

### Semantic Rules for $QL^f$

- (Q0) For atomic *wff*  $A$  ( $n$ -place predicate followed by  $n$  terms):
- (a) If  $n = 0$ , then  $A \Rightarrow_q T$  if the  $q$ -value of  $A$  is true. Otherwise  $A \Rightarrow_q F$ .
- (b) If  $n > 0$ , then  $A \Rightarrow_q T$  if the  $n$ -tuple formed by taking, in order, the references under  $q$  (objects in the domain) of the terms in  $A$  is an element of the  $q$ -value of  $A$  (the extension of the predicate in  $A$ ). Otherwise  $A \Rightarrow_q F$ .
- (Q0<sup>+</sup>) An atomic *wff* of the form  $t_1 = t_2$  is true under  $q$  just if the terms  $t_1$  and  $t_2$  are assigned the same objects by  $q$ .
- (Q1) For any *wff*  $A$ :  $\neg A \Rightarrow_q T$  if  $A \Rightarrow_q F$ . Otherwise  $\neg A \Rightarrow_q F$ .
- (Q2) For *wffs*  $A, B$ :  
 $(A \wedge B) \Rightarrow_q T$  if both  $A \Rightarrow_q T$  and  $B \Rightarrow_q T$ . Otherwise  $(A \wedge B) \Rightarrow_q F$ .
- (Q3) For *wffs*  $A, B$ :  
 $(A \vee B) \Rightarrow_q F$  if both  $A \Rightarrow_q F$  and  $B \Rightarrow_q F$ . Otherwise  $(A \vee B) \Rightarrow_q T$ .
- (Q4) For *wffs*  $A, B$ :  
 $(A \supset B) \Rightarrow_q F$  if  $A \Rightarrow_q T$  and  $B \Rightarrow_q F$ . Otherwise  $(A \supset B) \Rightarrow_q T$ .
- (Q5) For *wffs*  $A, B$ ,  $(A \equiv B) \Rightarrow_q T$  if  $A \Rightarrow_q T$  and  $B \Rightarrow_q T$ , or if  $A \Rightarrow_q F$  and  $B \Rightarrow_q F$ ; otherwise  $(A \equiv B) \Rightarrow_q F$ .
- (Q6) For *wff*  $C(\dots v \dots v \dots)$  with variable  $v$  free,  $\forall v C(\dots v \dots v \dots) \Rightarrow_q T$  if  $C(\dots v \dots v \dots) \Rightarrow_{q^+} T$  for every  $v$ -variant  $q^+$  of  $q$ . Otherwise  $\forall v C(\dots v \dots v \dots) \Rightarrow_q F$ .
- (Q7) For *wff*  $C(\dots v \dots v \dots)$  with variable  $v$  free,  $\exists v C(\dots v \dots v \dots) \Rightarrow_q T$  if  $C(\dots v \dots v \dots) \Rightarrow_{q^+} T$  for at least one  $v$ -variant  $q^+$  of  $q$ . Otherwise  $\exists v C(\dots v \dots v \dots) \Rightarrow_q F$ .

### Some Results

- (V1) If a  $q$ -valuation  $q$  makes  $\forall v C(\dots v \dots v \dots)$  true, then, if  $c$  is a constant in  $q$ 's vocabulary, then  $q$  makes  $C(\dots c \dots c \dots)$  true. If  $c$  is not a constant in  $q$ 's vocabulary, then there is an extension  $q^+$  of  $q$  with a vocabulary that contains  $c$  and that makes  $C(\dots c \dots c \dots)$  true. (If a  $q$ -valuation makes a universal quantifier *wff* true, it makes all its instances true, too.)
- (V2) If a  $q$ -valuation  $q$  makes  $\exists v C(\dots v \dots v \dots)$  true, and  $c$  is a constant that does not occur in  $q$ 's vocabulary, then there is an extension  $q^+$  of  $q$  with a vocabulary that contains  $c$  and that makes  $C(\dots c \dots c \dots)$  true. (If a  $q$ -valuation makes an existential quantifier *wff* true, then it has an extension that makes an instance of the existential *wff* true, too.)
- (V3) If a  $q$ -valuation makes  $\neg \forall v C(\dots v \dots v \dots)$  true, then it also makes  $\exists v \neg C(\dots v \dots v \dots)$  true.
- (V4) If a  $q$ -valuation makes  $\neg \exists v C(\dots v \dots v \dots)$  true, then it also makes  $\forall v \neg C(\dots v \dots v \dots)$  true.
- (V5) Suppose a  $q$ -valuation  $q$  has a vocabulary  $V$  that does not contain the constant  $c$ ; and suppose  $q^+$  is an extension of  $q$  that assigns to  $c$  some object in the domain. Let  $W$  be a *wff* using symbols in  $V$ . Then if  $W \Rightarrow_q T$ , then  $W \Rightarrow_{q^+} T$ . (Extending a  $q$ -valuation to cover a new constant  $c$  doesn't affect the truth values of *wffs* that do not contain  $c$ .)