Assignment #9-key.

1.(a) $\exists x Lmx \Rightarrow_q$ T. There is at least one *x*-variant of *q* that makes Lmx true; namely, the *x*-variant that assigns x to *Juliet*.

(b) $(\exists x Lmx \supset Lmn) \Rightarrow_q$ T. Since the consequent Lmn is true under *q*, the conditional is true under *q*.

(c) $\forall x(Gx \supset \exists y Lxy) \Rightarrow_q$ T. All *x*-variants of *q* make the consequent $\exists y Lxy$ true (all 4 ways of assigning x to a member of the domain make $\exists y Lxy$ true because all four members of the domain stand in the L relation to some other member), Thus all *x*-variants of *q* make the conditional $(Gx \supset \exists y Lxy)$ true. Thus *q* makes the universal $\forall x(Gx \supset \exists y Lxy)$ true.

(d) $\exists x(Fx \wedge \forall y(Gy \supset Lxy)) \Rightarrow_q$ F. There are no *x*-variants of *q* that make $(Fx \wedge \forall y(Gy \supset Lxy))$ true. There are 4 *x*-variants, but only two are relevent here: the one that assigns x to Romeo and the one that assigns x to Benedick (because only Romeo and Benedick have the F property). But it's not true that, in addition to having the F property, Romeo also loves everyone with the G property (i.e., he does love Juliet, but he doesn't love Beatrice). And it's not true that, in addition to having the F property, Benedick also loves everyone with the G property (i.e., he does Beatrice, but he doesn't love Juliet).

2(a)

$\forall x(Fx \supset Gx) \Rightarrow_q$ T	
$\neg\forall x(Gx \supset Fx) \Rightarrow_q$ T	
$\exists x\neg(Gx \supset Fx) \Rightarrow_q$ T	(V3)
$\neg(Ga \supset Fa) \Rightarrow_{q+}$ T	(V2)
$\forall x(Fx \supset Gx) \Rightarrow_{q+}$ T	(V5)
$(Fa \supset Ga) \Rightarrow_{q+}$ T	(V1)
$Ga \Rightarrow_{q+}$ T	
$\neg Fa \Rightarrow_{q+}$ T	(Q1), (Q4), (Q3)
$\neg Fa \Rightarrow_{q+}$ T | $Ga \Rightarrow_{q+}$ T	(Q4)

Contradiction doesn't occur, so the argument is not *q*-valid.

Countermodel *q*: Let $D = \{0\}$, $a \Rightarrow 0$, $F \Rightarrow \{\}$, $G \Rightarrow \{0\}$.

<u>*Then*</u>: $Ga \Rightarrow_q$ T, because the object named by a under *q* has the property named by G.

<u>*And*</u>: $\neg Fa \Rightarrow_q$ T, because the object named by a under *q* does not have the property named by F.

<u>*And*</u>: $\forall x(Fx \supset Gx) \Rightarrow_q$ T, since there's only one *x*-variant of *q*, and it makes Fx false, so it makes $(Fx \supset Gx)$ true.

<u>*And*</u>: $\forall x(Gx \supset Fx) \Rightarrow_q$ F, since there's only one *x*-variant of *q*, and it makes Gx true and Fx false, so it makes $(Gx \supset Fx)$ false.

2(b)

$\forall x(Fx \supset Gx) \Rightarrow_q$ T	
$\neg\forall x(\neg Gx \supset \neg Fx) \Rightarrow_q$ T	
$\exists x\neg(\neg Gx \supset \neg Fx) \Rightarrow_q$ T	(V3)
$\neg(\neg Ga \supset \neg Fa) \Rightarrow_{q+}$ T	(V2)
$\forall x(Fx \supset Gx) \Rightarrow_{q+}$ T	(V5)
$(Fa \supset Ga) \Rightarrow_{q+}$ T	(V1)
$\neg Ga \Rightarrow_{q+}$ T	
$\neg\neg Fa \Rightarrow_{q+}$ T	(Q1), (Q4), (Q3)
$\neg Fa \Rightarrow_{q+}$ T * | $Ga \Rightarrow_{q+}$ T *	(Q4)

Contradiction occurs, so the argument is *q*-valid.

3. $\neg\forall x((Fx \wedge Gx) \supset (Fx \vee Gx)) \Rightarrow_q T$
$\exists x\neg((Fx \wedge Gx) \supset (Fx \vee Gx)) \Rightarrow_q T$ (V3)
$\neg((Fa \wedge Ga) \supset (Fa \vee Ga)) \Rightarrow_{q+} T$ (V2)
$(Fa \wedge Ga) \Rightarrow_{q+} T$
$\neg(Fa \vee Ga) \Rightarrow_{q+} T$ (Q1), (Q4), (Q3)
$Fa \Rightarrow_{q+} T$
$Ga \Rightarrow_{q+} T$ (Q2)
$\neg Fa \Rightarrow_{q+} T$
$\neg Ga \Rightarrow_{q+} T$ (Q1), (Q3), (Q2)

Contradiction occurs, so there's no *q*-valuation that makes the negation of the *wff* true. Thus all *q*-valuations must make the *wff* true. Thus it's a *q*-logical truth.