

Assignment #2-key.

1a)

P	$\neg(\neg P \wedge \neg P)$	P
T	T	T
F	F	F

The *wffs* are truth-functionally equivalent.

1b)

P	Q	R	$((P \vee Q) \wedge R)$	$(P \vee (Q \wedge R))$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

The *wffs* are not truth-functionally equivalent. The valuations in rows 2, 4 assign them different truth values.

1c)

P	$(P \vee \neg P)$	$\neg P$
T	T	F
F	T	T

The *wffs* are not truth-functionally equivalent. The valuations in rows 1, 2 assign them different truth values.

2. Construct a *wff* in **PL** that is truth-functionally equivalent to the truth function $\#(P, Q, R)$ given by the following truth-table:

P	Q	R	$\#(P, Q, R)$
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

First: Construct the basic conjunction that corresponds to each valuation that makes $\#(P, Q, R)$ true.

Row 2: $(P \wedge Q \wedge \neg R)$

Row 3: $(P \wedge \neg Q \wedge R)$

Row 4: $(P \wedge \neg Q \wedge \neg R)$

Second: Form the disjunction of all these basic conjunctions:

$$((P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R))$$

Note: This *wff* is true so long as any of its disjuncts (components) are true, which is the case in Rows 2, 3, 4. For any other row, none of the basic conjunctions are true, so the disjunction will be false.

3a)

P	Q	premise1 $(\neg P \wedge Q)$	conclusion $\neg(P \wedge Q)$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	F	T

There is no row (valuation) that makes the premise true and the conclusion false. So the **PL** argument is tautologically valid.

3b)

P	Q	<i>premise1</i> P	<i>premise2</i> $\neg P$	<i>conclusion</i> Q
T	T	T	F	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

There is no valuation that makes the premises true and the conclusion false. So the **PL** argument is tautologically valid.

3c)

P	Q	R	<i>premise1</i> $(Q \vee \neg P)$	<i>premise2</i> $\neg Q \wedge \neg R$	<i>premise3</i> $(\neg R \vee \neg P)$	<i>conclusion</i> $\neg P$
T	T	T	T	T	F	F
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

In all valuations in which the conclusion is false, at least one premise is false. So there are no valuations that make all premises true and the conclusion false. So the **PL** argument is tautologically valid.