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Quantum theory and the electromagnetic world-view

IN THE WINTER of 1906, Munich's new professor of theoretical physics, Arnold Sommerfeld (1868-1951), began a series of lectures on "Maxwell's theory and electron theory," a topic described in a letter that December to H.A. Lorentz as the "burning questions of electrons."[1] In a manner that would become characteristic of his lecturing style, Sommerfeld introduced his students almost immediately to the current problems plaguing the subject area at hand.[2] After a short historical overview of the topic, he noted:[3]

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The following abbreviations are used: AS, Arnold Sommerfeld Papers, Deutsches Museum, Munich, NL 089 (028); EM, Michael Eckert and Karl Märker, eds., *Arnold Sommerfeld: Wissenschaftlicher Briefwechsel*, *1* (Berlin, 2000).

1. Sommerfeld to Lorentz, 12 Dec 1906 (EM, 257-258).

2. Sommerfeld's students often remarked on how up-to-date his lectures were. "It is characteristic of his way of teaching that many of the problems he discussed in his lectures for advanced students, and in his seminar, were those which he was just going to solve himself," Otto Scherzer has been quoted as saying. Gregor Wentzel also stressed the fact that Sommerfeld would emphasize those aspects of the theory that were weak or obscure, "rather than explain them away," while Linus Pauling recalled that "Sommerfeld would point out the places where the theory was still uncertain, in order that the student would know that his failure to understand was due to deficiency in the state of the science and not in his reasoning ability." All three quoted in Paul Kirkpatrick, "Address of recommendation to Arnold Sommerfeld upon the award of the 1948 Oersted Medal for Notable Contributions to the teaching of physics," *American journal of physics, 12* (1949), 312-314.

3. Arnold Sommerfeld, Lectures on *Maxwell'sche Th[eorie] und Elektronenth[eorie],*

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> Of course all this is only valid for the negative electron and the apparent mass bound to it. About the positive electron and the matter apparently inseparably bound to it, we know nothing. Also there are still serious difficulties to overcome regarding electro-optical phenomena, which should, according to the electron theory, show the influence of the earth's movement. Lorentz recently said, in reply to my question as to how the electrons were doing: badly. Kaufmann's most recent experiments have not removed the difficulties. Therefore Planck also was pessimistic.

The reference was to the experiments of Walter Kaufmann, who had attempted to distinguish between the two most prominent electron theories at the time: that of Max Abraham, a former student of Max Planck's, who assumed a rigid, spherical electron, and the so-called Lorentz-Einstein theory, which assumed a deformable one.[4] In his report on Kaufmann's results to the 78th meeting of German scientists and physicians in September 1906, Planck called the two possibilities respectively the "sphere" and the "relative" theories and concluded that the choice between them was still open:[5]

> Therefore no option remains but to assume that some essential gap remains in the theoretical interpretation of the measurments, which must be filled before they can be used for a definitive decision between the sphere theory and the relative theory. There are various other possibilities, but I do not want to discuss these further, because to me the physical foundations [*Grundlagen*] of the theories appear too uncertain.

Sommerfeld commented on this "pessimism" of Planck's in the discussion that followed. A strong supporter of Abraham's theory, Sommerfeld disliked in equal

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Wi[nter] Se[mester] 1906/07. Wi[nter] Se[mester] 1908/09 (AS). "Freilich gibt das alles nur von der negativen Elektr. u. der mit ihr verbundenen scheinbaren Masse. Über die positive Elektr. u. die mit ihr scheinbar untrennbar verbundene Materie wissen wir nichts. Auch sind noch ernstl. Schwierigkeiten zu überwinden bei den elektroopt. Erscheinungen, die den Einfluss der Erdbewegung nach der Elektronenth. zeigen sollten. Lorentz sagte kurzl. *auf meine Frage, wie es den Elektronen geht: schlecht*. Die Schwierigkeiten auf elektroopt. Gebiet lassen sich angesichts der neuesten Messungen von Kaufmann nicht überwinden. So war auch Planck pessimist." The italicized sentence was written in shorthand. I am indebted to the Bonner Steno-Club for translating it for me.

4. Towards the end of 1905, Sommerfeld asked Wien whether he knew that Kaufmann had completed measurements that had given the victory to the rigid electron; hence the "Lorentz formula for the deformable electron lay completely outside the [range of] possible observational errors." Sommerfeld to Wien, 5 Nov 1905 (EM, 250). But Wilhelm Röntgen, Sommerfeld's senior colleague and professor of experimental physics at Munich, did not believe that Kaufmann's measurements had been exact enough to rule out the Lorentz theory. Sommerfeld to Wien, 23 Nov 1906, and to Lorentz, 12 Dec 1906 (EM, 255-258).

5. Contributions to the meeting, including Max Planck, "Die Kaufmannschen Messungen der Ablenkbarkeit der γ-Strahlen für die Dynamik der Elektronen," *Physikalische Zeitschrift, 7* (1906), 753-759, on 758.

measure, as he wrote in a letter to Lorentz, both Lorentz's deformable electron and Einstein's "deformed" time.[6] The 38 year-old Sommerfeld's suggested explanation for the difference in opinions caused some merriment:[7]

> Sommerfeld (Munich): I would not, for the time being, like to ally myself with the pessimistic standpoint of Mr. Planck. In the extraordinary difficulties of measurement the deviations may still be traced to unknown sources of error. As to the question of principle formulated by Mr. Planck, I would suspect that men under forty will prefer the electrodynamic postulate, those over forty the mechanical-relativistic postulate. I prefer the electrodynamic. (laughter)

In an article written in 1970, Russell McCormmach explained Sommerfeld's hostility to the "mechanical-relativistic postulate" as deriving from his devotion to an "electromagnetic view of nature."[8] This *Weltanschauung* encompassed three related positions: a distaste for, and mistrust of, mechanical modelling, especially as applied to microscopic phenomena; a belief that the only physical realities are electromagnetic in origin; and a programmatic commitment to a "concentration of effort on problems whose solution promised to secure a universal physics based solely on electromagnetic laws and concepts."[9] The notion of an electromagnetic program is key. No doubt most physicists at the turn of the century made some use of the electromagnetic concept. However, the proponent of the electromagnetic world view, would prefer theories that used only electromagnetic properties (or those assumed electromagnetic in origin, like the electron's mass),[10] eschewing mechanical concepts like deformability. Not merely the problems chosen, but also the modes of solution deemed acceptable, marked Sommerfeld from his older colleagues.

In common with many physicists of the generation that completed their university studies in the last two decades of the 19th century, Sommerfeld had seen

6. Sommerfeld to Lorentz, 12 Dec 1906 (EM, 257-258).
7. Ref. 5, 753-761, on 761.
8. Russell McCormmach, "H.A. Lorentz and the electromagnetic view of nature," *Isis, 61* (1970), 459-497, on 489-490. The anti-Semitism prevalent in German academia at this time also may have contributed to Sommerfeld's initial hostility towards Einstein's theory. Sommerfeld to Lorentz, 26 Dec 1907 (EM, 318-320): "As brilliant as [Einstein's papers] are, there seems to be something unhealthy in this unconstructable, non-intuitive dogmatism. An Englishman would have found it difficult to produce such a theory. Perhaps there is something here that corresponds, as with Cohn, to the abstract-conceptual style of the Semite."
9. McCormmach (ref. 8), 459.
10. That Sommerfeld believed in an electromagnetic origin for the electron's (and other particles') mass can be seen from the text of the lectures he delivered on *Maxwell'sche Th. U. Elektronenth.* in 1906/07 and 1908/09. He wrote there (p. 2): "Cathode Rays. Microatoms. Their mass is alterable; an electromagnetic action. The mechanics of the smallest and simplest masses is really electromagnetically based."

the gradual failure of the mechanical world-view, which held that all physical phenomena could be explained in terms of the equations and concepts of mechanics. His generation had also witnessed both Hertz's discovery of electromagnetic waves and the later successes of Lorentz's electron theory—crowned with the discovery of the electron in the last years of the 19th century. In Sommerfeld's days as a student of mathematics and physics in Königsberg, Hertz had defined what it meant to be a physicist. On hearing of Hertz's death in 1894 Sommerfeld wrote to his mother:[11]

> It is Awful! The man began his brilliant experimental investigations five years ago. Half of all physicists at the moment are following in his footsteps and are working on Hertzian oscillations. There are few discoveries that can stand next to his electromagnetic light-waves. If it had to be a physicist that died, why couldn't it have been one of the useless Papes, Volkmanns, etc.

A return to mechanical explanation seemed to be a "throwback." "For the younger physicists," wrote McCormmach, "the electromagnetic concepts clearly pointed to the future of physics."[12]

Since McCormmach's article and the work of Tetu Hirosige,[13] the electromagnetic world view has been accorded a prominent position in the story of the reception of Einstein's relativity theory. Little attention, however, has been paid to the possibilities of a similar study for that other great pillar of modern physics, the quantum theory. Few, if any, of the substantial synthetic accounts of the history of the quantum theory discuss the electromagnetic world-view in detail. This asymmetry seems all the more surprising given the overlap in the cast of characters that worked in both areas. Sommerfeld, Lorentz, and Wilhelm Wien all participated at the Solvay Conference in Brussels in 1911, the first conference devoted entirely to the problem of the quantum; all were key respondents to Einstein's relativity theory; and all were enthusiastic proponents of the electromagnetic view of nature.[14]

Of course, even together the incipient quantum and relativity theories did not make up all (or even most) of physics at the turn of the century. A full treatment of the meanings and effects of the electromagnetic world-view would have, at the very least, to explore wider developments in fin-de-siècle microphysics. On offer here is a much more limited study, which aims to use the electromagnetic world-view as a means of probing what we now know as the quantum theory, and at the

11. EM, 18.
12. McCormmach (ref. 8), 489.
13. Christa Jungnickel and Russell McCormmach, *Intellectual mastery of nature: Theoretical physics from Ohm to Einstein,* Vol. 2 (Chicago, 1990). 211-253; Tetu Hirosige, "Electrodynamics before the theory of relativity, 1890-1905," *Japanese studies in the history of science, 5* (1966), 1-49; "Theory of relativity and the ether," ibid., *7* (1968), 37-53, *HSPS, 1* (1969), 151-209.
14. An early formulation of this Weltanschauung is Wilhelm Wien, "Über die Möglichkeit einer elektromagnetischen Begründung der Mechanik," *Annalen der Physik, 5* (1901), 501-513.

same time, to use the (limited) case of the quantum theory to demonstrate the practices of the electromagnetic program.

This paper focuses on the work of Sommerfeld as one of the leading theorists of the old quantum theory.[15] By 1911, the year he presented a paper on the "Quantum of action" at the Solvay Conference, Sommerfeld espoused the necessity of some form of a quantum hypothesis. In his earlier lectures, however, he hesitated to accept Planck's position. I argue in Section 1 that his opposition to Planck's derivation of the black-body law, and his support for the result achieved by James Jeans and re-derived using the electron theory by Lorentz, can be traced to his commitment to the programmatic aims of the electromagnetic world view. In Section 2 I argue that this conclusion has deep implications for our understanding of the "conversion" of several leading physicists to the quantum theory after 1908. In his text on *Black-body theory and the quantum discontinuity*, Thomas Kuhn claimed that a lecture that Lorentz gave in Rome in 1908 marked a turning point in the history of the early quantum theory and led to a growing acceptance of the idea of a quantum discontinuity. While I grant the importance of the Rome lecture, I hold that the acceptance of discontinuity followed what was, in fact a more profound realization. In 1906, Sommerfeld had assumed that electromagnetic theory was untroubled by the problems that plagued mechanics. He then expected that a mechanical description of the electromagnetic ether should produce inconsistency, as it did with the incorrect black-body curve. Lorentz's lecture of 1908 was the first statement by one of its leading proponents that the electromagnetic world view must fail for the case of radiation. What was at stake for a significant part of the theoretical physics community was, far more critically than the question of discontinuity, the question whether the electromagnetic view of nature could incorporate Planck's results, or whether the universalizing dream of the electromagneticists had to be abandoned.

The latter view prevailed. Lorentz's public renunciation of the hope that the electron theory could reproduce the close match of Planck's equation to experimental data seems to have prompted many exponents of the electromagnetic view to do the same. In Section 3, however, I trace what may be seen as a partial continuation of the program. Sommerfeld's Solvay paper may be best understood as an attempt to reconcile the programmatic aims of the electromagnetic world view with the necessity of recourse to the quantum hypothesis. For Sommerfeld, the greatest achievements of this paper were two fold. First, he was able to explain the phenomena at question without recourse to mechanical explanation, and second,

15. In my doctoral thesis, "Principles and problems: Constructions of theoretical physics in Germany, 1890-1918" (Princeton University, 2003), I argue in contrast to the "physics of principles" put forward by Planck and Einstein, with its emphasis on abstract, de-anthropomorphized, de-historicized, "pure" principles, Sommerfeld's approach (a "physics of problems") focused on specific problems. Sommerfeld's stress on electromagnetic theory provided the rationale behind the selection of the particular problems upon which he and his students would focus, problems like wireless telegraphy, x-ray production and diffraction, and electron theory as well as the quantum theory.

both the electromagnetic theory and the quantum hypothesis were required *in the same calculation,* in order to achieve the desired result. No longer a universalizing vision, the attempt to prove the necessity of electromagnetic theory at all levels of explanation remained a key element of Sommerfeld's research agenda until (and even beyond) the advent of Niels Bohr's model atom in 1913.

1. TEACHING PLANCK'S *LECTURES*

Sommerfeld was called to Munich to fill the chair for theoretical physics in 1906. The position had been empty for a dozen years, ever since Ludwig Boltzmann had left it to return to Vienna. The high standards required by the Munich faculty and the paucity of practitioners in theoretical physics led to an almost comical situation in the intervening years, as the job was repeatedly offered to Boltzmann. Failing also to win Lorentz, the search moved on to younger men. Of the three candidates then considered (the geophysicist Emil Wiechert, Emil Cohn, and Sommerfeld), only Cohn held a position in physics. When Wiechert declined the position, the ministry offered it to Sommerfeld, who had come highly recommended by both Lorentz and Boltzmann.[16] Sommerfeld's work on x-ray diffraction and the electron theory had probably also attracted the attention of Wilhelm Röntgen, Munich's professor of experimental physics. Röntgen signalled his approval and Sommerfeld jumped at the opportunity to occupy a full professorship at the prestigious university.

The opportunity, however, brought with it a major challenge. "In Munich I had for the first time to give lectures on the different areas of theoretical physics and special lectures about current questions. From the beginning I plugged away at—and would not let any difficulty divert me from—the founding through a seminar and colloquium of a nursery for theoretical physics in Munich."[17]

These early lectures, written in Sommerfeld's 'hand and delivered during a critically formative period in the development of theoretical physics, provided a means for him to perfect his methods, update his knowledge, and educate a new generation of students and researchers. His lectures on heat radiation, delivered first in 1907 show mastery of Planck's *Vorlesungen über die Theorie der Wärmestrahlung*, published the year before.[18]

16. Jungnickel and McCormach (ref. 13), 149-160, 274-287; Michael Eckert and Willibald Pricha, "Boltzmann, Sommerfeld und die Berufungen auf die Lehrstühle für theoretische Physik in München und Wien, 1890-1914," Österreichischen Gesellschaft für Geschichte der Naturwissenschaften, *Mitteilungen, 4* (1994), 101-119.
17. Arnold Sommerfeld, "Autobiographische Skizze," in F. Sauter, ed., *Arnold Sommerfeld: Gesammelte Schriften* (4 vols., Braunschweig, 1968), *4,* 672-682, on 677. Cf. Michael Eckert, *Die Atomphysiker: Eine Geschichte der theoretischen Physik am Beispiel der Sommerfeldschule* (Brannschweig, 1993), 38-41.
18. *Theorie der Strahlung* (Sommer, 1907) (AS). Passages in the *Lectures* are difficult to date since Sommerfeld revised them year after year. However, the lectures on Planck's radiation theory seem to have remained largely unaltered, perhaps because he gave them only once or twice.

Planck's *Wärmestrahlung* constituted both a summary of his work on radiation theory since the turn of the century and a continuation and re-examination of it. Sommerfeld appears to have gone over the text with a fine-tooth comb. After beginning with the hastily scrawled claim that "radiation is a focus of modern research," Sommerfeld divided previous approaches, much as Planck had, into three types: thermodynamic, electrodynamic, and statistical methods.[19] In the thermodynamic category, he placed the work of Kirchhoff, Stefan and Boltzmann, and Wien; in the electrodynamic, that of Helmholtz, Maxwell, Rayleigh and Jeans, and Lorentz; while the statistical principally dealt with the methods of Boltzmann, Gibbs, and Planck. An outline of the structure of Sommerfeld's lectures is given in table 1. The column on the left represents the proposed structure of the course, as laid out in Sommerfeld's first lecture. The column on the right provides the topic

Table 1: Structure of Sommerfeld's Lectures

Sommerfeld's Lecture **First Outline**	**Sommerfeld's Lectures on** ***Theorie der Strahlung*** **Sommer 1907**
§1. Kirchoff 1859 §2. Stefan-Boltzmann 1879 u. 1884 §3. W. Wien. 1893 Das Wien'sche Verschiebungsgesetz	Einleitung u. Übersicht (Introduction and Overview) §1. Kirchhoff §2. Stefan-Boltzmann'sches Gesetz (Stefan-Boltzmann Law) §3. Wien'sches Verschiebungsgesetz (Wien's Law of Displacement)
Elektrodyn. Methoden §4. Helmholtz 1856 Reciprocitätssatz. Umkehrbarkeit des Strahlenganges. §5. Maxwell 1873 Strahlungsdruck §6. Rayleigh-Jeans 1905 §7. Lorentz 1903	Elektrodynamisches Teil §4. Maxwell'sches Strahlungsdruck (Maxwell's Radiation Pressure) §5. Bewegter Spiegel (Moving Mirror) §6. Jeans Ableitung eines Grenzfalles des Strahlungsgesetzes (Jeans Derivation of a Limiting-Case of the Radiation Law) §7. Lorentz Ableitung derselben Grenzformel aus der Elektronenth (Lorentz's Derivation of the Same Limit-Formula from the Electron Theory)
Statistische Methoden §8. Verteilungssatz der Energie §9. Wahrscheinlichkeit und Entropie, nach Boltzmann §10. Der Planck'sche Oscillator §11. Das Planck'sche Strahlungsgesetz §12. Das Planck'sche Elementarquantum h der Energie. Folgerungen von Einstein	Dritter Abschnitt. Statistisches. §8. Beispiel aus der Gastheorie §9. Planck'sche Theorie

19. The original is "Strahlung ein Brennpunkt moderner Forschung. Drei Strahlen kommen darin zusammen: Thermod., Electrod., Statistische Methoden."

headings for the course as actually delivered. The last three sections of the proposed course were compressed into a single one.

The difference between Sommerfeld's discussion of previous treatments and his analysis of Planck's own contribution jumps to the eye. Whereas his summary of earlier research (§§ 1 to 8) was in some cases as detailed as Planck's, his discussion of Planck's theory in § 9 is remarkably concise. Sommerfeld achieved this by concising almost entirely the discussion of the production of radiation by Hertzian resonators, a topic that took up almost a third of Planck's text. Instead, within half a page of writing out an expression for the energy of a Hertzian dipole, given in terms of its total energy *U*, the electromagnetic moment *f*, and two constants *K* and *L*,

$$U = \frac{1}{2}Kf^2 + \frac{1}{2}L\dot{f}^2, \tag{1}$$

Sommerfeld merely stated the relation Planck derived between the total energy of a resonator and its average energy *u* at frequency *v*:

$$U = \frac{c^3}{8\pi\nu^2}\bar{u}. \tag{2}$$

A parenthetical note following the equation indicates that a proof would follow, perhaps as an exercise, since no such proof appears in the lecture notes themselves.

From equation 2 onward, Sommerfeld followed Planck closely, reproducing in detail the now well-known combinatorial argument that results in Planck's equation relating energy to frequency for a black-body at a temperature, T:

$$U = \frac{h\nu}{\left(e^{\frac{h\nu}{kT}} - 1\right)} \tag{3}$$

Immediately following this result, Sommerfeld made some "Critical remarks" that hint at the reason for the curt exposition of Planck's resonator approach. Sommerfeld appears to have paid close attention to remarks made by Paul Ehrenfest, published in the *Physikalische Zeitschrift*, the year Planck's book appeared.[20] Planck

20. Paul Ehrenfest, "Zur Planckschen Strahlungstheorie," *Physikalische Zeitschrift, 7* (1906), 528-532, reproduced in Martin J. Klein, ed., *Paul Ehrenfest: Collected scientific papers* (Amsterdam, 1959), 120-124.

had introduced the resonators into radiation theory to obtain a parallel to the Maxwell-Boltzmann treatment of kinetic theory. Just as interaction between molecules brought about the Maxwell-Boltzmann distribution as an equilibrium distribution of velocities, the interaction of resonators would ensure that an initially arbitrary distribution of energies in a black-body would result in an equilibrated radiation. Ehrenfest quashed that possibility by showing that the resonators could not do what was required. Since they emitted and absorbed energy at characteristic frequencies, only resonators at the same frequency interacted, producing an equilibrium distribution of intensity and polarization for each color. For resonators at different frequencies, however, no interaction was possible, so any arbitrary frequency distribution would persist. Ehrenfest's argument:[21]

> 1) The frequency distribution of the radiation introduced into the model [described by Planck] will not be influenced by the presence of arbitrarily many Planck resonators, but will be preserved permanently.
> 2) A stationary radiation state will [nevertheless] result from emission and absorption by the oscillators in that the intensity and polarization of all rays of each color will be simultaneously equilibrated in magnitude and direction.
> In short: radiation enclosed in Planck's model may in the course of time become arbitrarily disordered, but it certainly does not become blacker. For the discussion to come, the following formulation is especially suitable: Resonators within the reflecting cavity produce the same effect as an empty reflecting cavity with a single diffusely reflecting spot on its wall.

As Kuhn has noted, Planck had made similar remarks at the end of his lectures, realizing, in his book's conclusion, that much of his analysis had been fruitless.[22]

It is clear that Sommerfeld drew his inspiration from Ehrenfest's critique. His first objection, under the title "The role of the resonators" reads:[23]

> The resonators only operate like a reagent, strips of litmus paper, not like a catalyst [*Ferment*], coal-dust. The non-black radiation remains non-black. The resonators can only increase the disorder of directions, not the color distribution. Because the resonator only works in the region (*v, dv*) to which it is allotted

21. Ibid., 121. Translation in T.S. Kuhn, *Black-body theory and the quantum discontinuity, 1894-1912* (Oxford, 1978), 159-160.
22. Max Planck, *Vorlesungen über die Theorie der Wärmestrahlung* (Leipzig, 1906), 220. cites an earlier paper by Ehrenfest, "Über die physikalischen Voraussetzungen der Planck'schen Theorie der irreversiblen Strahlungsvorgänge," *Wiener Berichte, 114* (1905), 1301-1314.
23. Sommerfeld, *Theorie der Strahlung,* (AS), §9: "Die Resonatoren wirken nur wie ein Reagenz, Streifen Lakmuspapier, ~~ohne d~~ nicht wie ein Ferment, Kohlenstaäubchen. Die nicht-schwarze Strahlung bleibt nichtschwarz. Die Resonatoren können nur die Unordnung der Richtungen vermehren, nicht die Farbenverteilung. Denn ~~je~~der Resonator wirkt nur auf den Bereich (*n, dn*) auf den er abgestimmt ist. ~~Wegen seiner~~ Der Resonator leistet nicht mehr als ein diffuser Spiegel (Vgl. §6 Jeans)."

> [*abgestimmt*]. The resonator does nothing more than a diffusing mirror. (Cf § 6 Jeans)

Another comment referred to the dissimilarity between the methods of Boltzmann and Planck. While Boltzmann had proved that the entropy, *S,* was a maximum for the Maxwell-Boltzmann distribution Planck had skipped this step. Sommerfeld noted, apparently again following Ehrenfest, that the "substitution for this unfortunately missing consideration" was the "auxiliary assumption" [*Hilfsannahme*] that we now know as Planck's hypothesis, $\varepsilon = h\nu$. It was only with this hypothesis that Planck was able to get to a result that provided the requisite dependence of the total energy on both temperature and frequency.[24]

Although Ehrenfest rejected the resonator approach, he did not reject the recourse to combinatorics. Rather, he explained the fundamentally different assumptions that led to the different results of Boltzmann (his former teacher) and Planck. For Ehrenfest, Planck's hypothesis was an additional (if peculiar) constraint that led to an experimentally verifiable result. He was willing to accept a version of Planck's thermodynamical and statistical approach without the appeal to resonators. For Sommerfeld, in contrast, the failure of Planck's resonators seems to have been emblematic of the problem with Planck's method in general. Sommerfeld treated the "auxiliary assumption" as little more than a gimmick. "I think it is very possible," Sommerfeld wrote in the lecture, "that Planck's formula is only a good approximation."[25]

As an approximation, Planck's equation had competitors. Sommerfeld described Jeans's result as an "approximation" as well. Jeans had assumed that energy could be distributed equally among the eigenvalues of vibrations within a cube of side *L*. Doing so, however, resulted in a curve that was not in accordance with the experimental data of researchers like Planck's friend at the Berlin *Technische Hochschule*, Heinrich Rubens.[26] Sommerfeld explicitly compared the assumptions implicit in Jeans's derivation to those of Planck in §6 of his lectures:[27]

> The most interesting question is now this: Why do we only obtain an approximate formula?

24. Ehrenfest (ref. 20), 532. Sommerfeld, *Theorie der Strahlung,* §9 (AS).
25. Sommerfeld, ibid. (AS).
26. Ehrenfest later used the term "Ultra-violet catastrophe" to refer to the fact that Jeans' model predicts a rush of energy into higher frequency vibrations. According to Kuhn (ref. 21, 195), the recognition of this problem occurred only after Lorentz's lecture in 1908.
27. Sommerfeld, *Theorie der Strahlung,* §6 (AS): "Die interessanteste Frage ist nun die: Warum erhalten wir hier nur eine Näherungsformel? [Italics mine]

1) Der Satz der gleichen Energieverteilung gilt nicht für den Äther allgemein, ist mechanisch abgeleitet. Es ist sozusagen Zufall, dass es noch fur ~~kurze~~ lange Wellen gilt. Lang heisst dabei nicht: gross gegen l, den l fällt heraus.

2) Standp. von Planck. Die Grosse h ist das Wirkungsquantum der Energie. Die Energie kann nicht beliebig unterteilt werden. Wäre die kleinste Energiemenge h=0, so würde ~~sich~~ auch ~~aus~~ die Planck'sche Formel in die Jeans'sche degenerieren.

~~3) Standp. von Jeans.~~

1. The assumption of the equipartition of energy is not generally valid for the Aether, *it is derived mechanically*. It is, so to speak, [mere] chance that it is still valid for long waves. Long thereby means nothing: Size depends on L, L drops out.
2. Standp. of Planck. The quantity h is the quantum of action of energy. The energy cannot be divided arbitrarily. If the smallest amount of energy were $h = 0$, then Planck's formula would also reduce to that of Jeans.

In deciding which theory to reject, Sommerfeld gave greater weight to the impotence of Planck's resonators than to the failure of Rayleigh-Jeans equation to match available experimental results. Sommerfeld regarded the choice between Planck and Jeans's formulas as a choice between two distinct methods. Jeans's result, as Jeans had derived it, did not receive Sommerfeld's support; Jeans had assigned a mechanical property (the equipartition of energy) to what was—for a proponent of the electromagnetic world view—a fundamentally non-mechanical ether. But Lorentz had derived Jeans's formula "from the electron theory," as Sommerfeld made clear in §7.

Lorentz's derivation thus provided, in Sommerfeld's eyes, a positive endorsement for Jeans's formula. On the other hand, Sommerfeld saw significant problems in Planck's approach to the theory of radiation. He laid these out in a series of "General comments" toward the beginning of the lectures. The thermodynamical approach to radiation, he noted, was at once "the most secure but the least satisfying." It did not provide understanding. Mechanism, or the kinetic theory, had eliminated thermodynamics by founding its laws on statistical mechanics. Along similar lines, "The program ~~offered by Planck~~ of radiation th[eory] should offer: to explain thermod[ynamics] electro-statistically." [28]

Planck failed because, while utilizing the statistical techniques of the kinetic theory, he came out firmly on the side of thermodynamics. As for pure electrodynamics, it could not achieve a single-valued expression for radiation intensity but infinitely many solutions. Mechanics served no better: "The temporal course of a thermodynamic process cannot be calculated on the mechanical heat theory or the electrodynamic theory of heat radiation under the [same] initial and boundary conditions that completely suffice in thermodynamics for the single-valued determination of the process.[29]

For Sommerfeld, the fact that Planck did not seek to explain radiation solely in electro-statistical terms spoke against his methods: "Planck's theory is therefore not ideal; the theories of Jeans and Lorentz are better in principle."[30] Here was the

28. Sommerfeld, *Theorie der Strahlung*, under the heading "Allgemeine Bemerkungen dazu" (AS): "Das Planck'sche Elementarquantum h der Energie"]. "Die Thermod. ist die sicherste Grundlage aber die am wenigsten befriedigende. Im Gegensatz zur Energetik verlangt man Verständnis des Mechanismus oder Elektrodynamismus. In der Gastheorie hat man die Thermod. eliminirt, mechanisch-statistisch erklärt. Das Program ~~von Planck lautete~~ der Strahlungsth. sollte lauten; die Thermod. elektr.-statistisch zu erklären."
29. Planck (ref. 22).
30. Sommerfeld (ref. 28): "Die Planck'sche Th. ist also nicht ideal; die Theorien von Jeans u. Lorentz principiell besser."

programmatic aim of the electromagnetic view of nature in operation. "Programmatic" because Sommerfeld had specific objections to Lorentz's particular version of the electron theory, preferring Abraham's. Nonetheless, he clearly deemed either better than one that did not seek to reduce all other explanatory means to electrodynamics. Jeans's result, as derived through the electron theory, was to be preferred over any result following from a system of thought that might seek to deny the unifactory capacities of electromagnetism. No doubt, like Lorentz himself, Sommerfeld hoped that a more complete electromagnetic theory would result in an expression in better accordance with experience and experiment. Until then, an "approximation" derived along correct programmatic lines to one derived in a manner he deemed "not ideal."

While usually not effusive, Sommerfeld waxed lyrical over the explanatory possibilities and unifying capacity of electrodynamics.[31]

> Heat (radiated) is light, therefore electr[icity?]; but heat is, on the other hand, molecular motion. ~~How should it~~ It must convert electr[ical] action into inertial action; as it does so, the theory shows the apparent degree to which kinetic energy actually might be electromagn[etic] energy of charged matter. Therefore in short: From the ident[ity] of light Leslie Prevost Rumford 18th Cent. and heat, the id[entity] of light and electr[icity] Maxwell Hertz end of the 19th Cent. and the id[entity] of heat and molecular mechanics Clausius Maxwell Boltzmann 19th Cent follows necessarily the id[entity] of molecular mech[anics] and electrodynamics (20th Century).

If Boltzmann had shown that thermodynamics reduced to mechanics, this last identity showed that both thermodynamics and mechanics could be reduced to electrodynamics. This conclusion, in turn, suggested a point by point refutation of Planck's "introductory theses":[32]

30. Ibid.: "Die Elektrodyn. schafft auch hier die höchste Einheit. Wärme (gestrahlt) ist Licht, also Elektr., aber Wärme ist andererseits Molekularbewegung. ~~Wie soll sich~~ Es muss sich Elektr. Wirkung in Tragheitswirkung umsetzen; wie sie das tut, zeigt die Theorie der scheinbaren Masse, wonach kinetische Energie tatsächlich elektromagn. Energie der geladener Materie sein soll. Also kurz: Aus der Ident. von Licht u. Wärme Leslie Prevost Rumford 18 Jahrh., der Id. von Licht u. Elektr. Maxwell Hertz Ende d. 19. Jahrh. und der Id. von Wärme u Molekular mechanik Clausius Maxwell Boltzmann 19 Jahrh. folgt mit Notwendigkeit die Id. von Molekularmech. u. Elektrodynamik (20. Jahrh.)"

31. Ibid.: "Daraufhin werden wir zu den Planck'schen Einleitungsthesen gerade das Gegenteil aussagen können:

1) Wärme pflanzt sich auf 2 versch. Artens fort, Leitung u. Strahlung.

1a) Wärme pflanzt sich nur auf eine Art fort, elektrod., bei der Leitung sind die elektr. Felder an Ladungen gebunden, bei der Strahlung breiten si sich frei im Äther aus.

2) Die Wärmestr. ist viel compl. wie die Wärmeleitung, weil sich dort der Zustand nicht durch einen Vektor charakterisieren lasst

2a) Die Wärmestr. ist viel einfacher wie die Wärmeleitung, weil die Besonderheiten der Ladungsverteilung (Materie) nicht mitspielen. Im Äther allein die Strahlungsrichtung u. Intensitäten, im Wärmeleiter ausserdem die Bewegungsrichtung der Molekule."

32. Kuhn (ref. 21), 189.

1) Heat diffuses [*fortpflanzt sich*] in two different ways, conduction and radiation.
1a) Heat diffuses in only one way, electrod[ynamic], in conduction the electr[ic] fields of the charges are bound to the molecule, in radiation they spread out freely in the Aether.
2) Heat rad[iation] is much more compl[icated] than heat conduction, because in that case the state cannot be characterized by a vector.
2a) Heat rad[iation] is much easier than heat conduction, because the particulars of the charge distribution (matter) don't play a part. In the Aether only the direction and intensity of the radiation [figure], in heat conduction the directions of movement of the molecule as well.

Black-bodies in an electromagnetic world

Kuhn's argument that Lorentz's lecture in Rome in 1908 marked the beginning of the acceptance of the "quantum discontinuity" runs as follows:[33]

> During 1908 Lorentz produced a new and especially convincing derivation of the Rayleigh-Jeans law. Shortly thereafter he was persuaded that his results required his embracing Planck's theory, including discontinuity or some equivalent departure from tradition. Wien and Planck quickly adopted similar positions, the former probably and the latter surely under Lorentz's influence. By 1910 even Jeans's position on the subject had been shaken, and he publicly prepared the way for retreat. These are the central events through which the energy quantum and discontinuity came to challenge the physics profession.

In the Rome paper Lorentz proved that the electron theory *must* lead to Jeans' result: an electromagnetic approach could not avoid the problems that followed from the equipartition theorem.[34] Without at this point making a choice between them, Lorentz then stated the difference between the Rayleigh-Jeans and the Planck case as boldly as possible. Accepting Planck would bring theory in line with experiment, but "we can adopt it only by altering profoundly our fundamental conceptions of electromagnetic phenomena." Accepting Jeans, on the other hand, would "oblige us to attribute to chance the presently inexplicable agreement between observation and the laws of Boltzmann and Wien."[35] For experimentalists, Kuhn

33. Lorentz wrote, "I admit that, when Jeans published his theory, I hoped that by examining it more closely one would be able to demonstrate the inapplicability to the ether of the theorem of "equipartition of energy" on which it is based....The preceding considerations seem to me to prove that that is not the case and that one cannot escape Jeans's conclusion, at least not without profoundly modifying the fundamental hypotheses of the theory." H.A. Lorentz, "Le partage de l'énergie entre la matière pondérable et l'éther," *Atti del IV Congresso Internazionale dei Matematici,* IV, *Atti (*3 vols., Rome, 1909), *1*, 145-165; reproduced in H.A. Lorentz, *Collected papers*, *7* (The Hague, 1934), 317-343.
34. Kuhn (ref. 21), 191-192.
35. H.A. Lorentz, "Zur Strahlungstheorie," *Physikalische Zeitschrift, 9* (1908), 562-563, in Lorentz, *Papers* (ref. 33), 344-346.

suggested, the issue was now clear. Jeans' equation did not work at all. If the choice was between it and Planck's, then the latter had to be accepted. In a paper published a few months after the Rome lecture, Lorentz acknowledged that he had been convinced by the arguments of experimentalists like Wien, Otto Lummer, and Ernst Pringsheim to abandon all hope for Jeans's equation. The final step, Kuhn claimed, owed much to Lorentz's "great personal authority," under which the gospel spread to the rest of the physics community.[36]

What gospel? Participants in the discussion referred, variously, to the "Rayleigh-Jeans," the "Jeans," and the "Jeans-Lorentz" formula. The two former do not imply an association with the electron theory, the latter definitely does. Kuhn smudges the difference.[37] Proponents of the electromagnetic world view (like Sommerfeld, Lorentz, and Wien) may not have regarded the choice between continuity and discontinuity as the key dilemma. Rather, the issue that "came to challenge" them, the issue over which they struggled, concerned the capacity of an electron theory to produce a Planck-like formula. Once it was accepted that this was impossible, discontinuity was adopted quite readily by this group.

Lorentz wrote Wien early in June 1908, "ceaselessly racking his brains over the last few years" trying to derive Planck's formula (or something similar) from the electron theory. Contrasted to this language of struggle, Lorentz's description of Planck's alternative solution, the introduction of elementary quanta of energy, seems almost casual: "In and of itself, I have nothing against it; I concede at once that much speaks in its favor and that it is precisely with such novel views that progress is made. I would, therefore, be prepared to adopt the hypothesis without reservation if I had not encountered a difficulty."[38]

Kuhn highlights this difficulty to explain Lorentz's hesitancy in accepting discontinuity, but the problem Lorentz outlined is not that of discontinuity, but rather of an asymmetry between the (continuous) absorption and emission of energy by resonators in interaction with the ether, and discontinuous emission and absorption otherwise. This specific question would continue to bother those who had accepted the idea of a quantum discontinuity for some time, and would eventually lead Planck to his so-called second and third theories, each of which posited different mechanisms (one continuous, one discontinuous) for resonator emission and absorption. Lorentz did not have a problem with the idea of discontinuity "in and of itself." What counted was whether the electromagnetic world view could include it. Lorentz wrote in the *Physikalische Zeitschrift:* "I can only conclude that a derivation of the radiation law from electron theory is scarcely possible without profound changes in its foundation. I must therefore regard Planck's theory as the only tenable one."[39]

36. Kuhn (ref. 21), 195.
37. Ibid., 193.
38. Lorentz to Wien, 6 June 1908, quoted in Kuhn (ref. 21), 194.
39. Lorentz, *Papers* (ref. 33), 345; Kuhn (ref. 21), 193.

Wien did not perceive immediately after the Rome lecture that Lorentz was suggesting abandonment of the electromagnetic world view, and his route towards Planck's theory can be understood as the mirrored inversion of Lorentz's. Whereas Lorentz tried to obtain Planck's result by beginning with the electron theory, Wien—after dismissing Jeans's result on experimental grounds—began with Planck's energy elements and sought to understand them in electromagnetic terms. He regarded Lorentz's re-derivation of Jeans's result in Rome as "sad." "The lecture Lorentz gave in Rome," he wrote to Sommerfeld, "has disappointed me greatly."[40]

> That he presented nothing more than the old Jeans theory without bringing in any sort of new viewpoint I find a little sad. Besides, the question of whether one should regard the Jeans theory as tenable lies in the realm of experiment. His opinion is not tenable here because observations show enormous deviations from the Jeans formula in a range in which the experimenter can easily control how far the radiation source deviates from a black-body. What's the point in presenting these questions to the mathematicians, who can make no judgement on precisely this point?
>
> It seems, in addition, a little peculiar to seek the advantage of the Jeans formula, in spite of the fact that it corresponds with nothing, in the fact that it can preserve the whole unlimited multiplicity of electron oscillations. And the spectral lines? Lorentz has not shown himself to be a leader of science this time.

Ruling out Jeans's result on experimental grounds was easier than jettisoning it for methodological reasons. It was not until reading Lorentz's second paper that Wien realized, with some dismay, what giving up Jeans's result meant for electromagnetic theory. Writing again to Sommerfeld, he noted that:[41]

> Lorentz has recognized his error over radiation theory and that Jeans' hypothesis is untenable. Now, however, the situation is not so simple, since in fact it appears that Maxwell's theory must be abandoned for the atom. Hence I have a question to pose to you again. Namely, to check how far Lorentz's statistical mechanics and proof is founded on the fact that a system obeying Maxwell's equations (including electron theory) must also obey the supposition of the "equipartition of energy," from which Jeans' law is deduced. A restriction of the degrees of freedom, as required by Planck's energy element, must also require an electromagnetic interpretation. Now it seems to me almost as if such [an interpretation] would be impossible, that precisely this restriction requires additional forces (fixed connections and the like) that don't fit in with a Maxwellian system. If that's really the case, it is no longer necessary to rack one's brains over an interpretation of the energy element and a representation of spectral series on an electromagnetic basis, but rather must seek to find an extension of Maxwell's equations within the atom.

40. Wien to Sommerfeld, 18 May 1908 (EM 338-339); Kuhn (ref. 21), 192.
41. Wien to Sommerfeld, 15 June 1908 (EM, 340-341); Kuhn (ref. 21), 203.

Standing almost as bookends, outlining first the problem and then the proposed solution, are the statements, "it appears as if Maxwell's theory must be abandoned for the atom" and "we rather must seek to find an extension of Maxwell's equations within the atom." Between the two, Wien translates the question of equipartition and the question of the meaning of Planck's energy elements into the language of electromagnetic theory. The resultant contradictions led him to echo and reject the lines written to him less than two weeks earlier by Lorentz: "it is not necessary to rack one's brains any more." The effort to save the electron theory and the electromagnetic world view in its entirety now seemed fruitless and Wien pointed quite calmly to the need for an inter-atomic extension of Maxwell's equations.

Sommerfeld's reply on June 20 was less pessimistic, since he had not found Lorentz's electrostatistic derivation of Jeans's result conclusive.[42] He promised Wien that he would communicate his objections to Lorentz and did so the same day. Rather than accepting Lorentz's calculations as a proof that Planck's result undercut the electromagnetic world view, Sommerfeld merely used the opportunity to stress what was at stake in such a question: "At one time, when I lectured on the theory of radiation, I believed Jeans' paradox could be overcome by saying that electrodynamics is not subject to mechanical laws. Your present remarks seem to me to be an excellent foundation for the resolution of this question."[43]

Fixing a date for Sommerfeld's acceptance of the necessity of discontinuity is not easy.[44] In November 1908 he wrote to Lorentz urging him to ignore his earlier criticisms, but did not explicitly retract his objections to Lorentz's theory in general.[45] In the latter part of 1908 Sommerfeld attended Minkowski's lectures on

42. Sommerfeld to Wien, 20 June, 1908 (EM, 341-343). Lorentz assumed that electron motions could be treated as quasi-stationary and hence restricted the number of degrees of freedom that needed to be considered to six. Sommerfeld rejected the assumption. He argued that "free electron vibrations" provided an infinite spectrum of free periods of vibration. "In the distribution of energy one must pay exactly as much attention to these infinitely many degrees of freedom as to the Eigen-vibrations of the box. Thus an equilibrium of energy appears to be possible, and a part of the energy for small l [wavelength], which Lorentz gives to the ether alone, must go over to the electrons....Lorentz actually only considers the equilibrium between the ether and uncharged atoms. This is not permissible, because the electrons mediate the energy exchange in the first place. In this energy exchange a part of the energy transferred remains stored up in the electrons."
43. "Als ich einmal über die Theorie der Strahlung vortrug, glaubte ich dem Jeans'schen Paradoxon dadurch entgehen zu können, daß ich sagte, die Elektrodynamik ist nicht den mechanischen Gesetzen unterworfen. Ihre jetzigen Ausführungen scheinen mir ein vorzügliches Fundament zur Entscheidung dieser Frage." EM, 342.
44. Kuhn (ref. 21, 188-194, 202-204), locates Lorentz's and Wien's public espousals of the quantum discontinuity in 1909.
45. Sommerfeld to Lorentz, 16 Nov 1908 (EM, 348-349): "You will respond that the small wavelengths—ultra-ultra-violet—cannot play a role in the region of heat [*Wärmegebiet*]. But I am lacking the proof for that." Sommerfeld congratulated Lorentz on the "victory" of the relativity theory brought by Bucherer's experiments, but lamented that "a great deal of

relativity and was "converted" by them;[46] he had therefore to reject Abraham's rigid spherical electron theory, which he had favored over Lorentz's, as not relativistically invariant. If Sommerfeld applied the relativity theory to the choice between competing electron theories, he would have been induced to accept Lorentz's some time after 1908. By late 1909, he had made this point explicitly, in lectures that mark the first classes taught anywhere in the world on relativity theory. In introductory comments, Sommerfeld noted that the hypothesis of the rigid electron "was dropped because it includes the hypothesis of absolute space. The deformable electron follows from the concept of relative space-time, which experience requires."[47] It follows, or almost, that Sommerfeld then accepted Lorentz's conclusion that the electron theory and the electromagnetic world view were incapable of dealing with the theory of radiation.

For physicists not committed to the electromagnetic world-view, the issue of discontinuity was a key means of understanding Planck's result. Einstein and Ehrenfest, who approached the issue from the perspective of Boltzmann's statistical mechanics, were the first, Kuhn argues, to "discover" the quantum discontinuity, and that some years before the Rome lecture. Jeans, on the other hand, initially denied the force of experimentalists' arguments, only conceding their validity in 1910. His phrasing of the choice on offer then does not include discussion of electron theory. Rather, Jeans placed the issue of discontinuity front and center:[48]

> Planck's treatment of the radiation problem, introducing as it does the conception of an indivisible atom of energy, and consequent discontinuity of motion, has led to the consideration of types of physical processes which were until recently unthought of, and are to many still unthinkable. The theory put forward by Planck would probably become acceptable to many if it could be stated physically in terms of continuous motion, or mathematically in terms of differential equations.

For the proponents of the electromagnetic world-view, the key issue introduced by black-body theory was the apparent failure of electron theory to incorporate or duplicate Planck's experimentally confirmed result. The acceptance of discontinuity followed with comparatively little struggle after that blow to their shared *Weltanschauung* had been assimilated. For those who were not wedded to the electromagnetic picture, however, discontinuity became the most troubling thing about

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the clarity and causality of the physical foundations of your original theory is lost."

46. The language of conversion is Sommerfeld's: "Ich bin jetzt auch zur Relativtheorie bekehrt; besonders die systematische Form und Auffassung Minkowski's hat mir das Verstaendnis erleichtert." Sommerfeld to Lorentz, 9 Jan [1910] (EM, 375-376). Minkowski's enthusiasm for the electromagnetic world-view no doubt appealed to Sommerfeld as well. Peter Galison, "Minkowski's space-time: From visual thinking to the absolute world," *HSPS, 10* (1979), 85-121, on 93.

47. Sommerfeld, "Elektronentheorie, II Teil" (AS).

48. J.H. Jeans, "Non-Newtonian mechanical systems, and Planck's theory of radiation," *Philosophical magazine, 20* (1910), 943-954, quoted in Kuhn (ref. 21), 204.

Planck's energy elements. If we wish to stay with Kuhn's religious language, we should speak not only of "converts" to discontinuity, but also of "lapsed" or at least disillusioned electromagneticists.

Sommerfeld's Solvay paper

Hostility towards purely mechanical explanation persisted in Sommerfeld's work, as did a preference, where possible, for electrodynamic explanation. In the paper he delivered at the Solvay conference in Brussels in 1911, the traces of his old allegiance remain strong even when coupled with modifications and limitations required by the hypothesis of action-quanta. He found satisfaction in the apparently necessary "symbiosis" between his preferred electromagnetism and the essential quantum.

The Solvay conference was one of several public meetings in 1911 that signalled Sommerfeld's first substantial public engagements with the quantum theory.[49] His Solvay paper was long and difficult. Yet, perhaps because he had not been one of those to accept the quantum discontinuity early (unlike, say Einstein or Ehrenfest), his paper did not engage with the black-body problem, specific heats, the existence of atoms, the kinetic theory, nor other standard problems. It applied Planck's ideas to a completely new class of problems.[50]

Sommerfeld began his paper with a criticism of the state of the restricted development of quantum theory, comparing it to a mechanics that only examined periodic circular motion. The quantum had figured only in treatments of periodic phenomena. This worked well for the problems of radiation theory and for specific heats, but failed in the light of larger questions of physics. Precisely because of its "special development," it had led to puzzles difficult to reconcile with the "very secure foundations of the electromagnetic field."[51] A more general standpoint was needed.

Rather than seeing Planck's quantum as a restriction on the way energy could be parcelled out (a quantum of energy), one could take the dimensions of the constant as reflective of a deeper meaning. The quantum of action, which had the dimensions of Energy x Time, limited the product, rather than either component

49. Armin Hermann, *The genesis of the quantum theory, 1899-1913* (Cambridge, 1971), 107, gives evidence that "the decisive time span during which Sommerfeld became a convert [to the quantum hypothesis] must lie between February and December 1910."
50. Ibid., 102-122; Sigeko Nisio, "The formation of the Sommerfeld quantum theory of 1916," *Japanese studies in the history of science, 12* (1973), 39-78; Max Jammer, *The conceptual development of quantum mechanics* (New York, 1966), 42-45. Jagdish Mehra and Helmut Rechenberg, *The historical development of quantum theory,* Vol. 1 (New York, 1982), 132-135.
51. Arnold Sommerfeld, "Die Bedeutung des Wirkungsquantums für unperiodische Molekularprozesse in der Physik," in Arnold Zucken, ed., *Die Theorie der Strahlung und der Quanten: Verhandlungen auf einer von E. Solvay einberufenen Zusammenkunft (30 Oktober bis 3. November 1911)* (Berlin, 1913), 252-300, on 252.

separately. This fact was hidden in analyses that took the period of a given motion as a constant. "Phrased completely generally," Sommerfeld wrote, "a large quantity of energy in a shorter time, a smaller in a longer time is taken up and given out by matter, so that the product of energy and time, or (closer to the definition) the time integral of the energy, is determined by the magnitude of *h*."[52] For example, high-velocity cathode-rays produced strong x-rays; lower-velocity cathode-rays, weaker ones. The strength of an x-ray, however, was taken as an inverse measure of the duration of the braking-time for the electrons impinging on the anode. So, high energy x-rays went with short braking-times, low energy with longer ones. The product should be a constant, dependent on *h*. A similar result could be inferred from the energy emission: large quantities of energy are emitted by radioactive substances in a short time, smaller quantities take a longer time.

The rule contradicted everyday experience, since a bullet travelling rapidly takes longer to slow down when it hits a wall than a slowly moving one. Sommerfeld suggested that the peculiarity of the quantum of action might explain this anomalous behavior for particles considerably smaller than those considered in ballistics problems. Expressing his result mathematically, Sommerfeld connected the quantum of action with the least action principle, which he "with Helmholtz-Planck, [saw] as the deepest foundation [*Grundsatz*] of mechanics and physics," and offered the "following fundamental hypothesis about the general meaning of *h:* with every purely molecular process a fixed, universal amount of action is taken up or given out from the atom."[53] That amount was given by $\int_0^\tau Hdt = h/2\pi$. (4)

Following a proof of the relativistic invariance of this result, Sommerfeld qualified his analysis as "in many points hypothetical and incomplete." He made an effort to avoid ruffling the feathers of his colleagues by too iconoclastic an approach:[54]

> Regarding the general comparison of the energy quantum and the action quantum...I would like to emphasise the wider consequences of the action quantum, but at the same time would only like to place my view with all caution alongside those of other researchers, who have concerned themselves so much longer and more fundamentally with these questions, and have so far supported the viewpoint of the energy quantum with such great success.

Having displayed the requisite modesty, Sommerfeld then turned to the pursuance, in the following sections of the "greater consequences of the action quantum," each of which dealt with aperiodic phenomena in terms of specific problems: the production of x-rays and γ-rays, the photoelectric effect, and ionization. The first two belonged to a pet project of Sommerfeld's. Early in 1911 he had presented a paper in Munich, "On the structure of γ-rays," which introduced much

52. Ibid., 253.
53. Ibid., 254.
54. Ibid., 257.

of the analysis he would present at Solvay. There he offered a quantum theoretical interpretation of the radiation given off when fast cathode rays strike the anticathode of the discharge tube. Two components may be distinguished: *Bremsstrahlung* (braking radiation), which is polarized and continuous in frequency, and fluorescent radiation, which is unpolarized and made up of rays of discrete frequencies apparently produced by the vibrations of the atoms of the anticathode. The sum of the energies from these two processes gave the total radiation energy ($E_{pol} + E_{unpol} = E_r$).

For Sommerfeld, only one of these components (the polarized) could be determined by electromagnetic theory. The unpolarized part was a problem in mechanics, relating to intra-atomic processes.[55] True to the programmatic aims of the electromagnetic world view, Sommerfeld did not consider the mechanical problem. Sommerfeld's result (Eq. 12) for *Bremsstrahlung* derived entirely from electromagnetic considerations.

To secure it, Sommerfeld began by noting that, since the unpolarized radiation spreads out uniformly in all directions, a simple expression connects its energy (E_{unpol}) and intensity (S_{unpol}),

$$E_{unpol} = 4\pi r^2 S_{unpol} \tag{5}$$

He then obtained two initial equations, for the energy and the intensity of the polarized part of the radiation (E_{pol} and S_{pol}, respectively), through a purely electrodynamic argument that made use of an expression derived by Abraham in his *Theorie der Elektrizität*.[56] Sommerfeld had derived this equation in his earlier paper on γ-rays, and merely wrote the results into his Solvay report,

$$E_{pol} = \frac{e^2\dot{v}}{6\pi c^2}\frac{\beta}{\sqrt{1-\beta^2}}, \tag{6}$$

$$S_{pol} = \frac{e^2\dot{v}}{16\pi^2 c^2 r^2}\int_0^\beta (1-\beta^2)^{\frac{3}{2}} d\beta$$

$$= \frac{e^2\dot{v}}{16\pi^2 c^2 r^2}\beta\left(1-\frac{1}{2}\beta^2+\frac{3}{40}\beta^4+\ldots\right). \tag{7}$$

55. Ibid., 258-259.
56. Arnold Sommerfeld, "Über die Struktur der γ-Strahlen," Mathemat.-physikal. Klasse,

(Here e = the elementary charge on the cathode ray electrons, v = their velocity, $\beta = v/c$, and $\dot{v}$ the deceleration during the braking). Rewriting (6) in terms of (7) Sommerfeld obtained

$$E_{pol} = \frac{2}{3} 4\pi r^2 S_{pol} (1 + \beta^2 + \frac{11}{20}\beta^4 \ldots) . \tag{8}$$

Dividing (8) by (5) one obtains:

$$\frac{E_{pol}}{E_{unpol}} = \frac{2}{3} \frac{S_{pol}}{S_{unpol}} (1 + \beta^2 + \frac{11}{20}\beta^4 \ldots) . \tag{9}$$

The ratio of the polarized to the unpolarized intensity had been determined experimentally by one of Röntgen's students in Munich, Walter Friedrich,[57] as

$$\frac{S_{pol}}{S_{unpol}} = \frac{3}{7} \tag{10}$$

Sommerfeld assumed a value for β of 0.4, inserted (10) into (9), and arrived at a numerical result for the ratio of polarized to unpolarized energy.

$$\frac{E_{pol}}{E_{unpol}} = \frac{2}{7} \times 1.17. \tag{11}$$

He removed the unpolarized part via the relation $E_r = E_{pol} + E_{unpol}$, thus eliminating the term representing the energy of the unpolarized radiation and therewith the mechanical portion of the analysis.

$$\frac{E_{pol}}{E_r} \approx \frac{1}{4} . \tag{12}$$

Akademie der Wissenschaft, Munich, *Sitzungsberichte*, 1911, 1-60; reproduced in Sauter (ref. 17), 377-436. The reference to Abraham's paper is on 382.

57. In his report to the Solvay conference Sommerfeld had originally used the results of Bassler (a student under Roentgen) and Wien; in the published report, he used the results of Friedrich and Wien's student Edna Carter.

His final step was to express (12) in terms of the ratio of the polarized energy to the energy of electrons impinging on the cathode (E_k).Wien and one of his students at Wurzburg had measured E_r/E_k.[58] Sommerfeld scaled it down to match the data Friedrich had obtained. He then had

$$\frac{E_r}{E_k}\cdot\frac{E_{pol}}{E_r}=\frac{E_{pol}}{E_k}=\frac{1}{6}\cdot 10^{-3}=1.7\times 10^{-4} \tag{13}$$

The quantum hypothesis now could be brought to bear an expression for E_k. Assuming that the potential energy of the electrons *(U)* was vanishingly small compared to their kinetic energy *(T),* he set $H = T-U$ equal to T. The hypothesis concerning the quantum of action became

$$\int_0^\tau T\,dt = \frac{h}{2\pi}\,. \tag{14}$$

If the deceleration of the electrons, $dv/dt = \dot{v}$, can be taken as constant during their braking, then the braking-time, τ, is $\beta c/\dot{v}$. Introducing the variable v' to represent the instantaneous velocity of the braking electrons, Sommerfeld had

$$dt = -\frac{dv'}{\dot{v}}, \qquad T = \frac{m}{2}v'^2 \tag{15}$$

and the desired condition,

$$\int_0^\tau T\,dt = \frac{m}{2\dot{v}}\int_0^{v_i} v'^2\,dv' = \frac{1}{3}\frac{m}{2}v_i^{\,2}\cdot\frac{v_i}{\dot{v}} = \frac{h}{2\pi} \tag{16}$$

With $E_k = mv^2/2$, and $E_k\tau = 3h/2\pi$. (17)

Manipulating Eq. (6) to obtain E_{pol} in terms of τ gave

$$E_{pol} = \frac{e^2}{6\pi c\tau}\frac{\beta^2}{\sqrt{1-\beta^2}}\,. \tag{18}$$

58. Wilhelm Wien, “Ueber die Energie der Kathodenstrahlen im Verhältnis zur Energie der Röntgen- und Sekundär-strahlen,” in *Festschrift Adolph Wüllner gewidmet zum 70.*

Sommerfeld divided both sides by E_k and inserted Eq. (17) on the right hand side of Eq. (18):

$$\frac{E_{pol}}{E_k} = \frac{e^2}{9hc} \cdot \frac{\beta^2}{\sqrt{1-\beta^2}} \tag{19}$$

Using known values for h, c, e and assuming again that $\beta = 0.4$, Sommerfeld thus obtained another numerical value for the ratio of the polarized radial energy to the kinetic energy of the impinging cathode rays:

$$\frac{E_{pol}}{E_k} = 2.7 \cdot 10^{-4}\,. \tag{20}$$

Eq. 20 agreed well with the value given in (13), especially in view of approximating assumptions made along the way.

The basis of his result should be clear. On the one hand, Sommerfeld used electrodynamics to calculate the ratio of the polarized energy to that of the total radiation energy. He avoided the energy contribution of the unpolarized radiation and thus the mechanics of atoms. On the other hand, he appealed to Planck's quantum theory to provide a value for the braking-time of the electrons, τ, which could not be obtained through the electrodynamic theory. This inter-reliance appealed to him. Where Wien had attempted both approaches, the electromagnetic in 1905, the radiation-theoretic in 1907 and had assumed them to be antithetical, Sommerfeld stressed that each needed the other to solve the radiation problem:[59]

> For Wien no connection existed between his electromagnetic and radiation-theoretical calculation of τ; each appeared to exclude the other. On the contrary, we have seen that the two approaches can be joined together. For the calculation of the energy of the polarised Röntgen radiation we proceed from the *purely electromagnetic formula* (4) [our (6)]. In this a quantity remains undetermined, $\dot{v}$, the deceleration corresponding to the co-dependent braking time τ. About this the electromagnetic theory can teach us nothing, because it is a function of the braking molecule. In its place the concept of the quantum of action now intervenes.…Not until the application of the radiation theory can the electromagnetic theory of Röntgen radiation be completely determined.

Geburtstage, 13 Juni 1905 (Leipzig, 1905), 1-14. Sommerfeld helped edit this *Festschrift,* so he would have known Wien's paper in detail.
59. Sommerfeld (ref. 51), 267, italics mine.

Sommerfeld saw the main benefit of his formulation of the quantum concept over that of Planck as lying in the easy connectability of his version to electromagnetic theory: "The opposition between our application of the quantum of action and Planck's method of the energy quantum has claimed much of our attention. Both depictions are foreign to classical electrodynamics and mechanics. But while our version is reconcilable with electrodynamics, the original depiction by Planck stands in unmistakable opposition to it."[60] The radiation theory may have put an end to the dream of an all-encompassing electromagnetic world-view, but the world-view would rise again as the partner of its would-be assassin: "Radiation theory and electromagnetism do not exclude, but rather complement one another."[61] Mechanics had been superseded.

This fruitful fusion could then be applied to a range of problems that otherwise threatened electromagnetic theory. The photoelectric effect, for example, "doubtless posed one of the greatest difficulties for customary electrodynamics." Again, as with the case of *Bremsstrahlung*, Sommerfeld's quantum of action could save the situation: "the difficulties seem to disappear as soon as one depicts the freeing of the electrons from the atomic bond as a Planck-style action-process and apply to it our fundamental hypothesis."[62] The same logic applied to the problem of ionization. In each case, the problem could be divided into two sections, one of which was electrodynamic, the other ruled by the quantum. The split, that is, was not (as it would eventually become) between "classical" and "modern" physics, but between quantum and electrodynamic theory. Unlike the classical/modern dichotomy, the electromagnetic/quantum one was explicitly and necessarily not an either/or. In Sommerfeld's vision, both the quantum and the electromagnetic field were required to understand the physical world.

Conclusion

McCormmach depicted the decline of the electromagnetic view of nature as a gradual one. Few physicists, he noted, began to work on the quantum theory until after around 1910, nor did many before then distinguish between Einstein's relativity and Lorentz's electron theory. It was "in the long run [that] the quantum and relativity theories worked against the electromagnetic program."[63] Yet we have seen that those who did accept the quantum hypothesis—unlike those who ceded validity to the relativity postulate—rapidly abandoned the universalizing vision of electrodynamics.[64] From being a vociferous opponent of Planck's theory in 1907,

60. Ibid., 294.
61. Ibid., 267.
62. Ibid., 276.
63. McCormmach (ref. 8), 496.
64. Gustav Mie, who saw no benefit in quantum theory, continued to advocate a form of the electromagnetic view of nature. By 1910 he had accepted Einstein's relativity postulate, but refused to accept that this might entail abandoning the notion of an all-pervading, but nonmechanical ether. McCormmach (ref. 8), 491-492.

Sommerfeld soon became reconciled to the idea that electrodynamics would have to share the task of describing all physical phenomena.

This sharing helps us to understand the diversity of the processes that led to the development of the old quantum theory. Olivier Darrigol, in his *From c-numbers to q-numbers,* has argued for the importance of "formal classical analogies" in the construction of what we now know as "modern" physics. "Not just a vague illustrative resemblance," he claims, these analogies "concerned entire pieces of logical and mathematical structures and were able to produce new laws and formalisms."[65] In the language used by Niels Bohr, the issue was one of translating classical concepts into a quantum language:[66]

> Using a metaphor, we may say that we are dealing with a translation of the electromagnetic theory into a language alien to the usual description of nature, a language in which continuities are replaced by discontinuities and gradual changes by immutability, except for sudden jumps, but a translation in which nevertheless every feature of the electromagnetic theory, however small, is duly recognized and receives its counterpart in the new conceptions.

By comparing this understanding of the role of electrodynamics in quantum theory with Sommerfeld's approach we can spot its distinctive nature and how it was framed by an early but continuing allegiance to the electromagnetic view of nature. Sommerfeld did not foresee the formal translation of electrodynamical theory into another language. Rather, what appealed to him was the need for both languages. His intent can be understood as preserving for the electromagnetic view of nature an independent space in the atomic realm it had once ruled over via the electron theory.

That this preservational purpose remained as part of Sommerfeld's quantum theory—and that it was electrodynamics, rather than mechanics or thermodynamics that he intended as the classical partner to the new physics—can be seen from the opening pages of a text that soon after its publication became known as the "Bible" for quantum spectroscopists. While Sommerfeld introduced electrodynamics in his *Atombau und Spektrallinien,* neither thermodynamics nor mechanics merit general discussion at all. Part 1, dealing with "Introductory facts" begins with a "Retrospect of the development of electrodynamics." From its beginning as a series of "disconnected laws" that functioned merely as an analogue to Newton's laws of gravitation, Sommerfeld described the rise of a new view in the second half of the 19th century that opposed a field theory to action at a distance. "The greatest triumph of this view," wrote Sommerfeld, occurred when Hertz proved the existence of electromagnetic waves.[67]

65. Olivier Darrigol, *From c-numbers to q-numbers: The classical analogy in the history of quantum theory* (Berkeley, 1992), xvi.
66. Ibid., xv.
67. Arnold Sommerfeld, *Atombau und Spektrallinien* (Braunschweig, 1919), 2; transl. Henry L. Brose, in Arnold Sommerfeld, *Atomic structure and spectral lines* (London, 1923), 2.

> From them an almost unbroken chain of phenomena leads by way of heat rays and infra-red rays to the true light rays, whose wavelengths are no more than fractions of μ [micrometers]. The greatest link in this chain came later as a direct result of Hertz's experiments, namely the waves of wireless telegraphy, whose wave-lengths have to be measured in kilometers…the smallest and most delicate link is added at the other end of the chain, as we shall see, in the form of Röntgen rays, and the still shorter γ-rays which are of a similar nature.

In *Atombau*, electrodynamics provided the gateway to and the foundation for the quantum theory. Describing his fine-structure theory, Sommerfeld would write of the "confluence of the three main currents of modern research in theoretical physics, namely, the theory of electrons, the theory of quanta, and the theory of relativity."[68] Neither heuristic, nor model, nor formal analogy, this last most fruitful construct of the electromagnetic view of nature possessed instead an independent existence in what for most others would be cast as an atomic empire ruled by the quantum hypothesis.[69]

> The original theory of Maxwell which had been perfected by Hertz retained its significance for phenomena on a large scale, such as in electrotechnics and wireless telegraphy….But to render possible deeper research leading to a knowledge of elementary phenomena, a deepened view became necessary. Maxwell's Electrodynamics had to give way to Lorentz's Dynamics of the Electron; the theory of the continuous field became replaced by the discontinuous theory, that of the atomicity of electricity. So the theory of action at a distance and the theory of action through fields were succeeded by the atomistic view of electromagnetism, the theory of electrons, which still holds today.

68. Ibid., 525.
69. Sommerfeld, *Atombau* (ref. 67), 3; *Structure* (ref. 67), 2-3.

SUMAN SETH

Quantum theory and the electromagnetic world-view

ABSTRACT

This paper has two goals: to use the electromagnetic world-view as a means of probing what we now know as the quantum theory, and to use the case of the quantum theory to explicate the practices of the electromagnetic program. It focuses on the work of Arnold Sommerfeld (1868-1951) as one of the leading theorists of the so-called "older" quantum theory. By 1911, the year he presented a paper on the "Quantum of action" at the Solvay Conference, Sommerfeld vocally espoused the necessity of some form of a quantum hypothesis. In his earlier lectures, however, his reservations about Max Planck's position were far more apparent. Section 1 argues that Sommerfeld's hostility towards Planck's derivation of the Black-body law, and his support for the result achieved by James Jeans and rederived using the electron theory by Lorentz, can be traced to his commitment to the programmatic aims of the electromagnetic world-view. Section 2 suggests that this conclusion has deep implications for our understanding of the "conversion" of several leading physicists to the quantum theory after around 1908. Section 3 traces a partial continuation of Sommerfeld's deeply held beliefs. Sommerfeld's Solvay paper is best understood as an attempt to reconcile the programmatic aims of the electromagnetic world-view with the necessity of recourse to the quantum hypothesis. No longer a universalizing vision, the attempt to prove the necessity of electromagnetic theory at all levels of explanation remained a key element of Sommerfeld's research agenda until (and even beyond) the advent of Niels Bohr's "planetary" model of the atom in 1913.