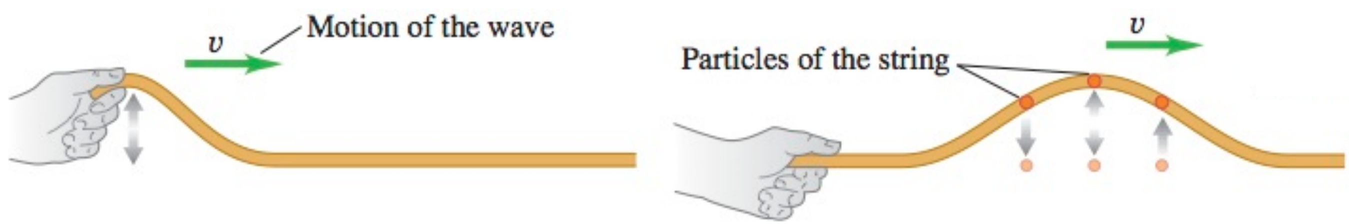
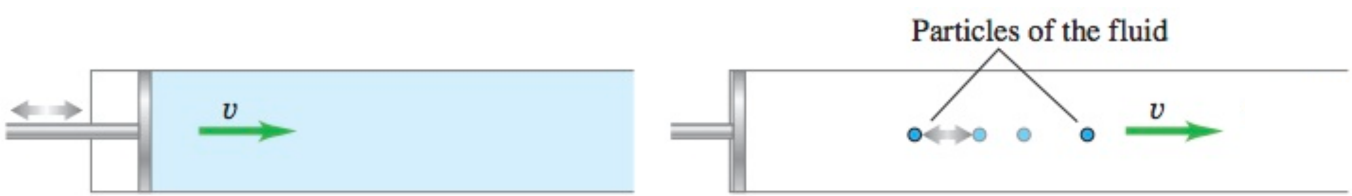
06. The Wave Theory: Further Developments. Buchwald (1989), Chaps 7-9.

A. Transverse and Longitudinal Waves (Contemporary View)

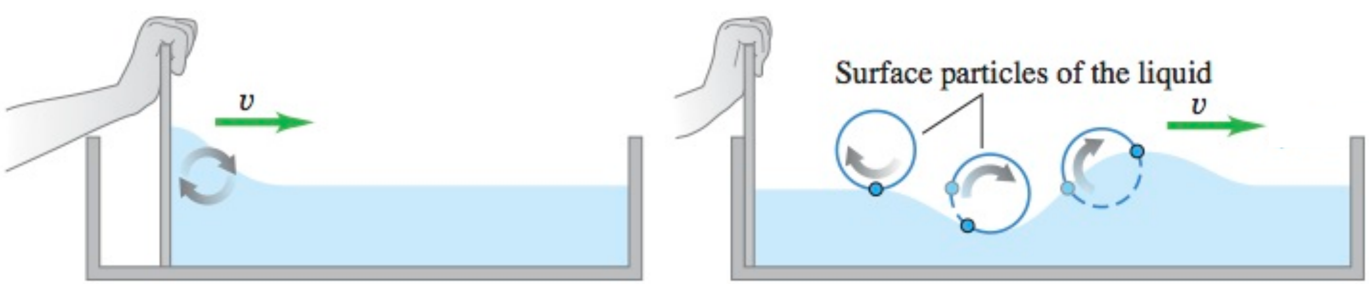
- *Transverse wave*: displacements of medium are perpendicular to direction of travel.



- *Longitudinal wave*: displacements of medium are in direction of travel.



- Surface wave: both transverse and longitudinal components.



- Are light waves transverse, longitudinal, or a combination of both?

B. The Puzzle of Polarization.

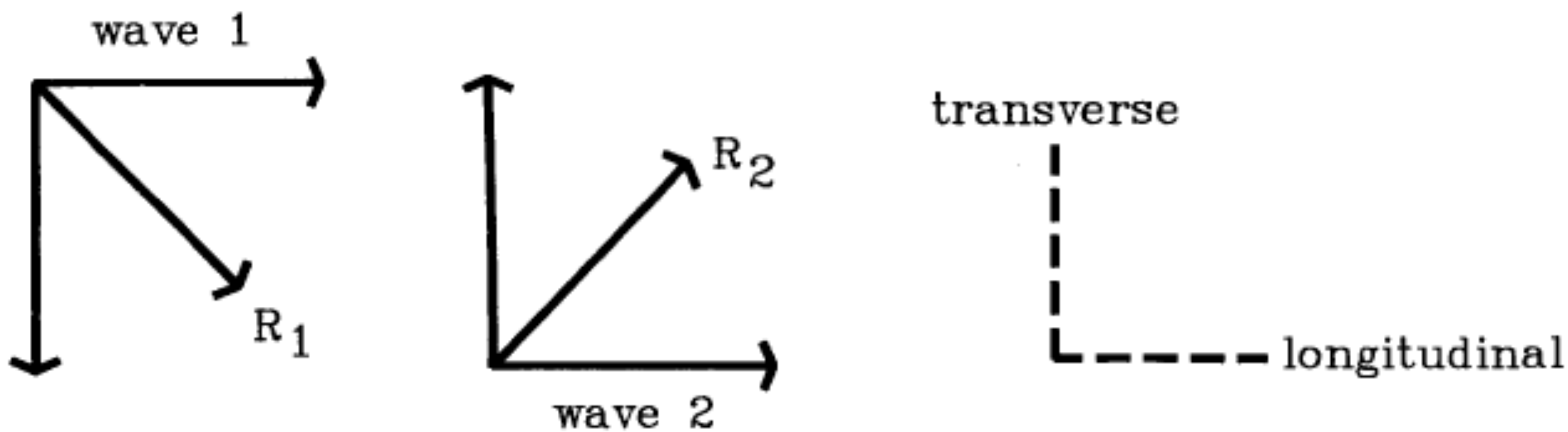
- 1816. Fresnel observes no interference of perpendicularly polarized beams that emerge from doubly refracting crystal.

"Two systems of waves in which the progressive [longitudinal] motion of the molecules of the fluid are modified by a transverse motion of going-and-coming, which would be perpendicular [to one another] and equal in intensity, could exert no action each on the other, when the discordance of the transverse motions corresponds to the accord of the progressive motion, or reciprocally, because then the resultants of these two forces in each system would be in perpendicular directions."

"There is another hypothesis that could explain the absence of fringes in circumstances otherwise favorable to their production: that would be transverse vibrations that would offer simultaneously condensed and dilated nodes on the same spherical surface, from which points of accord and discord would result [that are] so closely spaced that the eye, unable to distinguish them, would have the sensation of continuous light."

Fresnel's Two Possible Explanations:

(a) If longitudinal components have no phase difference, then transverse components must differ in phase by 180°; since then the resultant R_1 of first wave's components will be perpendicular to resultant R_2 of second wave's components.



Alternatively:

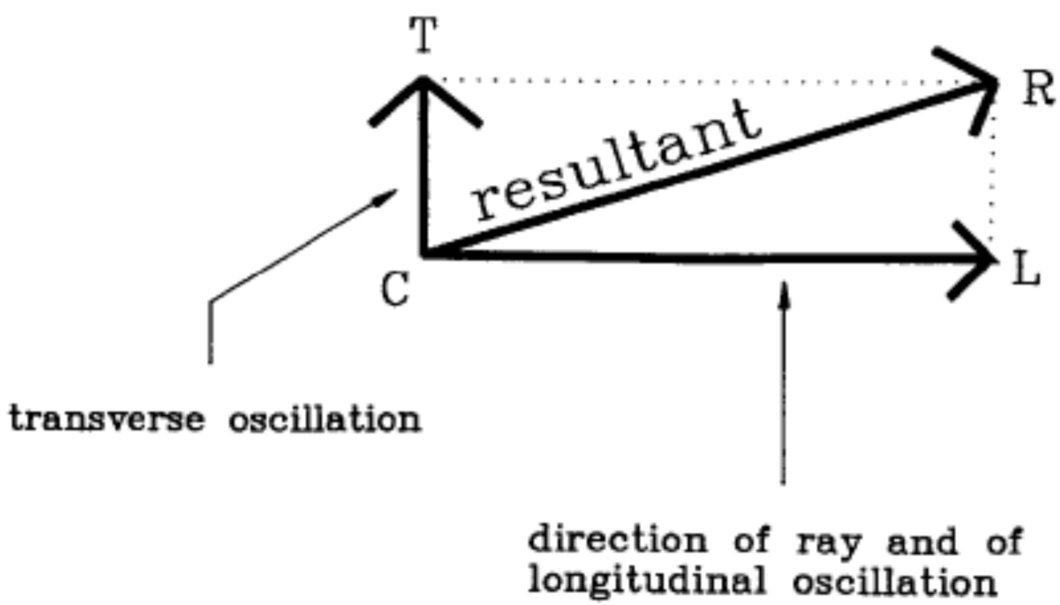
(b) Transverse components produce secondary interference pattern which obliterates visible fringes of interference pattern of longitudinal components.

Fresnel assumes:

- Unpolarized light must be longitudinal, since it lacks asymmetries.
- Polarized light either entirely lacks a longitudinal oscillation, or has the least possible longitudinal oscillation.

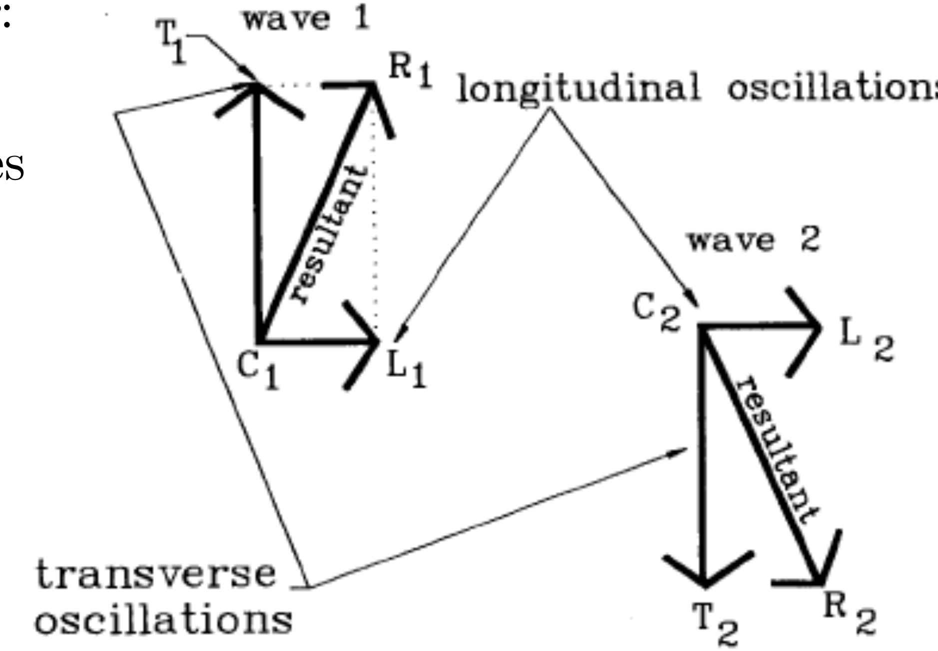
So:

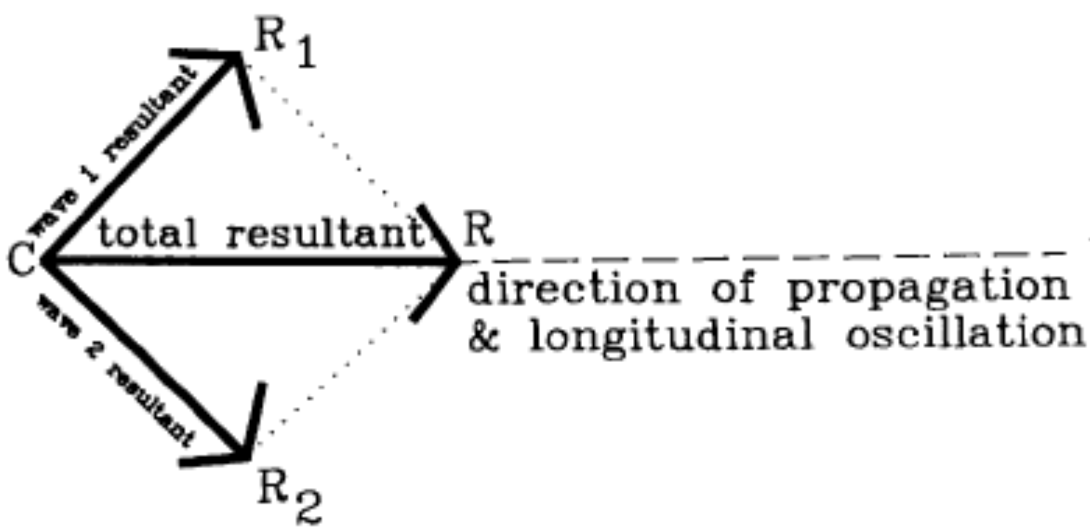
- *unpolarized wave*: $\measuredangle RCL$ is minimum.
- *polarized wave*: $\measuredangle RCL$ is maximum.
- *partially polarized wave*: $\measuredangle RCL$ lies somewhere between extremes.



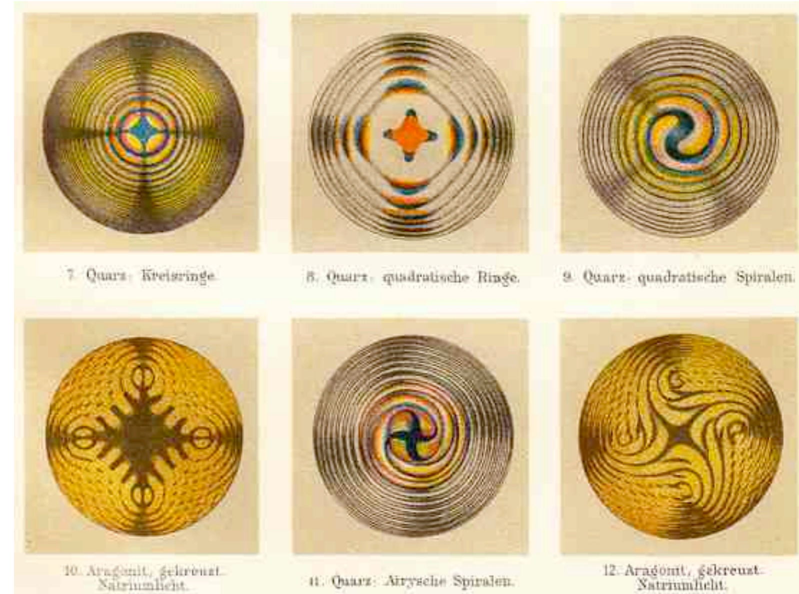
<u>*Case of perpendicularly polarized waves*</u>:

- CR (total resultant of compound of both waves) does not vanish, so waves do not interfere.
- <u>*Moreover*</u>: CR lies in direction of compound wave (it is purely longitudinal).
- <u>*Thus*</u>: We can imitate the lack of polarization of a single wave by compounding two polarized waves.



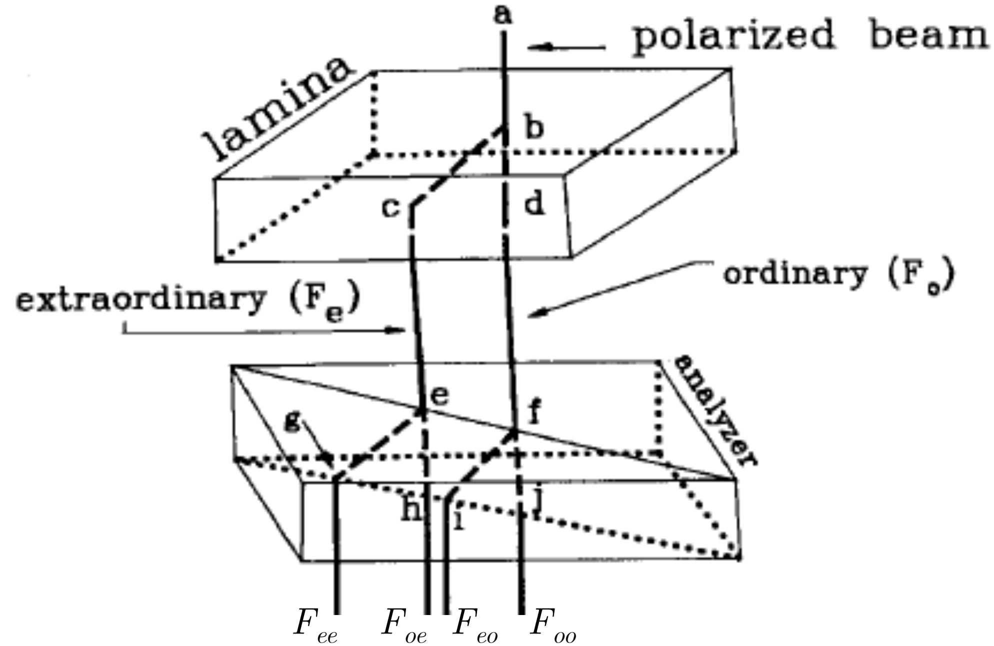


C. Fresnel's First Explanation of Chromatic Polarization.



- *Recall*: Chromatic polarization occurs when polarized white light passes through a doubly refracting lamina, yielding complementary-colored O and E beams polarized in directions perpendicular to each other.
- *Fresnel*: Since they are perpendicularly polarized, the O and E beams can't interfere *in the lamina* (contrary to a claim by Young).
- *But*: If they subsequently pass through a doubly refracting analyzing crystal, they are split each into O and E beams...
- *Thus*: *In the analyzing crystal*, the two resulting O beams can interfere, as can the two resulting E beams.

- $F =$ *intensity of incident polarized beam.*
- $i =$ *angle between plane of polarization of incident beam and optic axis of lamina.*
- $\alpha =$ *angle between plane of polarization of O beam and principle section of analyzer.*
- $\alpha - i =$ *angle between optic axis of lamina and principle section of analyzer.*
- *In lamina, Malus's Law gives*:
  - $F_o = F\cos^2 i$
  - $F_e = F\sin^2 i$



In analyzing crystal:

$$F_o \text{ splits into: } \begin{cases} F_{oo} = F\cos^2 i \cos^2(\alpha - i) \\ F_{eo} = F\sin^2 i \sin^2(\alpha - i) \end{cases} \qquad F_e \text{ splits into: } \begin{cases} F_{oe} = F\cos^2 i \sin^2(\alpha - i) \\ F_{ee} = F\sin^2 i \cos^2(\alpha - i) \end{cases}$$

- *Note*: Unlike Biot's selectionist formulas, Fresnel's wave expressions can't be added to each other.
  - "...the expressions take no direct account of the phase differences due to the different paths the rays have traversed..." (Buchwald, pg. 216.)
  - *No account (yet) of how waves with arbitrary phase difference can be combined.*

- <u>*Fresnel Claims*</u>: His account based on intereference is superior to Biot's because it explains why thick laminae do not exhibit chromatic polarization:

  - <u>*Suppose*</u>: Lamina thickness $= n\lambda$, for very large n.
  - <u>*Consider*</u>: λ' such that $n(\lambda' - \lambda) = \lambda'/2$ (*i.e.*, $\lambda' - \lambda$ is very small).
  - <u>*Then*</u>: $n\lambda = (n - \frac{1}{2})\lambda'$.
  - <u>*Which means*</u>: Where constructive interference occurs for λ, destructive interference occurs for λ'.
  - <u>*But*</u>: Since difference $\lambda' - \lambda$ is very small, colors that correspond to λ and λ' will be indistinguishable.
  - <u>*So*</u>: For thick laminae, we don't expect to see interference.
  - <u>*On the other hand*</u>: For thin laminae, n need not be so large, so $(\lambda' - \lambda)$ may be relatively large, hence intereference may be visible.

D. Fresnel's New View of Polarization.

- 1818: Fresnel's view of polarization not yet *fundamentally* changed:

"The oscillatory motions of the diverse points of the ether, in ordinary light, are all directed perpendicular to the wave or, if there are oblique motions, take place all round the normal at the same obliquity and with the same degree of energy in all azimuths. What characterizes, on the contrary, the vibrations of polarized light is that they do not take place in the same manner in all azimuths, and that the oblique motions I just mentioned do not have the same energy or the same obliquity all round the normal, or perhaps they take place only in a single plane, that of polarization."

- "Either natural light contains only a longitudinal motion, or -- and this was a new suggestion -- it consists of a simultaneous mixture of equally intense oblique oscillations in all directions round the normal." (Buchwald, pg. 226.)
- "The existence of interference required oscillations to take place to some extent at least in the same direction, whereas asymmetric behavior required that, also to some extent, they take place in different directions." (Buchwald, pg. 227.)

- 1819-21: Fresnel adopts a fundamentally new view of polarization.

"In effect, one may conceive direct light to be the assemblage, or more exactly the rapid succession of systems of waves polarized in all directions, and such that there is always as much polarized light in one plane as in the plane perpendicular to it..."

"...far from 'polarization' in the old sense of asymmetry being an unusual condition, light is *always* 'polarized'." (Buchwald, pg. 227.)

"So that direct light may be considered to be the reunion, or more exactly the rapid succession, of systems of waves polarized in all directions. According to this way of looking at things, the act of polarization consists not in creating transverse motions, but in decomposing them in two fixed, mutually perpendicular directions, and in separating the two components the one from the other; because, in each of them, the oscillatory motions will always operate in the same plane."

- *Buchwald*: A new "kinetic" understanding of polarization, as opposed to the selectionist "static" understanding.

Selectionist Static View:

- Required different spatial states to explain the differences between "polarized", "unpolarized", and "partially polarized" light.

"Light in one state differed in the nature of its asymmetry from light in another state, and the difference between the two states did not change over time -- it was a purely spatial difference." (Buchwald, pg. 230.)

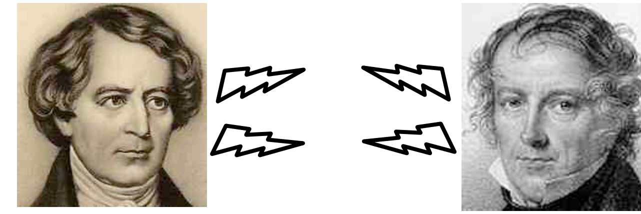
Fresnel's Kinetic View:

"Light in any of the three states is precisely as asymmetric as light in the other two, but the states are now supposed to differ in respect to what occurs over time." (Buchwald, pg. 230.)

"...one could always give the name *rectilinear polarizaiton* to what one has for a long time observed in the double refraction of calc-spar... and call *circular polarization* the new modification whose characteristic properties I have just described: it will naturaly divide into *circular polarization from left to right* and *circular polarization from right to left*... Between rectilinear polarization and circular polarization, there are a throng of intermediate degrees of diverse polarizations to which one could give the name *elliptical polarizations*, according to the same theoretical views."

- *Selectionism*: Whether a beam is unpolarized or partially polarized depends on whether it contains a set of rays with asymmetries oriented in nearly the same way. Approximate distinction.
- *Fresnel before 1821*: Whether a beam is unpolarized or partially polarized depends on the proportion in it of longitudinal to transverse oscillations. Approximate distinction.
- *New view*: The two orthogonal components in polarized light of any type have a fixed phase difference and a fixed amplitude ratio; both remain constant over time. But the phase difference between the two components in unpolarized light, as well as the amplitudes of the components, varies over time.
  - *Thus*: Absolute distinction between polarized and unpolarized light.

E. A Case of Mutual Misunderstanding.



- 1821. Big battle between Arago and Biot over chromatic polarization.
- *Arago and Ampere's report*: Misrepresents Fresnel's Prize Essay as an attack on Biot's theory of chromatic polarization.
- In response, Biot criticizes Fresnel's theory, bringing Fresnel into the debate.

- *Recall*: According to Biot, when it exists lamina, a ray is either polarized along 0 or $2i$, depending on whether lamina's thickness causes ray to oscillate an integral number of times from the 0 direction or the $2i$ direction.
- *Arago charges*: *Both* directions are observed.

- *Biot's response*:
  - Fresnel's formulas make same predictions.
  - Direction of polarization of emerging rays depends on thickness of lamina.

- Recall Biot's formula:

$$F_o = U\cos^2\alpha + A\cos^2(2i-\alpha)$$
$$F_e = U\sin^2\alpha + A\sin^2(2i-\alpha)$$

- Frenel's (modified) formula:

$$F_o = \cos^2\alpha - \sin(2i)\sin[2(i-\alpha)\sin^2[\pi(e-o)/\lambda]$$
$$F_e = \sin^2\alpha + \sin(2i)\sin[2(i-\alpha)\sin^2[\pi(e-o)/\lambda]$$

o = # wavelengths needed to produce half-wavelength phase shift for O beam.

e = # wavelengths needed to produce half-wavelength phase shift for E beam.

- These can be transformed into:

$$F_o = \cos^2[\pi(e-o)/\lambda]\cos^2\alpha - \sin^2[\pi(e-o)/\lambda]\cos^2(\alpha-2i)$$
$$F_e = \sin^2[\pi(e-o)/\lambda]\sin^2\alpha - \sin^2[\pi(e-o)/\lambda]\sin^2(\alpha-2i)$$

"... the total light $O + E$... behaves after its emergence... precisely as though it were composed of two distinct and complementary tints, of which the one O would conserve the polarization primitively imposed on it in the zero asimuth, and the other E would receive a new direction of polarization in the azimuth $2i$."

- *But:* Fresnel's formulas do not allow one to divide the emerging light into bundles.

- Biot observes: Whether or not both polarizations (0 and $2i$) occur is a matter of degree.

> "The entire issue reduced, in Biot's eyes, to the question of the width of the transition range between the two azimuths of polarization. And since Arago reported no data on the point, having only remarked the existence of intermediate thicknesses at which both azimuths must be present if we accept Biot's analysis, the issue remained undecided as far as Biot was concerned." (Buchwald, pg. 248.)

- Arago misses this point:

> "[Biot says]... in ten different places in his works that a polarized ray of simple [homogenous] light is polarized entirely at its exit either in the primitive plane or in the azimuth $2i$."

- Fresnel claims only that Biot's alternative is too complicated from a physical point of view.
- Buchwald: This is a result of Fresnel's conflating *emissionist* views with *selectionist* views.