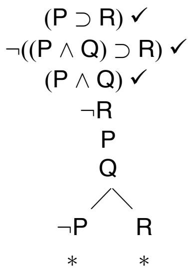
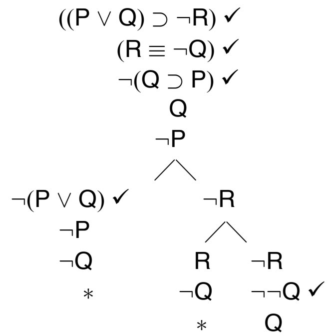
Assignment #5-key

1b)



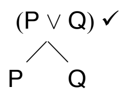
The completed tree closes.
So the argument is *tautologically valid*.

1a)



The completed tree has an open branch.
So the argument is *tautologically invalid*.

2a) If there is a complete tree for the **PLC** *wff A* all of whose branches are open, then *A* is a tautology.
Comment:
This is *not* a true claim about **PLC** trees. A branch on a complete tree represents an attempt to make the *wff*(s) on its trunk true. A closed branch represents a failed attempt, while an open branch represents a successful attempt. In other words, an open branch on a complete tree represents a *valuation* that makes the *wff*(s) on the trunk true. But the number of branches on a completed tree are not necessarily equal to the total number of possible valuations of the *wff*(s) on the trunk. There can be more valuations of the *wff*(s) on the trunk that there are branches in the completed tree. So just because all branches on a complete tree with a single *wff* on its trunk are open doesn't necessarily mean all valuations of the *wff* make it true. There could be one or more valuations that make it false. Here is an example:



All branches of this completed tree are open, but of course the *wff* (P ∨ Q) is not a tautology!

2b) If there is a complete tree for the **PLC** *wff A* all of whose branches close, then *A* is a contradiction.
Proof:
The tree rules *guarantee* that if a valuation that makes the *wff*(s) on the trunk true exists, then it will be found in the form of an open branch (the tree rules require you to unpack a *wff* into its "truth-makers"). So if there are no open branches on a complete tree with a single *wff* on its trunk, then there are no valuations that make that *wff* true. So the *wff* must be a contradiction.

2c) If there is a complete truth tree for the **PLC** *wff* $(A \supset B)$ all of whose branches are closed, then *A* is a tautology and *B* is a contradiction.
Proof:
Suppose: There's a complete truth tree for $(A \supset B)$ all of whose branches close.
Then: $(A \supset B)$ is a contradiction (see #2b).
So: All valuations make $(A \supset B)$ false.
So: All valuations make *A* true and *B* false.
So: *A* is a tautology and *B* is a contradiction.

3. For any *wffs* $A_1, ..., A_n, C$, if $\models ((A_1 \wedge ... \wedge A_n) \supset C)$, then $A_1, ..., A_n \models C$.
Proof:
Suppose: $\models ((A_1 \wedge ... \wedge A_n) \supset C)$
Then; No valuation makes $((A_1 \wedge ... \wedge A_n) \supset C)$ false.
So; No valuation makes $(A_1 \wedge ... \wedge A_n$ true and *C* false.
So: No valuation makes $A_1, ..., A_n$ true and *C* false.
So: If any valuation makes $A_1, ..., A_n$ true, it also makes *C* true.
Thus: $A_1, ..., A_n \models C$.