vee \exists xGx) \equiv \exists x(Fx \vee Gx)) \checkmark$
  2.  $\neg(\exists xFx \vee \exists xGx) \checkmark$   $(\exists xFx \vee \exists xGx) \checkmark$  (i) on 1.
  3.  $\exists x(Fx \vee Gx) \checkmark$   $\neg\exists x(Fx \vee Gx) \checkmark$
  4.  $\neg\exists xFx \checkmark$   $\forall x\neg(Fx \vee Gx)$  (c) on 2, ( $\neg\exists$ ) on 3.
  5.  $\neg\exists xGx \checkmark$
  6.  $\forall x\neg Fx$   $\exists xFx \checkmark$   $\exists xGx \checkmark$  ( $\neg\exists$ ) on 4, (c) on 3.
  7.  $\forall x\neg Gx$   $Fa$   $Ga$  ( $\neg\exists$ ) on 5, ( $\exists$ ) on 6, a/x.
  8.  $(Fa \vee Ga) \checkmark$   $\neg(Fa \vee Ga) \checkmark$   $\neg(Fa \vee Ga) \checkmark$  ( $\exists$ ) on 3, a/x, ( $\forall'$ ) on 4, a/x.
  9.  $Fa$   $Ga$   $\neg Fa$   $\neg Ga$  (c) on 8.
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- 2b)
1.  $\neg(\forall x(Fx \supset \neg(Gx/Hx)) \supset \neg\exists x(Fx \wedge Hx)) \checkmark$
  2.  $\forall x(Fx \supset \neg(Gx/Hx))$  (d) on 1.
  3.  $\neg\neg\exists x(Fx \wedge Hx) \checkmark$
  4.  $\exists x(Fx \wedge Hx)$  (a) on 3.
  5.  $(Fa \wedge Ha) \checkmark$  ( $\exists$ ) on 4, a/x.
  6.  $Fa$  (e) on 5.
  7.  $Ha$
  8.  $(Fa \supset \neg(Ga/Ha)) \checkmark$  ( $\forall'$ ) on 2, a/x.
  9.  $\neg Fa$   $\neg(Ga \vee Ha) \checkmark$  (g) on 8.
  10. \*  $\neg Ga$  (c) on 9.
  11.  $\neg Ha$
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