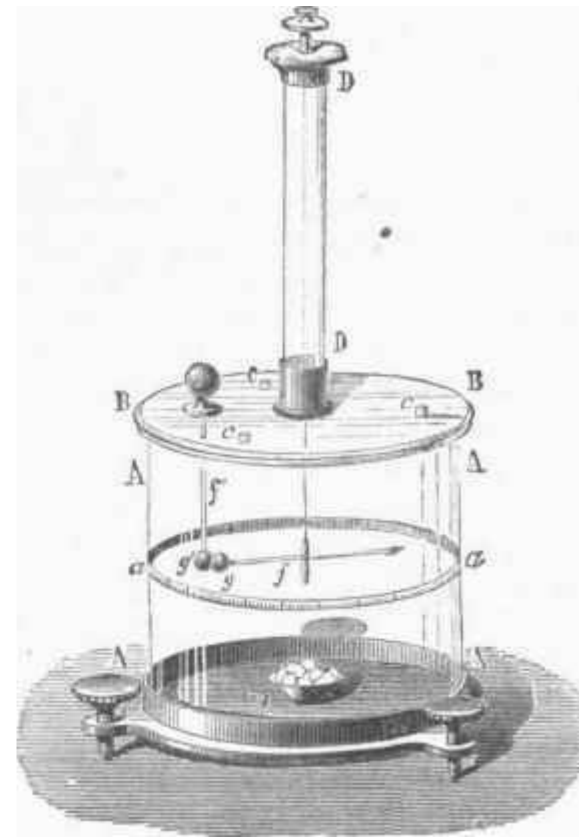
08. Ampère and Faraday

Darrigol (2000), Chap 1.

A. Pre-1820.

(1) Electrostatics (frictional electricity)



- 1780s. Coulomb's description:
  - Two electric fluids: positive and negative.
  - Inverse square law:

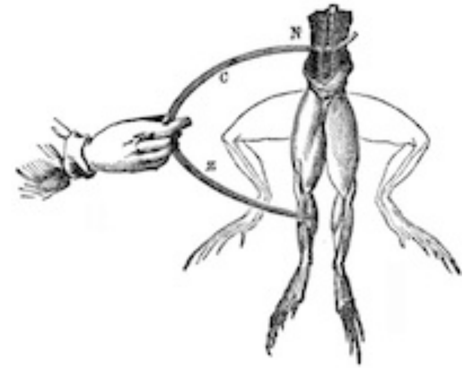
Charles-Augustin de Coulomb
(1736-1806)

> It follows therefore from these three tests, that the repulsive force that the two balls -- [which were] electrified with the same kind of electricity -- exert on each other, follows the inverse proportion of the square of the distance.""

(2) Magnetism: Coulomb's description:

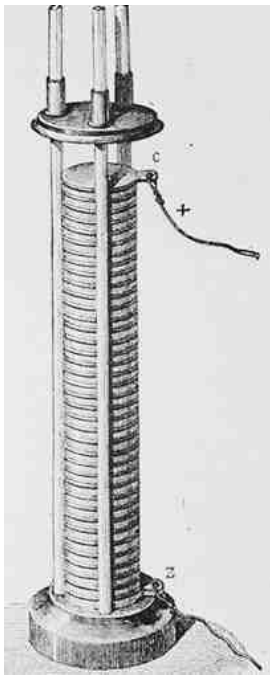
- Two fluids ("astral" and "boreal") obeying inverse square law.
- No magnetic monopoles: fluids are imprisoned in molecules of magnetic bodies.

(3) Galvanism

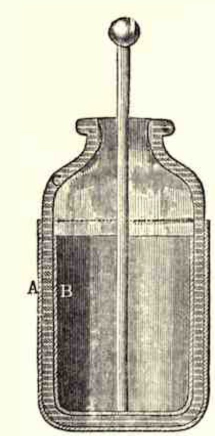


Luigi Galvani
(1737-1798)

- 1770s. Galvani's frog legs. "Animal electricity": phenomenon belongs to biology.
- 1800. Volta's ("volatic") pile.



- *Pile consists of alternating copper and zinc plates separated by brine-soaked cloth.*
- *A "battery" of Leyden jars that can spontaeously recharge themselves.*



1745 Leyden jar.

- *Charged rod connected to inner foil.*
- *Outer foil grounded.*
- *Inner and outer surfaces store equal but opposite charges.*

Alessandro Volta
(1745-1827)

- *Volta*: Pile is an electric phenomenon and belongs to physics.
- *But*: Nicholson and Carlisle use voltaic current to decompose water into hydrogen and oxygen. Pile belongs to chemistry!

- Are electricity and magnetism different phenomena?
  - *Electricity involves violent actions and effects: sparks, thunder, etc.*
  - *Magnetism is more quiet...*
- 1820. Oersted's *Experimenta circa effectum conflictus elecrici in acum magneticam* ("Experiments on the effect of an electric conflict on the magnetic needle").
  - *Galvanic current = an "electric conflict" between decompositions and recompositions of positive and negative electricities.*
  - *Experiments with a galvanic source, connecting wire, and rotating magnetic needle: Needle moves in presence of pile!*

Hans Christian Oersted
(1777-1851)

__Oersted's Claims__

- Electric conflict acts on magnetic poles.
- Electric conflict is not confined within the conductor, but also acts in the vicinity of the conductor.
- Electric conflict forms a vortex around the wire.

"Otherwise one could not understand how the same portion of the wire drives the magnetic pole toward the East when placed above it and drives it toward the West when placed under it. An opposite action at the ends of the same diameter is the distinctive feature of vortices."

B. Ampère's Attractions.

- 1820. Ampère's astatic needle: plane of rotation perpendicular to action of the Earth.
  - *Concept of a "circuit": closed "electric current".*
  - *"Galvanometer" = current detector.*

> "...for one who tries to explain the South-North orientation [of a magnetic needle], would not it be the simplest idea to assume in the Earth an electric current?"

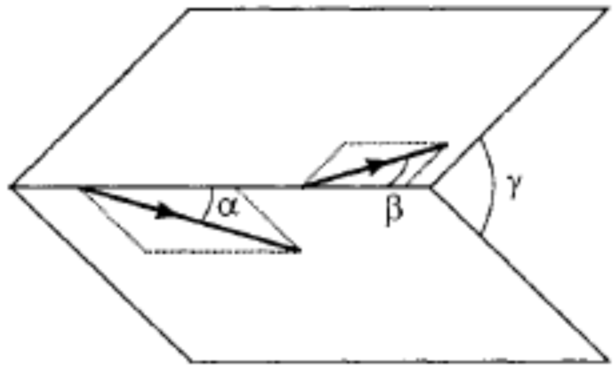
André-Marie Ampère
(1775-1836)

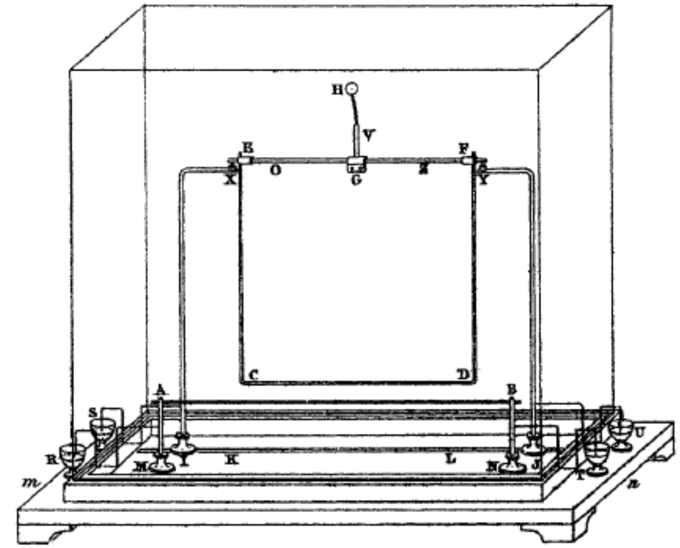
- *Analogously*: Every magnet owes its properties to the existence of closed currents in its mass ("Ampèrean currents").
- *And*: Electric currents have to possess all the properties of a magnet.

Analytic description of interaction between "current elements"

- *Experimental guess*: Force between elements AB and CD proportional to

$$\frac{\cos\gamma\sin\alpha\sin\beta}{r^2}$$





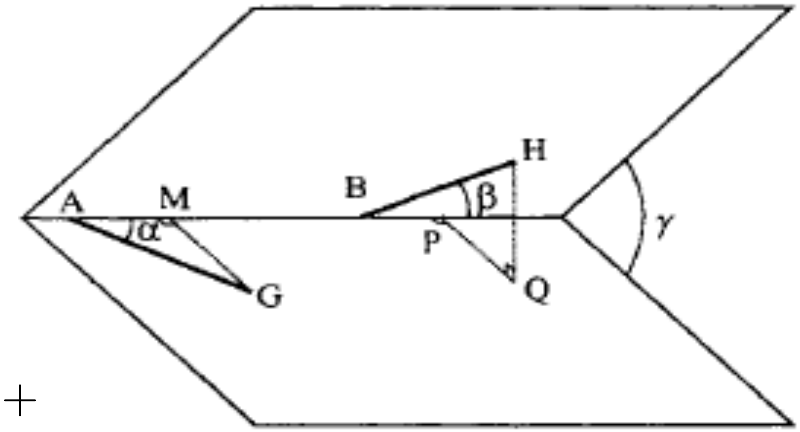
> *General Principle*:
>
> Any two infinitely short currents with the same extremities are equivalent, no matter how contorted they may be.

- *Claim*: This requires force law to be modified to:

> *Ampere's Force Law*:
>
> $$f \propto \frac{\sin\alpha\sin\beta\cos\gamma + k\cos\alpha\cos\beta}{r^2}, \qquad k = const.$$

Proof:

- Element AG decomposes into AM and MG.
- Element BH decomposes into BP, PQ, QH.



- *So*: General Principle entails that force

$$(AG \to BH) = (AM \to BP) + (AM \to PQ) + (AM \to QH) + (MG \to BP) + (MG \to PQ) + (MG \to QH).$$

- *Let*: $m =$ *force between parallel unit current elements that are perpendicular to line joining their center.*

  $n =$ *force between parallel unit current elements that are on line joining their center.*

- *Then*: $(AM \to BP) \propto n \cos\alpha \cos\beta$

  $(MG \to PQ) \propto m \sin\alpha \sin\beta \cos\gamma$

- *And*: All other forces vanish by symmetry.
- *So*: $(AG \to BH) \propto m \sin\alpha \sin\beta \cos\gamma + n \cos\alpha \cos\beta$.
- *Or*: $(AG \to BH) \propto \sin\alpha \sin\beta \cos\gamma + k \cos\alpha \cos\beta$, for $k = n/m$.

Characteristics of Ampère's Experiments

- "...the more definite devices were a direct expression of his theoretical beliefs within material constraints... In general, he knew the results of his experiments in advance." (Darrigol, pg. 13.)
- Experiments as reifications of preconceived theoretical ideas.
- *Reactions of others*:

Laplace

"Monsieur Ampère is so clumsy that when his apparatus does not move, he reportedly pushes to shift it."

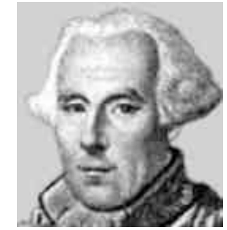
"Hardly any of his experiments succeeded... He is dreadfully confused and is equally unskillful as an experimenter and as a debater."

Oersted

- "[His] ingenious instruments required skilled workers and fairly high expenses... In my opinion we do a favor to Science when we try to diminish the material obstacles that we encounter in our researches andmake it possible for a great number of people to study a new experiment: we thus give better chances to new discoveries." (Charles-Gaspard de la Rive.)

Ampère's "most dangerous critic"

- 1820. Biot and Savart propose law governing action of a current-carrying wire upon a magnetic pole:



Félix Savart
(1791-1841)

Biot

> "When an indefinite connecting wire, traversed by the voltaic current, acts upon an element of austral or boreal magnetism... the resultant of the actions which it exerts is perpendicular to the shortest distance between the element and the wire."

> "...the total action of the connecting wire on the magnetic element is reciprocal to the rectilinear distance between the element and the wire."

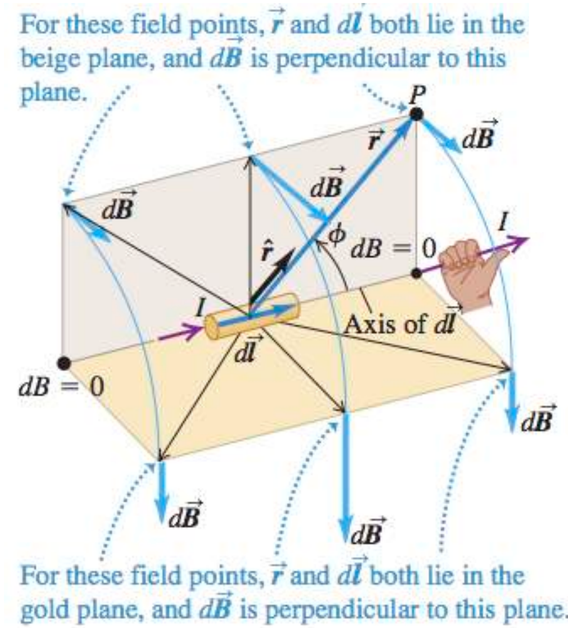
> "If an observer be supposed to occupy the place of the wire with his head at the copper and his feet at the zinc extremity, his face being turned towards the needle, the force which proceeds from the wire will cause the elements of austral magnetism to tend from the right hand of this observer to the left and the elements of boreal magnetism to tend towards his right..."

- *Laplace's infinitesimal version*: Strength of interaction between current element and magnetic pole is proportional to $\sin\phi / r^2$, where r is separation distance and ϕ is angle between direction of current and line joining element and pole.

Biot-Savart Force Law (modern rendition)

- The "Biot-Savart Law" determines the *magnetic field* at a point due to a current element Idl:

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} \qquad \text{or} \qquad d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$$



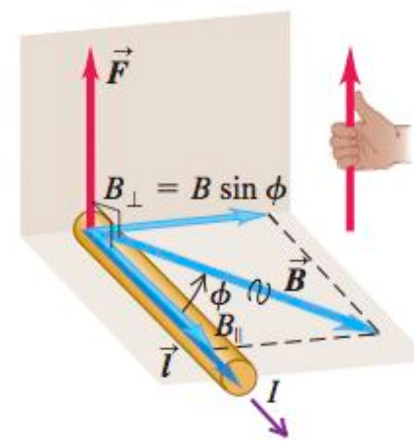
- The "Biot-Savart Force Law" determines the *magnetic force* on a current element Idl due to another current element $I'dl'$:

$$d\vec{F} = Id\vec{l}\times d\vec{B} = Id\vec{l}\times\frac{\mu_0}{4\pi}\frac{I'd\vec{l}'\times\hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{II'}{r^2}\left[d\vec{l}\times\left(d\vec{l}'\times\hat{r}\right)\right]$$

$$= \frac{\mu_0}{4\pi}\frac{II'}{r^2}\left[d\vec{l}'\left(d\vec{l}\cdot\hat{r}\right)-\hat{r}\left(d\vec{l}\cdot d\vec{l}'\right)\right]$$

Recall: BAC-CAB Rule:
$\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}\left(\vec{A}\cdot\vec{B}\right)$



C. Faraday's Rotations.

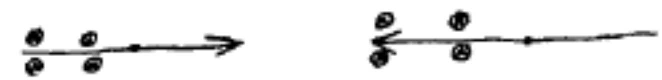
- 1820. Experiments with Humphrey Davy at Royal Institute.
- *Observes*: One pole of magnetic needle placed under current conducting wire is "strongly attracted" by the wire and remains in contact with it.
- *Claim*: Wire becomes magnetic.
  - *Why*? Iron filings sprayed on wire stick.

Michael Faraday
(1791-1867)

"There are many arguments in favour of the materiality of electricity, and but few against it; but still it is only a supposition; and it will be well to remember, while focusing on the subject of electro-magnetism, that we have no proof of the materiality of electricity, or of the existence of any current through the wire."

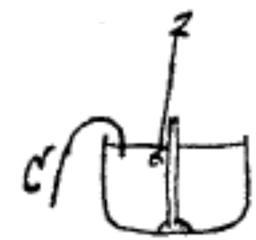
1821. Experiments with vertical wire and suspended magnetic needle.

- *Observes*: On a given side of the needle, attraction turns into repulsion (or *vice versa*) when wire passes a point between center and extermity of needle.



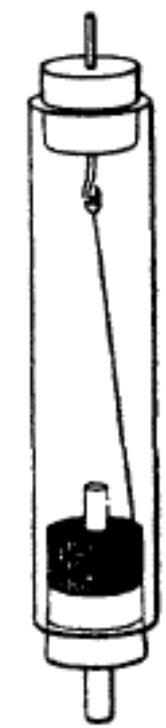
- *Suspects*: Motion of a free wire around a fixed magnetic pole is circular.

- *Confirms*: In certain configurations, continuous rotation of the wire is observed.



- *Infers*: Rotations connect the various facts of electromagnetism: All attractions and repulsions derive from the simple fact of rotation of a pole around a wire.

- *Marketing strategy*: Mails critics and supporters portable version of experiment.



New terms in Faraday's descriptions:

- "Pole" = center of action (no longer specific to magnets; can be produced by electric currents, too).
- "Power" = portion of space from which specific actions emante.

"It has been allowed, I believe, by all who have experimented on these phenomena, that the similar powers repel and the dissimilar powers attract each other; and that, whether they exist in the poles of the magnet or in the opposite sides of conducting wires."

"With regard to the opposite sides of the connecting wires, and the powers emanating from them, I have merely spoken of them as two, to distinguish the one set of effects from the other."

- "Faraday thus filled the space around wires and magnets with powers, virtual rotations, and eventually iron filings. In contrast, he left the internal state of wires and magnet undetermined." (Darrigol, pg. 20.)

D. Electro-dynamique.

"Metaphysics was filling my head. However, since Faraday's memoir has appeared, all my dreams are about electric currents."

- *But*:

"These facts comply with the general laws of physics, and one does not have to admit as *a simple primitive fact, a revolutive action* of which nature gives no other example and which we find it difficult to consider as such."

- Primitive rotations betray every principle of French Newtonian physics:
  - *No basis for calculations.*
  - *Heterogeneous entities (poles and currents).*
  - *Contradict principle of action and reaction (net torque acts on pole-current system).*

- *Faraday unfazed*:

"It is quite comfortable to find that experiment needs not quail before mathematics but is quite competent to rival it in discovery."

- Ampère's "electro-dynamique" = new science of interacting currents.

- 1822. Modified formula for force between two current elements:

$$d^2 f = ii' \frac{ds ds'}{r^2}\left(\sin\alpha \sin\beta \cos\gamma - \frac{1}{2}\cos\alpha\cos\beta\right)$$

- ds, ds' = elements; i, i' = currents.
- _Entails:_ Two current elements on the same straight line and with same orientation repel each other.

- _Can be rewritten as_:

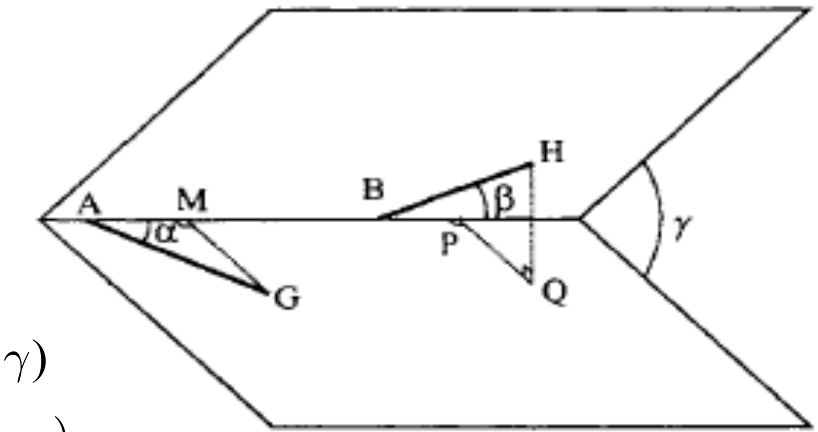
Ampère's Force Law (final version):

$$d^2 f = -ii' \frac{ds ds'}{r^2}\left(r\frac{\partial^2 r}{\partial s \partial s'} - \frac{1}{2}\frac{\partial r}{\partial s}\frac{\partial r}{\partial s'}\right)$$

Proof (contemporary):

- *Let*: $\vec{l}, \vec{l}' =$ position vectors of elements ds, ds'

  $\vec{r} = \left(\vec{l} - \vec{l}'\right)$



- *Then*: $\vec{r} \cdot d\vec{l} = r ds \cos\alpha$

  $\vec{r} \cdot d\vec{l}' = r ds' \cos\beta$

  $d\vec{l} \cdot d\vec{l}' = ds ds' (\cos\alpha \cos\beta + \sin\alpha \sin\beta \cos\gamma)$

- *So*: $$d^2\vec{f} = ii' \frac{ds ds'}{r^2}\left(\sin\alpha \sin\beta \cos\gamma - \frac{1}{2}\cos\alpha\cos\beta\right)\hat{r}$$

  $$= -ii'\left(\frac{d\vec{l} \cdot d\vec{l}'}{r^2} - \frac{3}{2}\frac{(\vec{r} \cdot d\vec{l})(\vec{r} \cdot d\vec{l}')}{r^4}\right)\hat{r} = -ii'\frac{\hat{r}}{r^2}\left(d\vec{l} \cdot d\vec{l}' - \frac{3}{2}(\hat{r} \cdot d\vec{l})(\hat{r} \cdot d\vec{l}')\right)$$

- *Now*: $\cos\alpha = \dfrac{\partial r}{\partial s}, \;\; \cos\beta = \dfrac{\partial r}{\partial s'}$

- *So*: $\vec{r} \cdot d\vec{l} = r ds \dfrac{\partial r}{\partial s}, \;\; \vec{r} \cdot d\vec{l}' = r ds' \dfrac{\partial r}{\partial s'}, \;\; d\vec{l} \cdot d\vec{l}' = ds ds' \left(\dfrac{\partial r}{\partial s}\dfrac{\partial r}{\partial s'} + r \dfrac{\partial^2 r}{\partial s \partial s'}\right)$

- *Thus*: $$d^2 f = -ii' \frac{ds ds'}{r^2}\left(r\frac{\partial^2 r}{\partial s \partial s'} - \frac{1}{2}\frac{\partial r}{\partial s}\frac{\partial r}{\partial s'}\right)$$

Comparison: Ampère's Force Law vs. Biot-Savart's Force Law

- Ampère's Force Law can be expressed as:

$$d\vec{F}_A = \frac{\mu_0 I I'}{4\pi}\frac{\hat{r}}{r^2}\left[3\left(\hat{r}\cdot d\vec{l}\right)\left(\hat{r}\cdot d\vec{l}'\right) - 2\left(d\vec{l}\cdot d\vec{l}'\right)\right]$$

- Biot-Savart's Force Law can be expressed as:

$$d\vec{F}_{B\text{-}S} = \frac{\mu_0}{4\pi}\frac{II'}{r^2}\left[d\vec{l}'\left(d\vec{l}\cdot\hat{r}\right) - \hat{r}\left(d\vec{l}\cdot d\vec{l}'\right)\right]$$

- dF_A obeys Newton's 3rd Law: $dF_A = -dF_A$.
- $dF_{B\text{-}S}$ doesn't: $dF_{B\text{-}S} \neq -dF_{B\text{-}S}$.
- $(dF_A - dF_{B\text{-}S})$ is an exact differential, thus it integrates to zero around a closed path.
- *So*: When source current element dl' is from a closed circuit, $dF_A = dF_{B\text{-}S}$.
- *Can also show*: For collinear and parallel current elements, dF_A is repulsive whereas $dF_{B\text{-}S} = 0$.

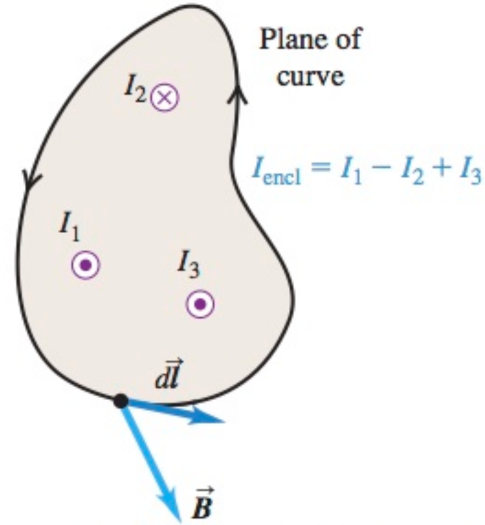
Ampère's Force Law vs. Ampère's Circuital Law

- Ampère's Force Law is *not* the same as what is known today as "Ampère's Law".

- Ampère's (Circuital) Law for magnetostatics:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \qquad \text{or} \qquad \oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

- Obtained from *Biot-Savart Law*: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$



Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$.

Ampère's (Circuital) Law from Biot-Savart Law:

- *Biot-Savart Law*: $\vec{B} = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{x}')\times\frac{\hat{r}}{r^2}\,d^3x'$ $Id\vec{l} \leftrightarrow \vec{J}d^3x$

- *Or*: $\vec{B} = \frac{\mu_0}{4\pi}\vec{\nabla}\times\int\frac{\vec{J}(\vec{x}')}{r}\,d^3x'$ $\hat{r}/r^2 = -\vec{\nabla}(1/r)$

- *So*: $\vec{\nabla}\times\vec{B} = \vec{\nabla}\times\left(\frac{\mu_0}{4\pi}\vec{\nabla}\times\int\frac{\vec{J}(\vec{x}')}{r}\,d^3x'\right)$ $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right) = \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right) - \vec{\nabla}^2\vec{A}$

$$= \frac{\mu_0}{4\pi}\vec{\nabla}\int\vec{J}(\vec{x}')\cdot\vec{\nabla}\left(1/r\right)d^3x' - \frac{\mu_0}{4\pi}\int\vec{J}(\vec{x}')\vec{\nabla}^2\left(1/r\right)d^3x'$$

$$= -\frac{\mu_0}{4\pi}\vec{\nabla}\underbrace{\int\vec{J}(\vec{x}')\cdot\vec{\nabla}'\left(1/r\right)d^3x'}_{=0} + \mu_0\vec{J}(\vec{x})$$

$\vec{\nabla}\left(1/r\right) = -\vec{\nabla}'\left(1/r\right)$ and $\vec{\nabla}^2\left(1/r\right) = -4\pi\delta(\vec{x}-\vec{x}')$

- *Why?* $\int\vec{J}(\vec{x}')\cdot\vec{\nabla}'\left(1/r\right)d^3x' + \int\left(1/r\right)\vec{\nabla}'\cdot\vec{J}(\vec{x}')d^3x' = \int_V\vec{\nabla}'\cdot\left(\vec{J}(\vec{x}')/r\right)d^3x'$

$$= \int_S\left[\left(\vec{J}(\vec{x}')/r\right)\cdot\hat{n}\right]d^2x'$$

- *So*: $\int\vec{J}(\vec{x}')\cdot\vec{\nabla}'\left(1/r\right)d^3x' = \int_S\left[\left(\vec{J}(\vec{x}')/r\right)\cdot\hat{n}\right]d^2x' - \int\left(1/r\right)\vec{\nabla}'\cdot\vec{J}(\vec{x}')d^3x'$

$= -\int\left(1/r\right)\vec{\nabla}'\cdot\vec{J}(\vec{x}')d^3x'$ *Surface integral vanishes (J = 0 on S).*

$= 0$ $\vec{\nabla}\cdot\vec{J} = -\partial\rho/\partial t = 0$

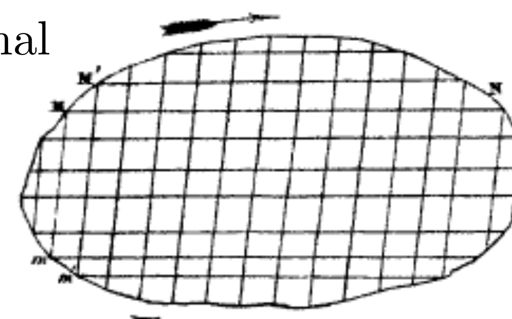
- 1823-26. Ampère's electrodynamics reaches maturity.
  - *Given Ampere's force law, can calculate every known magnetic or electromagnetic effect.*

> "I have solely consulted experiment to establish the laws of these phenomena, and I have deduced the only formula that can represent the forces to which they are due."

- No assumptions about the nature of electric currents:

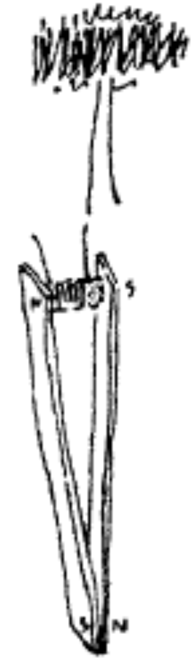
> "Whatever be the physical cause to which we may wish to relate the phenomena produced by this action, the formula obtained will always remain the expression of facts. "

- *<u>However</u>*:
  - *<u>Recall</u>*: Sucess of Fresnel's 1820s optical ether suggests phenomena can be reduced to local motions of a medium.
  - *<u>Thus</u>*: Electrodynamic forces may be seen as actions in the ether.
  - *<u>And:</u>* Equivalence of a closed circuit and a net of infinitesimal current loops suggests a rotary motion in the ether.
  - *<u>Suppose</u>*: Ether = neutral fluid resulting from combination of negative and positive electricity.



E. Electromagnetic Induction.

- *Oersted, et al*: Electric currents have magnetic effects.
- Do magnetic sources induce electrical currents?
  - 1821-22. *Ampère's initial experiments yield tentative results.*
  - 1825. *Faraday fails to detect induced currents.*
  - 1831. "Distinct conversion of Magnetism into Electricity."



- *Puzzle*: Induced current occurs both when closing the circuit and when breaking it. Doesn't occur for steady currents.
- *Solution*: New state of matter: the "electro-tonic" state.

> "Whilst the wire is subject to either volta-electric or magneto-electric induction, it appears to be in a perculiar state; for it resists the formation of an electrical current in it, whereas, if in its common condition, such a current would be produced; and when left uninfluenced it has the power of originating a current, a power which the wire does not possess under common circumstances... I have, after advising with several learned friends, ventured to designate [this state of matter] as the *electro-tonic* state."

- <u>*Further experiments*</u>: Observes currents induced radially in direction perpendicular to motion.

"The current of electricity which is excited in a metal when moving in the neighborhood of a magnet depends for its direction altogether upon the relation of the metal to the resultant of magnetic action, or to the magnetic curves."

- <u>*1832 Summary:*</u>

"If a terminated wire moves so as to cut a magnetic curve, a power is called into action which tends to urge an electric current through it."

- "Magnetic curve" = "the lines of magnetic forces... which would be depicted by iron filings or those to which a very small magnetic needle would form a tangent".
- Law suggests no further need for "electro-tonic" state:

"By rendering a perfect reason for the effect produced [the law seems to] take away any reason for supposing that perculiar condition, which I ventured to call the electro-tonic state."

- Two essential discoveries of electromagnetic rotation and induction are related:

"The power of inducing electric currents is circumferentially exerted by a magnetic resultant or axis of power, just as circumferential magnetism is dependent upon and is exhibited by an electric current."