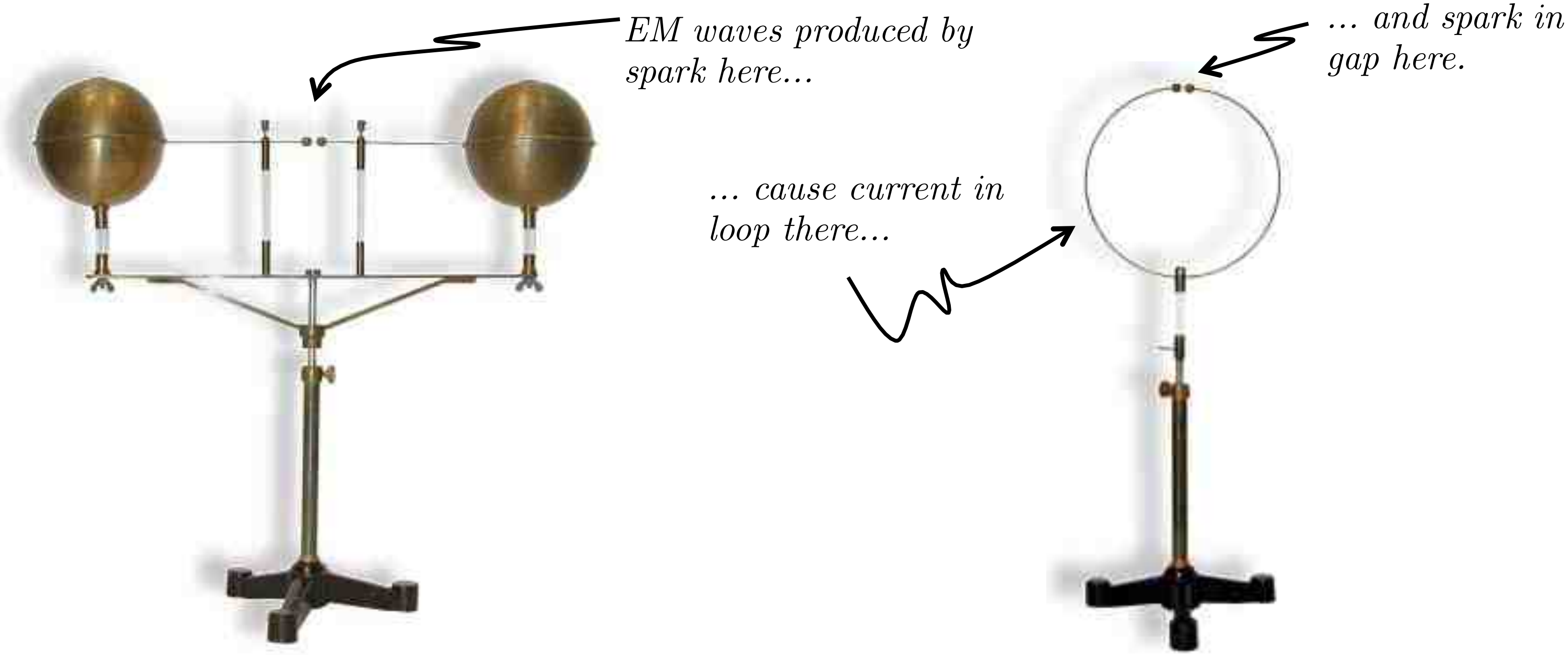
17. Quantum Theory and the Electromagnetic Worldview. Seth (2004)

1. The Electromagnetic Worldview.

- 1873: Maxwell's *A Treatise on Electricity and Magnetism.*
- 1888: H. Hertz generates and detects electromagnetic waves.

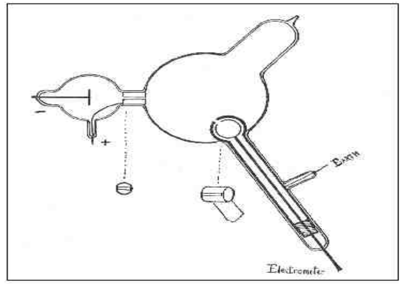
Heinrich Hertz



Hertzian radiator

Hertzian resonator

- 1892: H. A. Lorentz's electron theory.



H. A. Lorentz

- 1897-98: J. J. Thompson measures charge-to-mass ratio of electron.

- 1901: W. Wein

All mechanics can be reduced to electromagnetic theory.

<u>*Characteristics:*</u>

- ◦ A distaste for, and mistrust of, mechanical modeling, especially as applied to microscopic phenomena;
- ◦ a belief that the only physical realities were electromagnetic in nature;
- ◦ a programmatic commitment toward a "concentration of effort on problems whose solution promised to secure a universal physics based solely on electromagnetic laws and concepts".

2. Sommerfeld's 1907 Lectures on Planck's Radiation Theory.

Arnold Sommerfeld
(1868-1951)

Max Planck
(1858-1947)

- 1906: Planck's *Vorlesungen über die Theorie der Wärmestrahlung* (*The Theory of Heat Radiation*).

- ***Goal***: Derive a formula for the energy distribution of "black-body radiation".
  - *Black-body radiation* = thermal radiation emitted by a perfectly absorbing object.

- ***Initial question***: How should a black-body in thermal equilibirum be modeled?

Story to come:

- Rayleigh and Jeans use *electrodynamic analysis* and obtain a formula that is inconsistent with experimental data.
- Planck obtains a formula that fits the experimental data, and then works backwards to justify it on the basis of *thermodynamic and statistical* principles (*requires assumption of "discontinuity" about the nature of energy*).
- Sommerfeld's electromagnetic worldview orients him favorably towards Rayleigh-Jeans and away from the principled approach of Planck.

Rayleigh and Jeans (1900, 1905):

- Suppose black-body is a cavity that traps EM waves in the form of standing waves with different modes.
- *Then*:

$$\frac{\textit{number of modes per unit frequency}}{\textit{volume of cavity}} = \frac{8\pi\nu^2}{c^3}$$

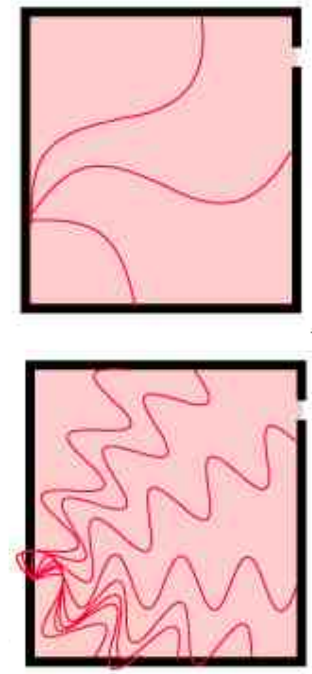
- *Now*: In equilibirum, the average energy per mode is kT.

> *Why*? The Equipartition Theorem says that a gas in *equilibrium* has an average energy of ½kT for each degree of freedom.
>
> *And*: R&J consider the trapped EM waves to have normal modes with two degrees of freedom.

- *So*:

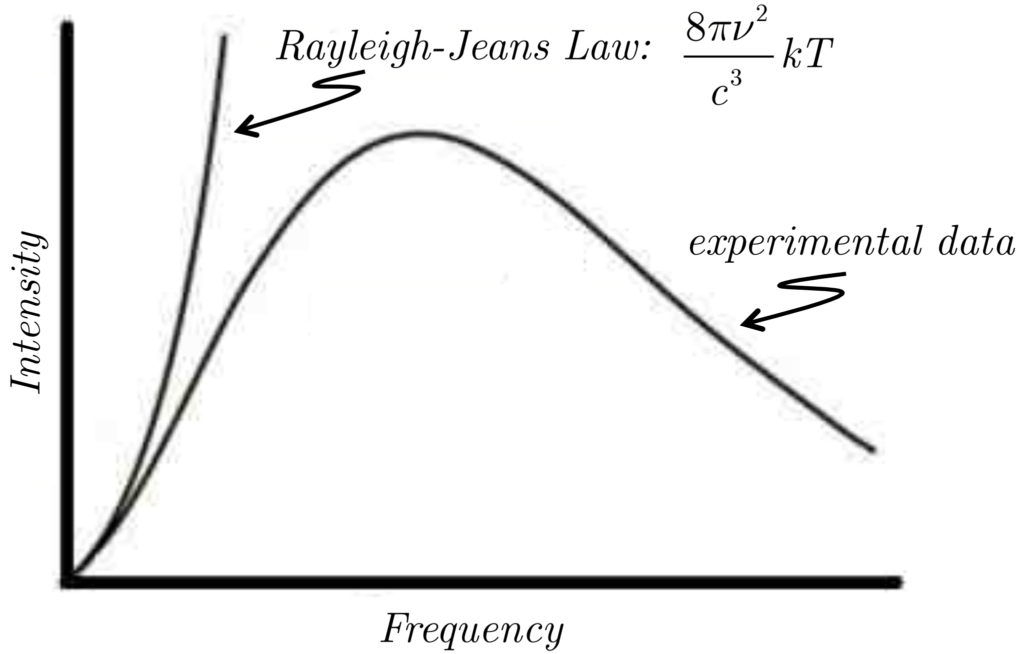
> **Rayleigh-Jeans Law**
>
> $$\frac{\textit{energy per unit frequency}}{\textit{volume of cavity}} = \rho(\nu, T) = \frac{8\pi\nu^2}{c^3} kT$$

- *But*: *Experimental data on black-body radiation indicate this law is wrong!*



*The higher the frequency, the more modes you can fit into the cavity.**

*Diagrams from http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html#c2



- *Planck*: The data fit the following distribution:

Planck Law

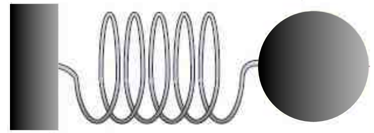
$$\frac{\textit{energy per unit frequency}}{\textit{volume of cavity}} = \rho(\nu,T) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT}-1}$$

$h = \textit{constant}$

- *And*: This requires that the average energy per mode is $\dfrac{h\nu}{e^{h\nu/kT}-1}$ and *not* kT.

- *How can this disregard for the Equipartition Theorem be justified?*

Planck's Model for Black-Body Radiation (1900)



- Suppose the walls of the black-body cavity are composed of "Hertzian resonators" (*i.e.*, little oscillators).

- ***Then (1899)***: $\rho(\nu, T) = \dfrac{8\pi\nu^2}{c^3} \times \left(\begin{array}{l} \textit{average energy} \\ \textit{of resonator} \end{array} \right)$

- ***So***: Planck Law entails the *average energy* of a resonator is $\dfrac{h\nu}{e^{h\nu/kT} - 1}$.

- The *average entropy* S of a resonator with this energy E is defined by the *thermodynamic* relation $1/T = \partial S/\partial E$.

- ***And***: This is the same form as the entropy for the system derived by *statistical* considerations (*a la* Boltzmann), provided that the following relation holds for the average energy of a resonator:

> **Planck's quantum hypothesis**
>
> $E = n\epsilon$, where $\epsilon = h\nu$ and $n = 1, 2, 3, \ldots$

- ***What this means***: Planck can derive the experimentally correct energy distribution for black-body radiation (Planck's Law) provided he assumes the energy of the resonators is "quantized" in discrete units of $h\nu$.

Planck's Derivation of Planck's Law.

- 1900 lecture:

"The most essential point of the whole calculation" is the postulate that the energy of N resonators of frequency ν is made up entirely of "an entirely determinate number of finite equal parts" the size of which is determined by the "natural constant" h so that the "energy element" ϵ is equal to $h\nu$.

- *Planck asks*: How many ways W can $P = E/\epsilon$ such energy elements be divided among N resonators?

$$\epsilon \mid \epsilon\epsilon\epsilon \mid \epsilon\epsilon \mid \epsilon\epsilon\epsilon\epsilon \ \ldots \mid \epsilon\epsilon$$

- *There are $N-1$ dividers $|$ and P energy elements ϵ.*
- *If all these symbols are distinguishable, then there are $(P+N-1)!$ ways of ordering them.*
- *But the ϵ's are indistinguable and so are the $|$'s; so we've over-counted by a factor of $(N-1)!P!$.*

- *So*: $$W = \frac{(N+P-1)!}{(N-1)!\,P!} \approx \frac{(N+P)^{N+P}}{N^N P^P}$$

Stirling's approximation:
$N! \approx N^N$, *for large N.*

- *Boltzmann sez*: The entropy S of an energy distribution among states with W possible arrangements of energy is given by $S = k\log W + \mathit{const.}$

- *So*: $S = k\log\left(\dfrac{(N+P)^{N+P}}{N^N P^P}\right)$

$$= N\left\{\left(1+\frac{E}{\epsilon}\right)k\log\left(1+\frac{E}{\epsilon}\right) - \frac{E}{\epsilon}k\log\left(\frac{E}{\epsilon}\right) + \mathit{const.}\right\}$$

where $P = NE/\epsilon$, and E is the average energy of a resonator.

- *Thus*: The average entropy $S_{ave} = S/N$ of a resonator is given by,

$$S_{ave} = \left(1+\frac{E}{h\nu}\right)k\log\left(1+\frac{E}{h\nu}\right) - \frac{E}{h\nu}k\log\left(\frac{E}{h\nu}\right) + \mathit{const.}$$

- *And*: This obeys the thermodynamic relation $dS/dE = 1/T$.

- Which entails $E = \dfrac{h\nu}{e^{h\nu/kT} - 1}$.

- *Recall:* $\rho = \dfrac{8\pi\nu^2}{c^3} \times E$

Planck's (1899) result for the energy distribution of the resonator system.

- *And:* This gives the Planck Law: $\rho = \dfrac{8\pi\nu^2}{c^3}\dfrac{h\nu}{e^{h\nu/kT} - 1}$

Sommerfeld's Critique of Planck

- Resonator model can't guarantee cavity is in equilibrium.
  - Interaction between resonators is meant to bring system into equilibrium.
  - *But*: Only resonators at same frequency can interact.
- To demonstrate that Planck's formula is the equilibrium energy distribution, it must be shown to maximize the entropy.
  - *But*: Planck's formula results from his quantum hypothesis alone.
- Moreover, Lorentz (1908) showed that the Rayleigh-Jeans formula is entailed by his electron theory.

"I think it is very possible that Planck's formula is only a good approximation."

"In deciding which theory to reject, the impotence of Planck's resonators outweighed the failure of the Rayleigh-Jeans equation to match available experimental results. ...The choice between Planck's and Jean's formulas was, rather, framed as a choice between two distinct *methods*." (Seth, pg. 77.)

- Kuhn (1987): Lorentz' derivation of the Rayleigh-Jeans formula from the electron theory marked the beginning of acceptance of the quantum discontinuity.

"We can adopt it [Planck's formula] only by altering profoundly our fundamental conceptions of electromagnetic phenomena."

"Proponents of the electromagnetic worldview... may not have regarded the choice between continuity and discontinuity as the central issue. Rather, the question that 'came to challenge' them... was whether the electron theory could produce a Planck-like formula. Once it was accepted that this was impossible, discontinuity was adopted quite readily by this group." (Seth, pg. 80.)

3. 1911 Solvay Conference in Brussels.

- Planck and Sommerfeld offer different approaches to the quantum hypothesis.

"Sommerfeld's statement leads to a finite element of action, Planck's statement to a finite element of phase space; both things seem to be fundamentaly different to me; one is of a dynamical, the other of a statistical nature." (Paul Langevin)

Sommerfeld's Dynamical Version of the Quantum Hypothesis.

- *Implicit motive*: Show how electromagnetism and the quantum hypothesis are *compatible.*
- *Recall*: Planck's (1900) quantum hypothesis states $E = nh\nu$, $n = 1, 2, 3, \ldots$
- *Which means*: Planck's constant $h = (\mathit{energy}) \times (\mathit{time})$.

"Phrased completely generally, a large quantity of energy in a shorter time, a smaller in a longer time is taken up and given out by matter, so that the product of energy and time, or (closer to the definition) the time intergral of the energy is determined through the magnitude of h."

- *Or*:

Quantum Hypothesis (Sommerfeld version)

$$\int_0^\tau H\,dt = \frac{h}{2\pi}$$

H = Hamiltonian function describing the energy of a system.

"...with every purely molecular process a fixed, universal amount of action [$(\mathit{energy}) \times (\mathit{time})$] is taken up or given out from the atom."

Characteristics of Sommerfeld's version of quantum hypothesis:

(a) Rooted in areas of research to which he had been devoted for more than a decade:

- 1911 Solvay paper: New version of quantum hypothesis used to calculate formula for ratio of polarized energy to total energy of X-rays.
- Extends earlier Habilitationschrift (1895) on diffraction of X-rays.

(b) Not a deduction from general axioms, nor a definition; but following from the consideration of specific problems.

(c) Used in conjunction with data in process of constructing mathematical expressions.

(d) Illuminates the shift in adherence to the electromagnetic worldview.

- No contradiction between Sommerfeld's quantum hypothesis and electrodynamics:

"The opposition between our application of the quantum of action and Planck's method of the energy quantum has claimed much of our attention. Both depictions are foreign to classical electrodynamics and mechanics. But while our version is reconcilable with electrodynamics, the original depiction by Planck stands in unmistakable opposition to it."

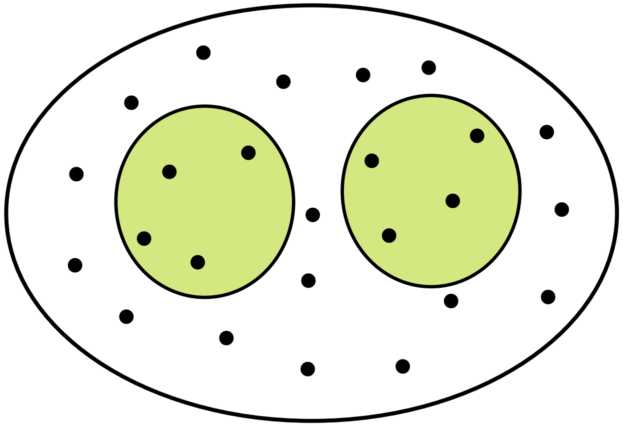
"Radiation theory and electromagnetism do not exclude, but rather complement one another."

"The split, that is, was not (as it would eventually become) between 'classical' and 'modern' physics, but between quantum and electrodynamic theory. Unlike the classical/modern dichotomy, the electromagnetic/quantum one was explicitly and necessarily not an either/or. In Sommerfeld's vision, both the quantum and the electromagnetic field were required to understand the physical world." (Seth, pg. 90.)

Planck's (1911) Statistical Version of the Quantum Hypothesis.

- *Phase space* = Space of all possible states of a physical system.

- *Liouville's Theorem:* Areas of equal size in phase space are equiprobable.



- *Planck:* Let $dqdp$ be an "elementary area of probability" (*i.e.*, the area of a very small region of phase space).

Each point represents a possible state, labeled by particular values of position q and momentum p.

  - If $dqdp$ is allowed to be infinitely small (if phase space is a continuum), then Rayleigh-Jeans' Law results from phase space analysis of black-body radiation.
  - If $dqdp$ is constrained to be no smaller than h, then Planck's Law can be derived.

- *So*:

Quantum Hypothesis (Planck 1911 version)

$$\int dqdp = h$$

Difference between 1900 and 1911 version:

"Above all things it is to be emphasized that, at least in my opinion, the quantum hypothesis is no energy hypothesis, but is an action hypothesis."

Planck on Sommerfeld's version:

"Until the definitive statement of such a dynamical law is achieved, it appears to me safer to restrict oneself to a statistical formulation of the quantum hypothesis..."