**Assignment #8-key.**

1. (a) "1 is not less than 1." True. (Under the $q$-valuation defined in #1, the object named by "m" does not stand in the relation named by "L" to itself.)

   (b) "There's a positive integer such that, if 1 is even, then it is even." True, since it's false that 1 is even. (All $x$-variants of our $q$-valuation make (Fm ⊃ Fx) true, since Fm is false.)

   (c) "For every positive integer, there's another that is less than it." False, since for 1, there is no positive integer less than it. (There is a $y$-variant of our $q$-valuation that does not make ∃xLxy true; namely, the $y$-variant that assigns y to the 1-object.)

   (d) "There's a positive integer such that every positive integer is less than it." False, since for any positive integer, there's always another one that is greater than it. (There is no $y$-variant of our $q$-valuation that makes ∀xLxy true. In other words, there is no way to assign a positive integer to y that makes ∀xLxy true.)

2a) If a $q$-valuation makes $\neg\exists v C(...v...v...)$ true, then it also makes $\forall v \neg C(...v...v...)$ true.

***Proof***: Suppose there's a $q$-valuation $q$ such that $\neg\exists v C(...v...v...) \Rightarrow_q$ T.

_Then_: $\exists v C(...v...v...) \Rightarrow_q$ F. (Q1)

_So_: There's no $v$-variant $q^+$ such that $C(...v...v...) \Rightarrow_{q+}$ T. (Q6)

_So_: For all $v$-variants $q^+$, $C(...v...v...) \Rightarrow_{q+}$ F.

_Thus_: For all $v$-variants $q^+$, $\neg C(...v...v...) \Rightarrow_{q+}$ T. (Q1)

_So_: $\forall v \neg C(...v...v...) \Rightarrow_q$ T. (Q5)

2b) Suppose $q$ is a $q$-valuation and $A$ and $B$ are **QL** wffs. Then $\neg(A \wedge B) \Rightarrow_q$ T if and only if $(\neg A \vee \neg B) \Rightarrow_q$ T.

***Proof***: The claim is an "if and only if" claim. So we need to prove two conditional claims:

_("⇒") Suppose_: There's a $q$-valuation $q$ such that $\neg(A \wedge B) \Rightarrow_q$ T.

_Then_: $(A \wedge B) \Rightarrow_q$ F. (Q1)

_Then_: Either (i) $A \Rightarrow_q$ F, or (ii) $B \Rightarrow_q$ F, or (iii) both $A \Rightarrow_q$ F and $B \Rightarrow_q$ F. (Q2)

_Now_: If (i) $A \Rightarrow_q$ F, then $\neg A \Rightarrow_q$ T (Q1). Hence $(\neg A \vee \neg B) \Rightarrow_q$ T (Q3).

_And_: If (ii) $B \Rightarrow_q$ F, then $\neg B \Rightarrow_q$ T (Q1). Hence $(\neg A \vee \neg B) \Rightarrow_q$ T (Q3).

_And_: If (iii) both $A \Rightarrow_q$ F and $B \Rightarrow_q$ F, then both $\neg A \Rightarrow_q$ T and $\neg B \Rightarrow_q$ T (Q1). So $(\neg A \vee \neg B) \Rightarrow_q$ T (Q3).

_So_: $(\neg A \vee \neg B) \Rightarrow_q$ T.

_("⇐") Suppose_: There's a $q$-valuation $q$ such that $(\neg A \vee \neg B) \Rightarrow_q$ T.

Then: Either (i) $\neg A \Rightarrow_q$ T, or (ii) $\neg B \Rightarrow_q$ T, or (iii) both $\neg A \Rightarrow_q$ T and $\neg B \Rightarrow_q$ T (Q3).

_Now_: If (i) $\neg A \Rightarrow_q$ T, then $A \Rightarrow_q$ F (Q1). Hence $(A \wedge B) \Rightarrow_q$ F (Q2). So $\neg(A \wedge B) \Rightarrow_q$ T (Q1).

_And_: If (ii) $\neg B \Rightarrow_q$ T, then $B \Rightarrow_q$ F (Q1). Hence $(A \wedge B) \Rightarrow_q$ F (Q2). So $\neg(A \wedge B) \Rightarrow_q$ T (Q1).

_And_: If (iii) both $\neg A \Rightarrow_q$ T and $\neg B \Rightarrow_q$ T, then both $A \Rightarrow_q$ F and $B \Rightarrow_q$ F (Q1). So $(A \wedge B) \Rightarrow_q$ F (Q2). So $\neg(A \wedge B) \Rightarrow_q$ T (Q1).

_So_: $\neg(A \wedge B) \Rightarrow_q$ T.

2c) Suppose $A$ is a **QL** *wff* that does not contain the variable $v$. If a $q$-valuation makes $(A \supset \forall v C(...v...v...))$ false, then it also makes $\forall v(A \supset C(...v...v...))$ false.

***Proof***: Suppose there's a $q$-valuation $q$ such that $(A \supset \forall v C(...v...v...)) \Rightarrow_q$ F.

_Then_: $A \Rightarrow_q$ T and $\forall v C(...v...v...) \Rightarrow_q$ F. (Q4)

_So_: There's a $v$-variant $q^+$ such that $C(...v...v...) \Rightarrow_{q+}$ F. (Q5)

_Now_: $A \Rightarrow_{q+}$ T, since $A$ doesn't contain $v$. (V5)

_So_: $(A \supset C(...v...v...)) \Rightarrow_{q+}$ F. (Q4)

_So_: Not all $v$-variants of $q$ make $(A \supset C(...v...v...))$ true.

_Thus_: $\forall v(A \supset C(...v...v...)) \Rightarrow_q$ F. (Q5)

**Extra Credit #2-key.**

1. (a) "For every positive integer, if it's even, then there's another that's less than it." True, since the smallest even positive integer is 2, and there's another less than it (namely, 1); and every other even positive integer is greater than 2. (All *x*-variants of the *q*-valuation in #1 make $(Fx \supset \exists yLyx)$ true.)

   (b) "For every positive integer, if there's another that's less than it, then it's even." False. 3 is a positive integer that is not even and for which there's another that's less than it (namely, 2 and 1). (There's an *x*-variant of our *q*-valuation that does not make $(\exists yLyx \supset Fx)$ true; namely, the *x*-variant that assigns $x$ to 3.)

   (c) "There's a positive integer such that, if there's another that's less than it, then it's even." True. 2 is an example. (There's at least one *x*-variant of our *q*-valuation that makes $(\exists yLyx \supset Fx)$ true; namely, the *x*-variant that assigns $x$ to 2.)

   (d) "There's a positive integer *x* such that, for all other positive integers *y*, if 1 is less than *y*, then *x* is less than *y*." True. 1 is the example. 1 is less than all positive integers which stand in the relation of 1 being less than them! (There's at least one *x*-variant of our *q*-valuation that makes $\forall y(Lmy \supset Lxy)$ true; namely, the *x*-variant that assigns $x$ to 1.)

   (e) "All positive integers are such that, for any other positive integer, if the first is less than the second, then the first is less than some other positive integer which is less than the second." ("Every positive integer is such that, for all positive integers for which it's less than, there's another that is in-between them.") False. Take 5 and 6 as examples. 5 is less than 6, but there is no positive integer less than 6 and more than 5. (Not all *x*-variants of our *q*-valuation make $\forall y(Lxy \supset \exists z(Lxz \wedge Lzy))$ true; namely, the *x*-variant that assigns $x$ to 5 doesn't make $\forall y(Lxy \supset \exists z(Lxz \wedge Lzy))$ true.)