18. The Light Quantum Hypothesis.

Brush (2007)

1. A Return to an Emissionist Theory of Light?

- Einstein (1905): "On a Heuristic Point of View Concerning the Production and Transformation of Light"

"The wave theory of light, which operates with continuous spatial functions, has proved itself superbly in describing purely optical phenomena and will probably never be replaced by another theory. One should keep in mind, however, that optical observations refer to time averages rather than instantaneous values, and it is quite conceivable, despite the complete confirmation of the theory of diffraction, reflection, refraction, dispersion, etc., by experiment, that the theory of light, operating with continuous spatial functions, leads to contradictions when applied to the phenomena of emission and transformation of light."

"Indeed, it seems to me that the observations of 'black-body radiation', photoluminescence, production of cathode rays by ultraviolet light, and other related phenomena associated with the emission or transformation of light appear more readily understood if one assumes that the energy of light is discontinuously distributed in space. According to the assumption considered here, in the propagation of a light ray emitted from a point source, the energy is not distributed continuously over ever-increasing volumes of space, but consists of a finite number of energy quanta localized at points of space that move without dividing, and can be absorbed or generated only as complete units."

- Planck's (1900) Quantum Hypothesis. Energy of *particular type* of radiation (black-body) is quantized in units $E = nh\nu$, $n = 1, 2, 3, \ldots$

- Einstein (1905) extends this across the board to high-frequency EM radiation.
  - *A return to an emissionist theory of light?*

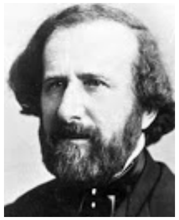
Wave theory dominance, pre-1905.

- 1810's-20's. Young, Fresnel: Interference, diffraction, polarization.

- 1850. Fizeau and Foucault: Velocity of light measured... Travels slower in water than in air!
  - *Recall: Wave theory predicts this; emission theory predicts faster in air than water.*

- 1873. Maxwell: Optical waves as EM waves.

- 1888. Hertz: EM waves detected.



Hippolyte Fizeau
1819-1896

Leon Foucault
1819-1868

- *Critics of Einstein*: Planck, Laue, Wien, Sommerfeld, Bohr.
  - *Wave theory is necessary to explain intereference, diffraction, refraction, etc.!*

"That he may sometimes have missed the target in his speculations, as for example, in his hypothesis of light quanta, cannot really be held against him." (1914.)

Max Planck

Robert Millikan
(1868-1953)

"I spent ten years of my life testing that 1905 equation of Einstein's, and, contrary to all my expectations, I was compelled in 1915 to assert its unambiguous experimental verification in spite of its unreasonableness since it seemed to violate everything that we knew about the interference of light." (1949.)

"It is as if his temperamental wishful thinking occasionally robs him of his capacity for calm deliberation, as once with his new theory of light, when he beleived he could construct a light beam capable of interference out of nothing but incoherent parts." (1916.)

Gustav Mie
(1869-1957)

Einstein's "year of miracles" (1905):

a. *Light quantum hypothesis (March)*: "On a Heuristic Point of View Concerning the Production and Transformation of Light."

b. *Dissertation (April)*: "A New Determination of Molecular Dimensions."

c. *Brownian motion (May)*: "On the Motion of Small Particles Suspended in Liquids at Rest Required by the Molecular-Kinetic Theory of Heat."

d. *Relativity (June)*: "On the Electrodynamics of Moving Bodies."

e. $E = mc^2$ *(Sept)*: "Does the Inertia of a Body Depend upon its Energy Content?"

- 1905a: 9 sections, last 3 are applications.
- Section 8 on an explanation of the photoelectric effect (won Einstein the Nobel Prize).
- Sections 1-6 formulate Einstein's argument for the light quantum hypothesis.

2. Einstein's (1905a) Argument for the Light Quantum Hypothesis.

Step 1. Consider a system of n independently moving points in volume V_0.

- Probability that all n points are located in subvolume V is $W = (V/V_0)^n$.

- "Boltzmann's Principle": $S = k \log W$
    - *Relates the probability of a microstate to its macroscopic entropy S.*

- *So*: A change in volume from V_0 to V produces a change in entropy of:

$$S - S_0 = k \log (V/V_0)^n$$

- *Note:* *Any* system with independent spatially localized components can be described by the same analysis (*e.g.*, an ideal gas of spatially localized molecules, *etc*...)

Step 2. Derive similar expression for entropy change of a quantity of high frequency radiation due to a change in volume

- *Wien Law*: $\rho = \alpha\nu^3 e^{-h\nu/kT}$ (*energy density of high-frequency black-body radiation*)

- $dS = (1/T)dE$ (*thermodynamic relation between entropy S, energy E, and temperature T*)
- $\partial\varphi/\partial\rho = 1/T$ (*induced relation between entropy density $\varphi(\nu) = S/Vd\nu$, energy density $\rho(\nu) = E/Vd\nu$, and temperature T*)

- *Combine with Wien Law to get*:

$$\varphi = -k\frac{\rho}{h\nu}\left\{\ln\frac{\rho}{\alpha\nu^3} - 1\right\} \qquad or \qquad S = -k\frac{E}{h\nu}\left\{\ln\frac{(E/Vd\nu)}{\alpha\nu^3} - 1\right\}$$

- *So*: A change in volume from V_0 to V produces a change in entropy of:

$$S - S_0 = -k\frac{E}{h\nu}\left\{\ln\frac{(E/Vd\nu)}{\alpha\nu^3} - 1\right\} + k\frac{E}{h\nu}\left\{\ln\frac{(E/V_0d\nu)}{\alpha\nu^3} - 1\right\}$$

$$= k\frac{E}{h\nu}\ln(V/V_0)$$

- *Thus*: $S - S_0 = k\log(V/V_0)^{E/h\nu}$

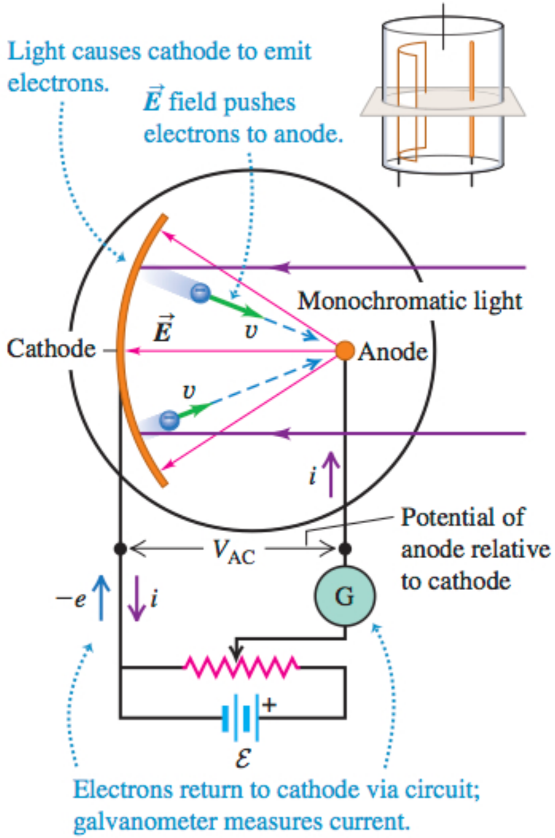
Step 3. Interpretation.

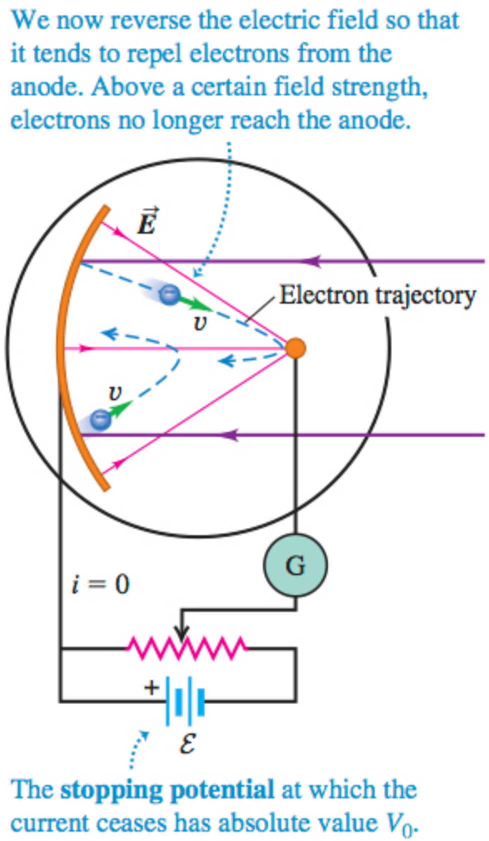
- This suggests (*via* Boltzmann's Principle) that there is a probability of $W = (V/V_0)^{E/h\nu}$ associated with the process whereby the entropy of high-frequency radiation changes due to a change in volume.
- *And*: This is identical to the probability $W = (V/V_0)^n$ associated with the process whereby the entropy of a system consisting of n independent spatially localized components changes due to a change in volume, *provided* $E/h\nu = n$*!*

"Monochromatic radiation of low density behaves--as long as Wien's radiation formula is valid [*i.e.* at high values of frequency/temperature]--in a thermodynamic sense as if it consisted of mutually independent energy quanta of magnitude [$h\nu$]."

3. The Photoelectric Effect.

- *Experimental setup*: Cathode and annode enclosed in an evacuated tube and connected through a battery. Shine light on cathode.





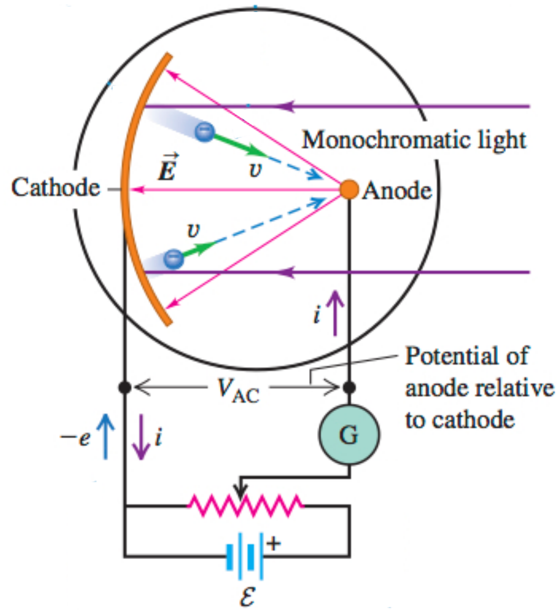
- If potential difference V_{AC} is great enough, electrons ejected from cathode can travel across to annode and produce a current.
- By reversing voltage can shut off current.
- *Stopping potential* V_0 = minimum voltage needed to shut off current = voltage needed to overcome max. kinetic energy of ejected electrons and prevent them from reaching annode: $eV_0 = K_{max}$.

How should the current depend on voltage and frequency and intensity of light?

Wave theory predictions:

- Energy depends on intensity, not frequency.

1. *Current should depend on intensity, not frequency.* **?**
   (Current should flow for any frequency.)
2. *For low-intensity light, should be time delay between when light is turned on and when current flows.* **?**
   (Electrons need time to absorb enough energy to overcome "escape energy" ("work function") ϕ associated with metal.)
3. *Stopping potential should increase with intensity.* **?**
   (Stopping potential should not depend on frequency.)



Experimental results:

1. *Current depends on frequency, not intensity.*
   (Only flows if $\nu \geq$ "threshold frequency" ν_0.)
2. *No measureable time delay between when light is turned on and when current flows.*
3. *Stopping potential depends on frequency, not intensity.*

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Experimental results:

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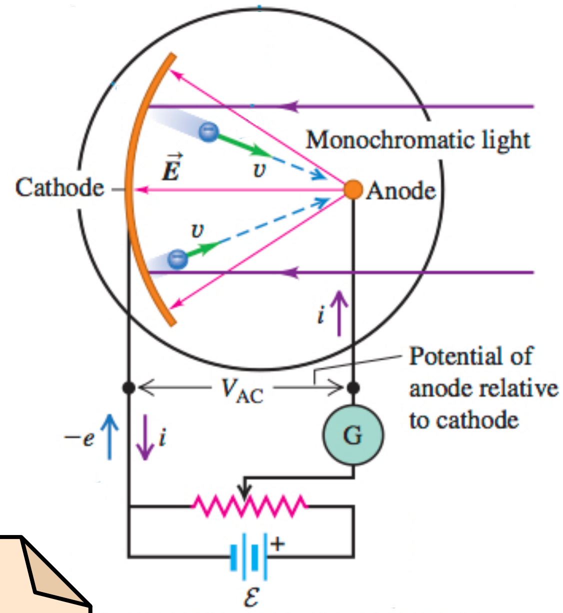


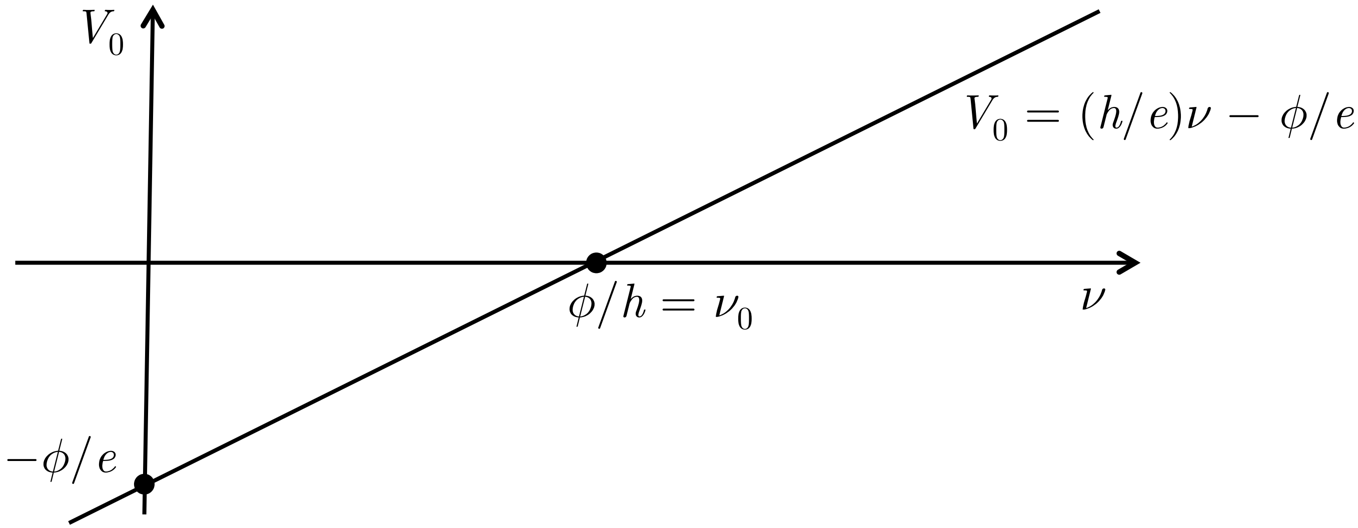
Einstein's Explanation:

- intensity = # photons (light quanta)
- frequency $\propto$ energy ($E = h\nu$)

1. "Threshold frequency" $\nu_0 = \phi/h$ is frequency of photon needed to match electron's "escape energy" ϕ.
2. No measureable time delay because as soon as a photon with threshold frequency hits an electron, it will be ejected.
3. Einstein's equation for the stopping potential:

$$eV_0 = K_{max} = h\nu - \phi$$

Millikan's (1916) Confirmation of Einstein's Explanation



"The semi-corpuscular theory by which Einstein arrived at this equation seems at present to be wholly untenable."

- 1921. Einstein's Nobel Prize: Awarded for his quantitative equation describing the photoelectric effect, and *not* the light quantum hypothesis (and not special relativity or general relativity!)

> "To have discovered the quantitative nature of the phenomenon was important enough to deserve the prize even if the discovery was made with the help of a dubious theory!" (Brush, pg. 219.)

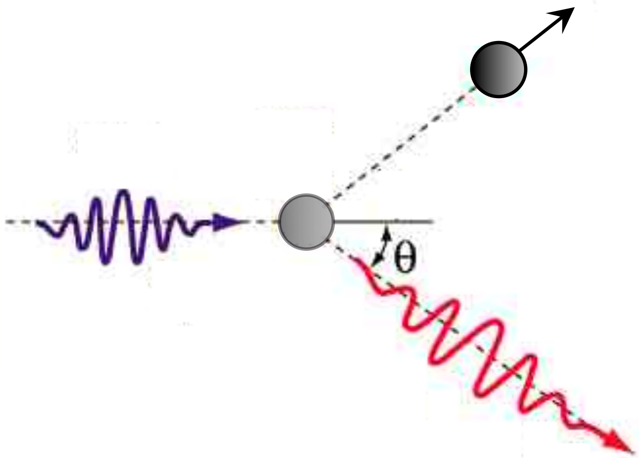
4. The Compton Effect.

Arthur Compton
(1892-1962)

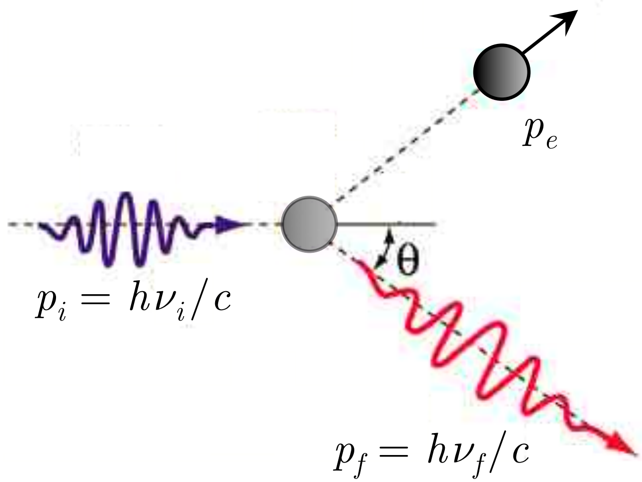
- 1922. "A Quantum Theory of the Scattering of X-rays by Light Elements".
- Describes scattering of X-rays by an electron.
- *In particular*: Electron *recoils*, and scattered X-ray has *longer wavelength* than incident X-ray.

Compton's Explanation:

- Suppose a quantum of light with frequency ν and energy E has momentum $p = E/c = h\nu/c$.
- Can now model the scattering of light by an electron as an elastic collision between a light quantum and an electron in which momentum and energy are conserved...



<u>*Conservation of momentum requires:*</u> $\mathbf{p}_i = \mathbf{p}_f + \mathbf{p}_e$



- <u>*So*</u>: $\mathbf{p}_e \cdot \mathbf{p}_e = (\mathbf{p}_i - \mathbf{p}_f) \cdot (\mathbf{p}_i - \mathbf{p}_f)$

- <u>*Or*</u>: $p_e{}^2 = p_i{}^2 + p_f^2 - p_i p_i \cos\theta$ (a)

<u>*Conservation of energy requires:*</u>

$$h\nu_i + mc^2 = h\nu_f + \sqrt{(mc^2)^2 + (p_e c)^2}$$

- <u>*Or*</u>: $p_e{}^2 = (h\nu_i/c - h\nu_f/c + m)^2 - m^2c^2$ (b)

- <u>*Now*</u>: Subtract (b) from (a):

$$\lambda_f - \lambda_i = \frac{h}{mc}(1 - \cos\theta) = \frac{2h}{mc}\sin^2(\theta/2)$$

Wavelength of light quantum $\lambda = c/\nu$

The "Compton wave-length" of an electron

- 1923. C. T. R. Wilson reports observations of recoil electrons using new cloud chamber method.

"In view of the fact that these recoil electrons were unknown at the time this theory was presented, their existence and the close agreement with the predictions as to their number, direction and velocity supplies strong evidence in favor of the fundamental hypotheses of the quantum theory of scattering." (1924.)

"Since the idea of light quanta was invented primarily to explain the photoelectric effect, the fact that it does so very well is no great evidence in its favor. the wave theory explains so satisfactorily such things as the reflection, refraction and interference of light that the rival quantum theory could not be given much credenceunless it was found to account for somenew theory for which it had not been especially designed. This is just what the quantum theory has recently accomplished in connection with the scattering of x-rays." (1925.)

- Are *novel predictions* a criterion for theory acceptance?

5. The Bohr-Kramers-Slater (BKS) Theory.

Niels Bohr
(1885-1962)

- 1924. An attempt to retain wave theory in face of Compton effect.
- *Motivation*: Similarity between Compton formula and (relativistic) Doppler formula:

Compton formula can be put into the form:

$$\nu_f = \frac{\nu_i}{1 + 2\alpha \sin^2(\theta/2)} \qquad \alpha = h\nu_i/mc^2$$

Relativistic Doppler formula can be put into the form:

$$\nu_f = \frac{\nu_i}{1 + \dfrac{2\beta}{1-\beta}\sin^2(\theta/2)} \qquad \beta = v/c$$

- *So*: Increase in wavelength of scattered X-ray can be interpreted as due to Doppler effect if $\alpha = \beta\,(1 - \beta)$ or $\beta = \alpha/(1 + \alpha)$.

- *But*: Recoil velocity of electron in Compton scattering gives:

$$\beta = 2\alpha \sin(\theta/2) \frac{\sqrt{1 + (2\alpha + \alpha^2)\sin^2(\theta/2)}}{\sqrt{1 + 2(\alpha + \alpha^2)\sin^2(\theta/2)}}$$

- *Which means*: X-ray scattering can't be due solely to Doppler effect...

- *BKS Claim*: An atom consists of virtual oscillators that oscillate at frequencies of all possible radiation that can be emitted (in form of waves).
- *And*: In Compton effect, interference between incident X-rays and virtual radiation field produces observed scattered X-rays.
- *But*: Since recoil electron's velocity is not given by Doppler effect:
  - *Energy and momentum are only conserved statistically (averaged over many electrons interacting with X-rays).*
  - *And*: *There's no correlation between scattered X-ray and recoil of electron.*
- Einstein is unamused...

"But I shouldn't let myself be pushed into renouncing strict causality before it had been defended altogether differently from anything done up to now. The idea that an electron ejected by a light ray can choose *of its own free will* the moment and direction in which it will fly off, is intolerable to me. If it comes to that, I would rather be a shoemaker or even an employee in a gambling casino than a physicist." (1924)

Experiments to test correlation between scattered X-rays and recoil of electrons:

- Bothe and Geiger (1924-1925). Use counters.
- Compton and Simon (1925). Use cloud chamber.

Result: Correlation always observed!

"These results do not appear to be reconcilable with the view of the statistical production of recoil and photoelectrons proposed by Bohr, Kramers, and Slater. They are, on the other hand, in direct support of the view that energy and momentum are conserved during the interaction between radiation and individual electrons." (1925.)

"I have received a letter from Geiger, in which he tells, that his experiment has given strong evidence of a coupling in the case of the Compton Effect. It seems thererfore that there is nothing else to do than to give our revolutionary efforts as hoourable a funeral as possible." (1925.)

6. Reception of the Light Quantum Hypothesis.

- *Question*: What role did the Compton effect play in the acceptance of the LQH?

Table 1. Supporters and Opponents of a Corpuscular Aspect of Electromagnetic Radiation Before 1923. (Numbers in parentheses indicate age in 1920, if known; * means may have died before 1920.)

Supporters	Opponents
William Henry Bragg (58)	Alfred Berthoud (46)
William Lawrence Bragg (30)	Niels Bohr (35)
Louis de Broglie (28)	Max Born (38)
Maurice de Broglie (45)	Leon Brillouin (31)
Norman Robert Campbell (40)	Arthur Holly Compton (28) CV
Daniel F. Comstock (87*)	Karl Taylor Compton (33)
James Arnold Crowther (37)	Peter Debye (36)
Arthur Stanley Eddington (38) B.S.	William Duane (48)
Paul Ehrenfest (40)	Franz Exner (71)
Albert Einstein (41)	G.W.C. Kaye (40) CV
C.D. Ellis	Max Laue (40)
Arthur Haas (36)	H.A. Lorentz (67)
Arthur Llewelyn Hughes (37)	Robert A. Millikan (52)
James Jeans (43)	J.W. Nicolson (39)
Abram Joffe (40)	Max Planck (62)
G.W.C. Kaye (40) CV	O.W. Richardson (41)
H.A. Kramers (26) B.S.	Arnold Sommerfeld (52)
Rudolf Ladenburg (38)	Siegfried Valentiner (44)
Oliver Lodge (69)	
D.V. Mallik (54)	
Walther Nernst (56)	
Fritz Reiche (37)	
Erwin Schrödinger (33) B.S.	
Johannes Stark (46)	
J.J. Thomson (64)	
Leonard T. Troland	
Mieczyslaw Wolfke	
Robert W. Wood (52)	

Table 2. Acceptance of Particle Nature of Radiation*

	Favorable			Unfavorable		Total
Monographs, technical reviews	++	+	0	–	--	N
1916	2	1	0	0	0	3
1917	0	0	0	1	0	1
1918	1	0	0	0	1	2
1919	0	0	2	0	1	3
1920	0	1	2	1	1	5
Subtotal for 1916–20	3	2	4	2	3	14
1921	2	4	2	3	0	11
1922	2	1	1	4	1	9
1923	0	4	0	1	0	5
1924	2	3	0	2	1	8
1925	1	2	0	1	0	4
Subtotal for 1921–25	7	14	3	11	2	37
1926	3	2	0	1	0	6
1927	1	3	2	1	1	8
1928	1	6	0	0	0	7
1929	0	5	0	0	0	5
1930	2	3	0	0	0	5
**Subtotal for 1926–30	7	19	2	2	1	31
1931	0	1	0	0	0	1
1932	0	4	0	1	0	5
1933	0	4	0	0	0	4
1934	0	1	0	0	0	1
1935	0	2	0	0	0	2
Subtotal for 1931–35	0	12	0	1	0	13
Textbooks, popular articles	++	+	0	–	--	N
1916	0	0	0	4	0	4
1917	0	1	2	2	0	5
1918	0	0	0	1	1	2
1919	0	0	1	3	0	4
1920	0	0	1	2	0	3
Subtotal for 1916–20	0	1	4	12	1	18

Table 2. (Continued)

	Favorable			Unfavorable		Total
1921	2	0	0	3	0	5
1922	0	0	0	0	1	1
1923	2	0	0	3	0	5
1924	0	3	0	5	3	11
1925	1	0	1	1	1	4
Subtotal for 1921–25	5	3	1	12	5	26
1926	3	7	0	1	0	11
1927	1	7	0	3	0	11
1928	1	4	3	5	0	13
1929	4	5	0	4	0	13
1930	3	10	0	3	0	16
**Subtotal for 1926–30	13	33	3	16	0	64
1931	1	4	0	1	0	6
1932	5	3	1	2	0	11
1933	1	8	0	1	0	10
1934	3	9	0	1	0	13
1935	2	10	1	2	0	15
Subtotal for 1931–35	12	34	2	7	0	55

*Symbols: ++ = strongly supports LQH; + = leans toward LQH; 0 = neutral; – = leans against LQH (or doesn't mention it but supports wave theory of light); -- = strongly rejects LQH

Table 3. Evidence for Particle Nature of Radiation**

	P	P > C	C = P	0	C > P	C	N
Monographs, technical reviews							
1916	3	0	0	0	0	0	3
1917	0	0	0	1	0	0	1
1918	1	0	0	1	0	0	2
1919	0	0	0	3	0	0	3
1920	1	0	0	4	0	0	5
Subtotal for 1916–20	5	0	0	9	0	0	14
1921	4	0	0	7	0	0	11
1922	3	0	0	6	0	0	9
1923	2	0	0	3	0	0	5
1924	3	0	0	3	2	0	8
1925	3	0	0	1	0	0	4
Subtotal for 1921–25	15	0	0	20	2	0	37
1926	0	1	1	1	1	2	6
1927	0	0	1	7	0	0	8
1928	1	2	1	0	0	3	7
1929	0	0	2	0	1	2	5
1930	0	1	1	0	0	3	5
Subtotal for 1926–30	1	4	6	8	3	9	31
1931	0	0	1	0	0	0	1
1932	0	1	1	2	1	0	5
1933	1	1	2	0	0	0	4
1934	0	0	0	0	1	0	1
1935	0	0	0	0	2	0	2
Subtotal for 1931–35	1	2	4	2	4	0	13
Textbooks, popular articles							
1921	2	0	0	3	0	0	5
1922	0	0	0	1	0	0	1
1923	2	0	0	3	0	0	5
1924	2	0	0	9	0	0	11
1925	0	0	0	4	0	0	4
Subtotal for 1921–25	6	0	0	20	0	0	26

Table 3. (Continued)

	P	P > C	C = P	0	C > P	C	N
Textbooks, popular articles							
1926	3	2	1	3	1	1	11
1927	7	0	0	4	0	0	11
1928	1	2	2	8	0	0	13
1929	2	0	3	3	4	1	13
1930	4	4	2	3	1	2	16
Subtotal for 1926–30	17	8	8	21	6	4	64
1931	2	0	0	2	2	0	6
1932	0	2	2	3	1	3	11
1933	3	0	3	1	1	2	10
1934	4	0	4	1	3	1	13
1935	4	3	3	2	2	0	15
Subtotal for 1931–35	13	5	12	12	9	6	55

**Symbols: P = only photoelectric effect is mentioned; P > C = photoelectric is stronger evidence than Compton effect; C = P, the two are equally strong; 0 = neither effect mentioned or particle nature of light rejected; C > P = Compton effect is stronger evidence than photoelectric; C = only Compton effect is mentioned.

- Are novel predictions (*viz.*, recoil electrons) a criterion for theory acceptance?

"...only a few authors even mentioned the fact that Compton had predicted recoil electrons, and none of them stated that his theory was more likely to be valid *because* he predicted them *before* they were discovered." (Brush, pg. 241.)

"I suggest three major facts, to which each physicist might give a different weight, but all of which were needed to explain the conversion of (almost) the entire community: (1) the Compton effect; (2) the photoelectric effect; (3) all the other phenomena, especially those involving *x*-rays, specific heats of solids at low temperatures, and atomic spectra, which could not plausibly be explained by a wave theory but could (more or less accurately) be explained by some kind of quantum theory." (Brush, pg. 245.)