

**Assignment #8. Due Thurs March 31.**

1. Indicate whether the **QL** wffs below are true or false under the following translation key.

Domain of discourse =  $\{1, 2, 3, \dots\}$  (the set of positive integers)

F means \_\_\_ is even

L means \_\_\_ is less than \_\_\_

m means 1

- (a)  $\neg Lmm$
- (b)  $\exists x(Fm \supset Fx)$
- (c)  $\forall y\exists xLxy$
- (d)  $\exists y\forall xLxy$

2. Construct proofs for the following claims:

(a) If a  $q$ -valuation makes  $\neg\exists vC(\dots v\dots v\dots)$  true, then it also makes  $\forall v\neg C(\dots v\dots v\dots)$  true.

(b) Suppose  $q$  is a  $q$ -valuation and  $A$  and  $B$  are **QL** wffs. Then  $\neg(A \wedge B) \Rightarrow_q T$  if and only if  $(\neg A \vee \neg B) \Rightarrow_q T$ .

(c) Suppose  $A$  is a **QL** wff that does not contain the variable  $v$ . If a  $q$ -valuation makes  $(A \supset \forall vC(\dots v\dots v\dots))$  false, then it also makes  $\forall v(A \supset C(\dots v\dots v\dots))$  false.

**(Hints:** Assume there's a  $q$ -valuation  $q$  such that  $(A \supset \forall vC(\dots v\dots v\dots)) \Rightarrow_q F$ . What does this mean? Look at Semantic Rule (Q4). You'll then need to know what  $\forall vC(\dots v\dots v\dots) \Rightarrow_q F$  means. See Rule (Q5). Note, also, that if  $A \Rightarrow_q T$ , then  $A \Rightarrow_{q^+} T$ , for any  $v$ -variant  $q^+$  of  $q$ . You should be able to explain why. Finally, remind yourself what you need to eventually demonstrate; namely,  $\forall v(A \supset C(\dots v\dots v\dots)) \Rightarrow_q F$ . What does this mean? See Rule Q5 again.)

**Extra Credit #2. Due Thurs March 31.**

1. Indicate whether the **QL** wffs below are true or false under the following translation key.

Domain of discourse =  $\{1, 2, 3, \dots\}$  (the set of positive integers)

F means \_\_\_ is even    L means \_\_\_ is less than \_\_\_    m means 1

- (a)  $\forall x(Fx \supset \exists yLyx)$
- (b)  $\forall x(\exists yLyx \supset Fx)$
- (c)  $\exists x(\exists yLyx \supset Fx)$
- (d)  $\exists x\forall y(Lmy \supset Lxy)$
- (e)  $\forall x\forall y(Lxy \supset \exists z(Lxz \wedge Lzy))$