Assignment #3-key.

1a) For any *wffs* A, B, if $A \vDash B$ and $B \vDash C$, then $A \vDash C$.

Proof: We're given $A \vDash B$ and $B \vDash C$, and we're supposed to show that $A \vDash C$.

Suppose: $A \vDash B$ and $B \vDash C$.

Then: There is no valuation that makes A true and B false, and there is no valuation that makes B true and C false.

So: All valuations that make A true also make B true, and all valuations that make B true also make C true.

Thus: All valuations that make A true also make C true.

Thus: $A \vDash C$.

1b) For any *wffs* A, B, if $A \vDash \neg B$, then it's not the case that $A \vDash B$.

Proof:

Suppose: $A \vDash \neg B$.

Then: There is no valuation that makes A true and $\neg B$ false.

So: There is no valuation that makes A true and B true.

Thus: All valuations that make A true also make B false.

So: It's not the case that $A \vDash B$.

1c) For any *wffs* A, B, if $\vDash(A \vee B)$ and $\vDash\neg(A \wedge B)$, then $A \vDash \neg B$ and $\neg B \vDash A$.

Proof:

Suppose: $\vDash(A \vee B)$ and $\vDash\neg(A \wedge B)$.

Then: All valuations make $(A \vee B)$ true, and all valuations make $\neg(A \wedge B)$ true.

Thus: All valuations make $(A \vee B)$ true, and all valuations make $(A \wedge B)$ false.

So: All valuations make either A true or B true, but not both simultaneously.

So: If a valuation makes A true, it must make B false, and if a valuation makes A false, it must make B true.

Thus: If a valuation makes A true, it must make $\neg B$ true, and if a valuation makes A false, it must make $\neg B$ false. (A and $\neg B$ are truth-functionally equivalent.)

Thus: There is no valuation that makes A true and $\neg B$ false, and there is no valuation that makes $\neg B$ true, and A false.

So: $A \vDash \neg B$ and $\neg B \vDash A$.

3a) The corresponding conditional is $((\neg P \wedge Q) \supset \neg(P \wedge Q))$.

P	Q	((¬	P	∧	Q)	⊃	¬ (	P	∧	Q))
T	T	F	T	F	T	**T**	F	T	T	T
T	F	F	T	F	F	**T**	T	T	F	F
F	T	T	F	T	T	**T**	T	F	F	T
F	F	T	F	F	F	**T**	T	F	F	F

The corresponding conditional is a tautology. Thus the argument is tautologically valid.

3b) The corresponding conditional is $((P \wedge \neg P) \supset Q)$.

P	Q	((P	∧	¬	P)	⊃	Q)
T	T	T	F	F	T	**T**	T
T	F	T	F	F	T	**T**	F
F	T	F	F	T	F	**T**	T
F	F	F	F	T	F	**T**	F

The corresponding conditional is a tautology. Thus the argument is tautologically valid.