

**Assignment #3. Due Thurs Feb 18.**

1. Explain why each of the following are true claims about **PLC**.

(a) For any *wffs*  $A, B$ , if  $A \vDash B$  and  $B \vDash C$ , then  $A \vDash C$ .

**(Hint:** This says "If  $A$  tautologically entails  $B$ , and  $B$  tautologically entails  $C$ , then  $A$  tautologically entails  $C$ ." To prove it, assume that  $A$  tautologically entails  $B$ , and  $B$  tautologically entails  $C$ . Then try to show that from this it follows that  $A$  tautologically entails  $C$ . To do this, you need to apply the definition of tautological entailment.)

(b) For any *wffs*  $A, B$ , if  $A \vDash \neg B$ , then it's not the case that  $A \vDash B$ .

(c) For any *wffs*  $A, B$ , if  $\vDash(A \vee B)$  and  $\vDash \neg(A \wedge B)$ , then  $A \vDash \neg B$  and  $\neg B \vDash A$ .

**(Hint:** This says "If  $(A \vee B)$  is a tautology and  $(A \wedge B)$  is a contradiction, then  $A$  tautologically entails  $\neg B$  and  $\neg B$  tautologically entails  $A$  (*i.e.*,  $A$  and  $\neg B$  are truth-functionally equivalent).

3. Recall that the corresponding conditional of an argument in **PLC** is a conditional *wff* whose consequent is the conclusion and whose antecedent is the conjunction of all the premises. Determine whether the following arguments in **PLC** are tautologically valid by evaluating their corresponding conditionals.

(a)  $(\neg P \wedge Q) \therefore \neg(P \wedge Q)$

(b)  $P, \neg P \therefore Q$