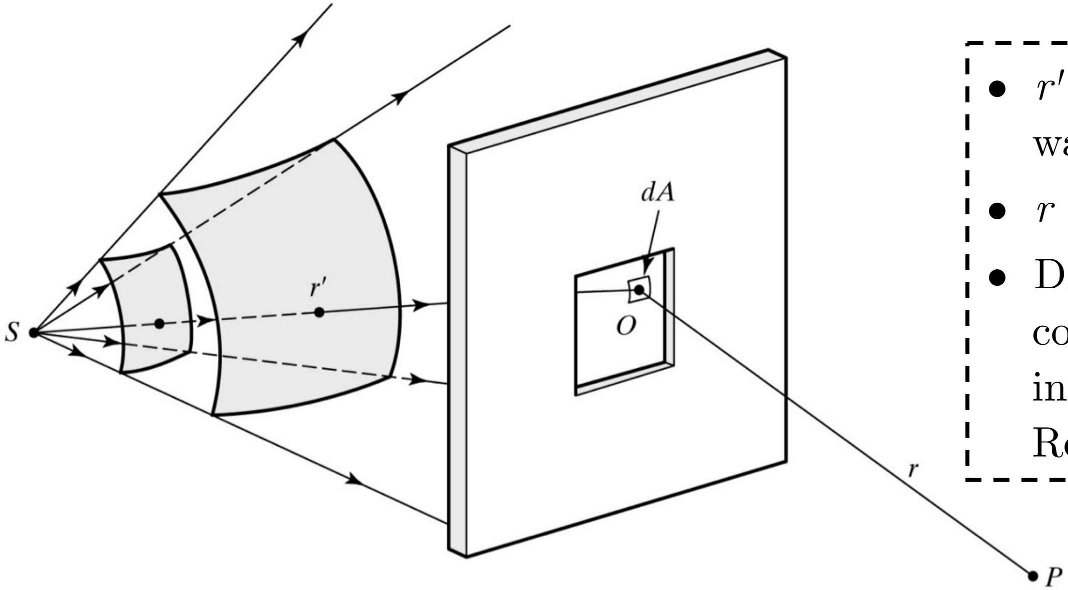
05. Huygens's Principle and the Wave Theory.

Buchwald (1989), Chap 6.

A. Fresnel Diffraction: Contemporary View



- r' = distance from source S to point O on wavefront at aperture.
- r = distance from O to P.
- Direction from points O to points P is not constant, so amplitude of wavelets originating at aperture is dependent on direction: Requires an "obliquity factor" correction.

- *Huygens-Fresnel Principle*: Wave at P is superposition of all wavelets from wavefront at aperture.
- Wavelet emanating from front area dA, produces spherical wave at P:

$$dE_P = \frac{dE_0}{r} e^{ikr}$$

- ○ Amplitude dE_0 of wavelet at dA is proportional to dA: $dE_0 \propto E_L dA$.
- ○ $E_L = (E_S/r')e^{ikr'}$ = *amplitude of wave centered at S.*

- *Thus*: $E_P = E_S \int \left(\frac{1}{rr'}\right) e^{ik(r+r')}\, dA$

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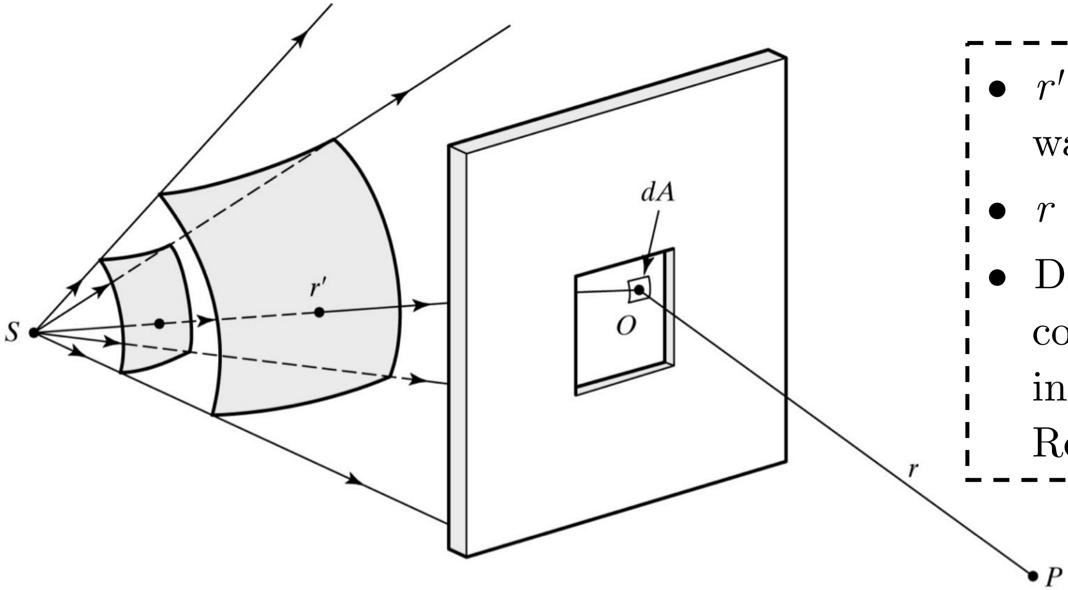

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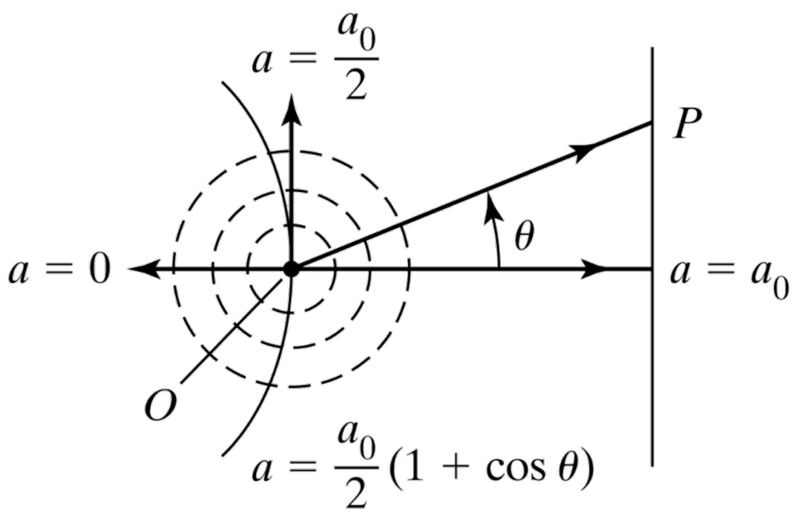
- r = distance from O to P.
- Direction from points O to points P is not constant, so amplitude of wavelets originating at aperture is dependent on direction: Requires an "obliquity factor" correction.

- *But*: Need obliquity factor, call it $F(\theta)$, where θ is angle between r and r'.



- *Huygens*: Source O of secondary wavelets radiates without regard to direction.
- *But*: Only forward wavelets exist.
- *So*: Require amplitude of wavelet to satisfy $a = a_0 F(\theta)$.
  - $F(\theta) = ½(1 + \cos\theta)$
  - a_0 is amplitude in forward direction.

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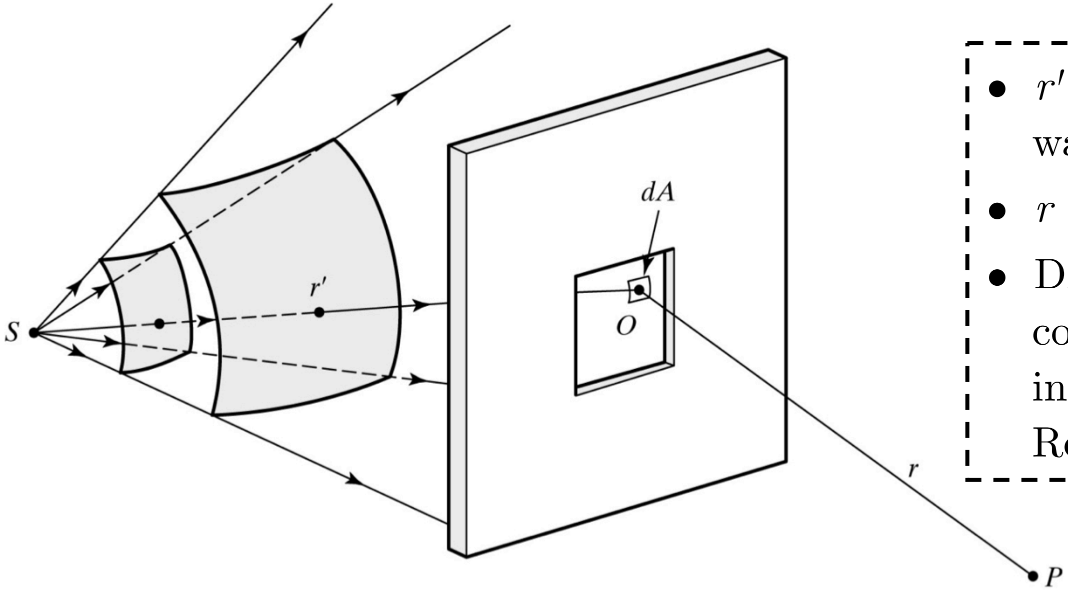


- r' = distance from source S to point O on wavefront at aperture.
- r = distance from O to P.
- Direction from points O to points P is not constant, so amplitude of wavelets originating at aperture is dependent on direction: Requires an "obliquity factor" correction.

- *And*: At O, incident and diffracted waves are 90° out of phase.

- *So*: Corrected "Fresnel-Kirchhoff Integral" is

$$E_P = \frac{e^{-i\pi/2} k E_S}{2\pi} \int \left(\frac{1+\cos\theta}{2rr'} \right) e^{ik(r+r')}\, dA$$

B. Fresnel on Superposition of Waves

- *1818. Problem*:

> Given the intensities [amplitudes] of an arbirary number of systems of [coherent] luminous waves and their respective positions, or their different degrees of accords and discords [phase differences], to determine the intensity of the total light.

- *Assume*: Ether particles oscillate harmonically.
- *Then*: Speed v of given particle at time t is speed of source at emission time $t - x/\lambda$, where $\lambda =$ wavelength, $x =$ distance from source, and for $i =$ phase:

$$v = a\sin\left[2\pi\left(t - \frac{x}{\lambda}\right) - i\right]$$

- *And*: This can be decomposed into:

$$R = a\cos(i)\sin\left[2\pi\left(t - \frac{x}{\lambda}\right)\right] - a\sin(i)\cos\left[2\pi\left(t - \frac{x}{\lambda}\right)\right]$$

- *Which means*: A single wave with arbitrary phase i can be considered to arise from the *interference* of two other waves with amplitudes $a\cos(i)$, $a\sin(i)$ that differ in phase by 90°.

C. The Fresnel Integrals

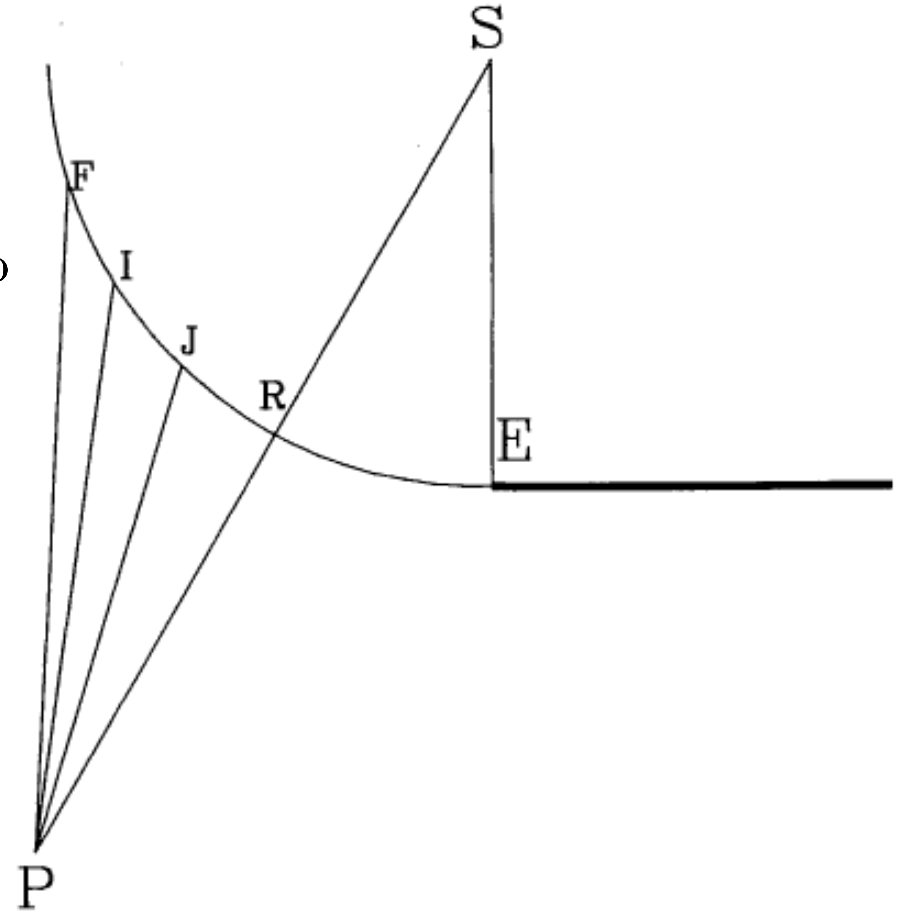
- 1818. Fresnel alters requirement of ray theory that oblique radiation only arises near obstacle edges:

"But [since] the effects produced by the rays that emanate from the primitive wave destroy one another nearly completely when the [rays] are sensibly inclined to the normal, the rays that appreciably influence the quantity of light received by each point P can be regarded as of equal intensity. In extending the integration to infinity, I suppose, for purposes of calculation, that this holds also for the other rays, inasmuch as the inexactitude of this hypothesis should not bring a sensible error in the results."

"... the intensity of radiation from a point on the front, he now assumes, decreases rapidly and continuously according to some unknown function of its inclination to the normal [an *inclination factor*]." (Buchwald, pg. 158.)

Fresnel's New Account:

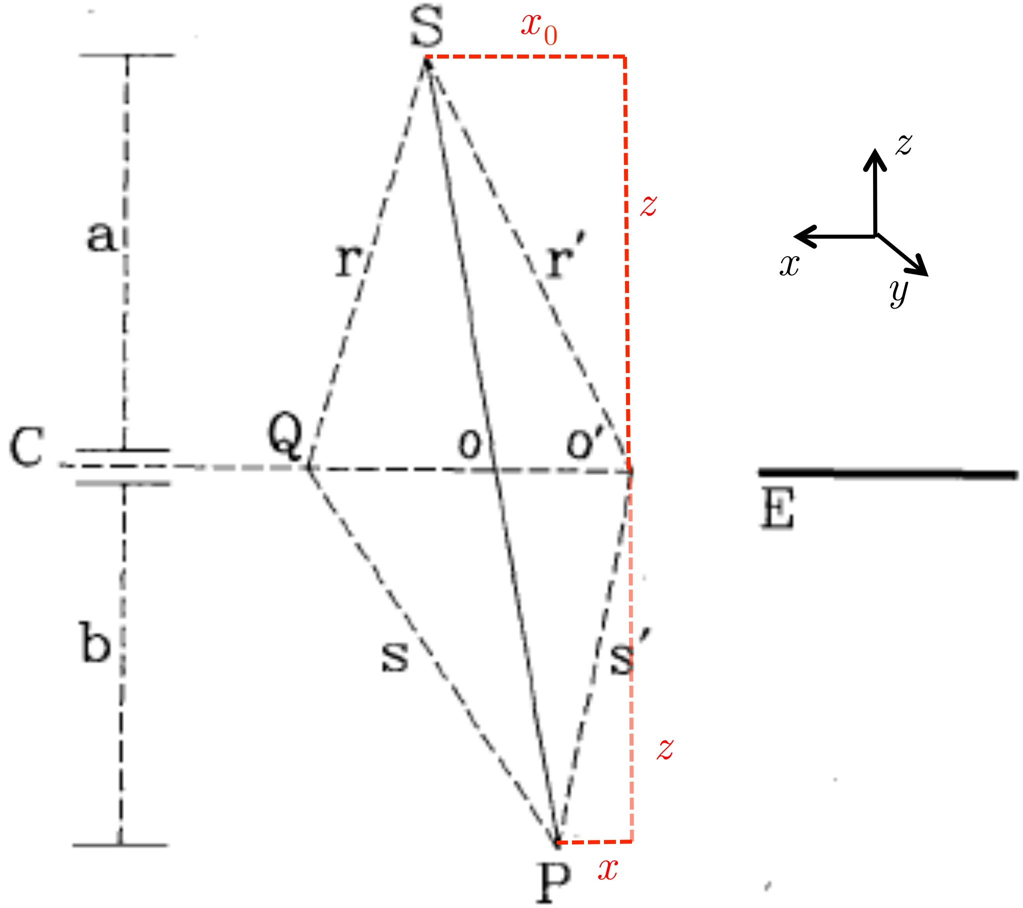
- Physically important point that determines a region that produces oblique radiation shifts from edge of diffractor to "pole" R = *intersection of front and line SP connecting source and field point.*
- Rays JP, IP, FP, ... differ from each other by a half-wavelength, and are nearly same length, except near pole.
- *So*: Arcs on wavefront far from pole have nearly same length, whereas arcs close to pole have unequal lengths.



- *So*: Waves from arcs far from pole cancel at P, whereas waves from arcs close to pole will not (even though they aren't near edge).
- *Upshot*: Fringe pattern is governed by interference of rays that are emitted from every point of the front: Combination of Huygens's principle and principle of interference.

How to sum contributions from all points on the front:

- Let origin of (x, y, z)-coord system be at $O' = (0, 0, 0)$.
- Let $Q = (\xi, 0, 0)$, $S = (x_0, 0, z)$, $P = (x, 0, z)$.



- *Then*:

$$r'^2 = {x_0}^2 + z^2$$
$$r^2 = (x_0 - \xi)^2 + z^2$$
$$= r'^2 + \xi^2 - 2x_0\xi$$

- *And*:

$$s'^2 = x^2 + z^2$$
$$s^2 = (x - \xi)^2 + z^2$$
$$= s'^2 + \xi^2 - 2x\xi$$

- *Now*: Apply binomial theorem to r^2 and s^2...

Binomial theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$$

- *So*: $r = r'^2\left[1 + \left(\frac{\xi^2}{r'^2} - \frac{2x_0\xi}{r'^2}\right)\right]^{1/2}$

$$= r'\left[1 + \frac{1}{2}\left(\frac{\xi^2}{r'^2} - \frac{2x_0\xi}{r'^2}\right) - \frac{1}{8}\left(\frac{\xi^4}{r'^4} - \frac{4x_0\xi^3}{r'^4} + \frac{4x_0^2\xi^2}{r'^4}\right) + \cdots\right]$$

$$\approx r' + \frac{\xi^2}{2r'} - \frac{x_0\xi}{r'} - \frac{x_0^2\xi^2}{2r'^3}$$

$$= r' + \frac{1}{2r'}\xi^2 - l_0^2\frac{\xi^2}{2r'} + l_0\xi \qquad l_0 = -x_0/r'$$

- *And similarly*: $s \approx s' + \frac{1}{2s'}\xi^2 - l^2\frac{\xi^2}{2s'} - l\xi + \cdots \qquad l = -x/s'$

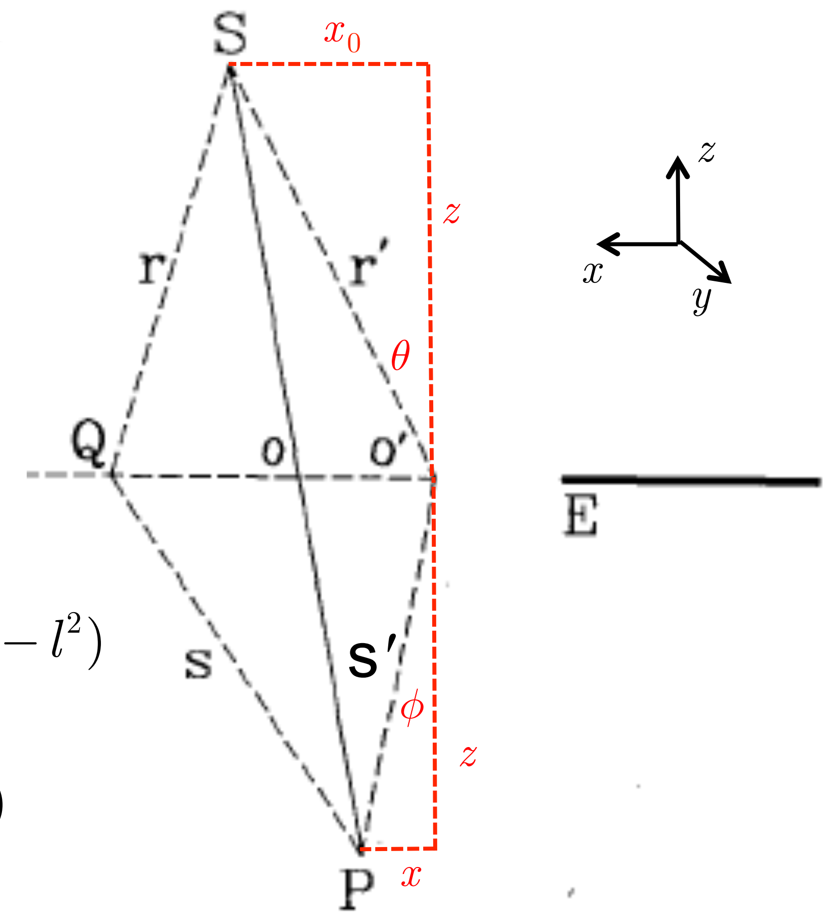
- *So*: $(r+s) - (r'+s') \approx (l_0 - l)\xi + \frac{1}{2}\left[\left(\frac{1}{r'} + \frac{1}{s'}\right)\xi^2 - l_0^2\frac{\xi^2}{r'} - l^2\frac{\xi^2}{s'}\right]$

Still too complicated to integrate!

$|l_0| = x_0/r' = \sin\theta = \cos(90° - \theta) \equiv \cos\alpha$

$|l| = x/s' = \sin\phi = \cos(90° - \phi) \equiv \cos\beta$

where α, β are the angles between SO', PO' and x-axis for arbitrary O'.



- *Claim*: $\cos\alpha = \cos\beta$ for $O' = O$.
- *So*: Move origin to pole O, for which $l_0 = l$.
- *Then*: $(r + s) - (r' + s') = \frac{1}{2}\xi^2\left(\frac{1}{r'} + \frac{1}{s'}\right)(1 - l^2)$

$$= \xi^2 \frac{r' + s'}{2r's'}(1 - l^2)$$

- *Inclination factor* $= (1 - l^2)$.
- *Now*: Assume field points P are near the normal: $\theta = \phi = 90°$, thus $l^2 = 0$.
- *Then:*

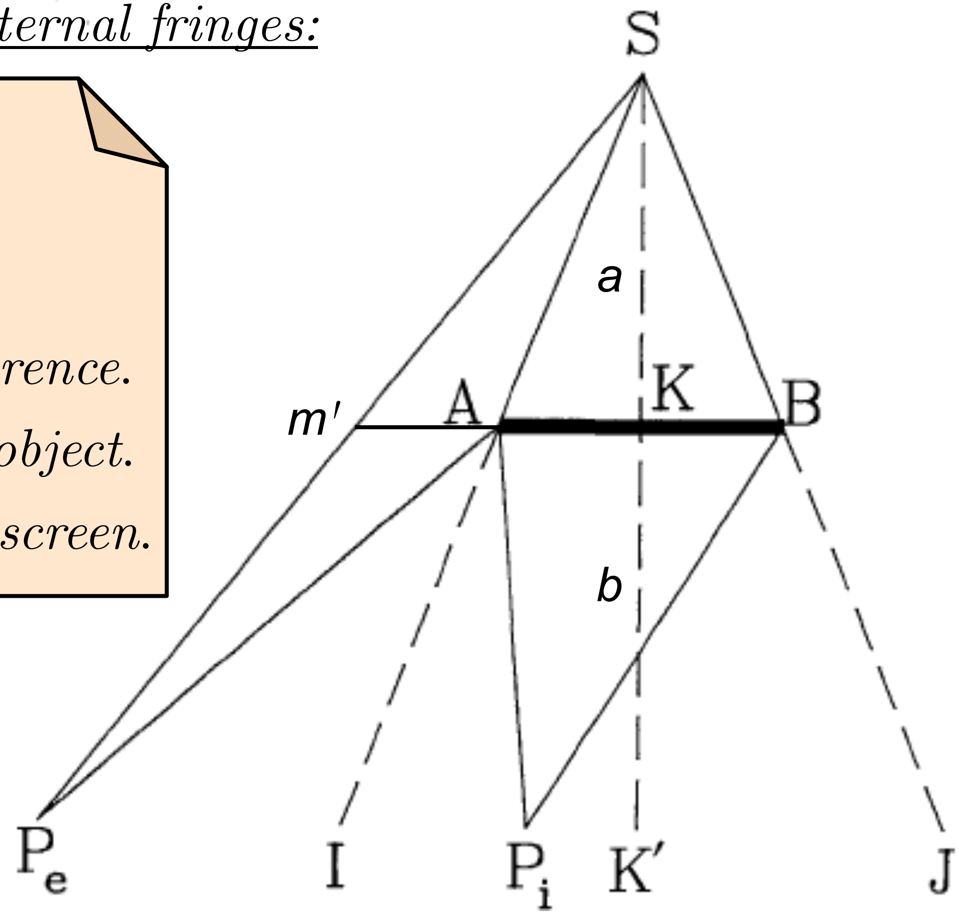
$$\textit{Path difference} = d = (r + s) - (r' + s') = \xi^2 \frac{r' + s'}{2r's'}$$

Fresnel's reasoning from formula for external fringes:

External Fringe Formula:

$$P_e I \approx \sqrt{\frac{2db(a+b)}{a}}$$

- $d = (SA + AP_e) - SP_e$ = *path difference.*
- $a = SK$ = *dist. between source and object.*
- $b = KK'$ = *dist. between object and screen.*



- $P_e I / m'A \approx (a+b)/a$
- Let $z = m'A$.
- *Then*:

$$P_e I = z\frac{(a+b)}{a} = \sqrt{\frac{2db(a+b)}{a}}$$

- *So*:

$$\text{Path difference} = d = z^2\frac{a+b}{2ab}$$

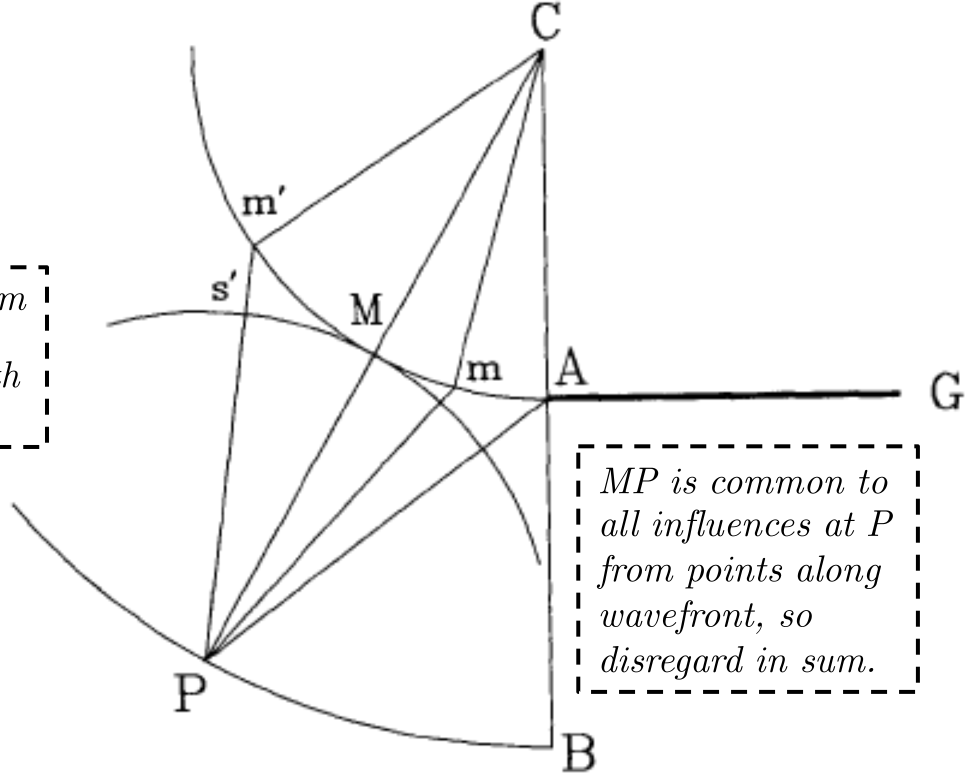
- Path difference $d = m's' = z^2 \dfrac{a+b}{2ab}$.
- *Now*: Decompose wave at P due wavelet centered at m':

$$\sin\left[2\pi\left(t - \frac{CM + m's'}{\lambda}\right)\right]$$

distance x from source to P associated with wavelet at m'

$$= \sin\left[2\pi\left(t - \frac{CM}{\lambda}\right) - \pi z^2 \frac{a+b}{ab\lambda}\right]$$

$$= \cos\left(\pi z^2 \frac{a+b}{ab\lambda}\right)\sin\left[2\pi\left(t - \frac{CM}{\lambda}\right)\right] + \sin\left(\pi z^2 \frac{a+b}{ab\lambda}\right)\cos\left[2\pi\left(t - \frac{CM}{\lambda}\right)\right]$$



MP is common to all influences at P from points along wavefront, so disregard in sum.

- Now consider infinitesimal arc dz and integrate over all such arcs

> "We see that the combination of Huygens's principle with the principle of interference solves the major problem that had been posed by the efficacious ray -- namely, to retrieve, within the limits of observational accuracy, the original formula for the external fringes in obstacle diffraction." (Buchwald, pg. 168.)

D. Objections to Huygens's Principle

- *1819. Arago's report*: "..presents what one might call a selectionist account of Fresnel's integral method -- one that avoids any mention whatever of Fresnel's 'elementary waves'". (Buchwald, pg. 183.)

"[The author]... supposes that, from each point of this surface, *elementary luminous rays* depart in all directions and with sensibly equal intensities as long as they do not deviate far from the normal; he does not take account of the *rays* that are very inclined [to the normal, since these], in his hypothesis, destroy one another..."

"It is entirely possible to think of Fresnel's integral theory in this way -- in terms of rays -- since the formulas encapsulate everything that can be known empirically about obstacle diffraction, and since in diffraction ray counts are not involved, insasmuch as each point of the front emits in all directions. By contrast, in partial reflection the assumption that rays exist as individuals leads to results entirely different from anything that can be obtained from the wave theory, because ray counts and ratios enter directly into the formulas." (Buchwald, pg. 184.)

Young's Objection (letter to Arago):

"...in fact I am still at a loss to understand the possibility of the thing; for if light has at all times so great a tendency to diverge into the path of the neighboring *rays* and to interfere with them as Huygens supposes, I do not see how it escapes being totally extinguished in a very short space, even in the most transparent medium... I cannot, however, deny the utility of Mr. Fresnel's calculations."

- *In other words*: Won't the secondary wavelets carry energy (*vis viva*) away in all directions so that the resultant wave should become small over time?
- *Young claims*: No radiation in oblique directions.
- *Fresnel's response*: Demonstrates that interference does not affect total *vis viva*.
- *But*: Young's objection is that *vis viva* is *dispersed*, not *destroyed*.

Fresnel to Young:

"Huygens's principle seems to me, just as much as that of interference, to be a rigorous consequence of the coexistence of small motions in the vibrations of fluids... one may therefore say, according to the principle of the coexistence of small motions, that the vibrations excited by this wave in an arbitrary point of the fluid situated beyond it are the sum of all the agitations that each of the disturbing centers would there give rise to acting in isolation."

- *Principle of composition*: The resultant amplitude at a given point may be calculated by adding together all the waves that would, considered individually, reach that point at a given time where the others not present.
- *Mathematically equivalent to*: Sum of solutions to the wave equation is itself a solution.
- *But not equivalent to Huygens's principle*: Huygens's principle says
  (a) Every point of a wavefront is a source of secondary wavelets; and
  (b) The wave at a given time is constructed by applying the principle of composition to the wavelets generated by the wavefront at an earlier time.

Poisson's Objection:

- Radiation must occur almost entirely along the normals to the front because:

Siméon Poisson
(1781-1840)

"it is only in this way that one may conceive, in the theory of undulations, the propagation of an isolated, thin streak of light, which the adversaries of that theory deny the possibility of."

- *Fresnel's response (1823)*:
  - No such thing as a beam in the sense Poisson means ("thin streak of light").
  - Inclination factor allows oblique radiation.

- *Now*: Show that a linearly oscillating fluid element will generate off-axis radiation...

- *Principle of composition sez*: Displacement Ab together with displacement Ad produce same effect as single displacement Ac.

- *Let*: $\measuredangle BAC = \measuredangle CAD = \alpha$.

- *Then*: $\dfrac{AP}{AR} = \dfrac{\sin\alpha}{\sin 2\alpha} = \dfrac{1}{2\cos\alpha}$

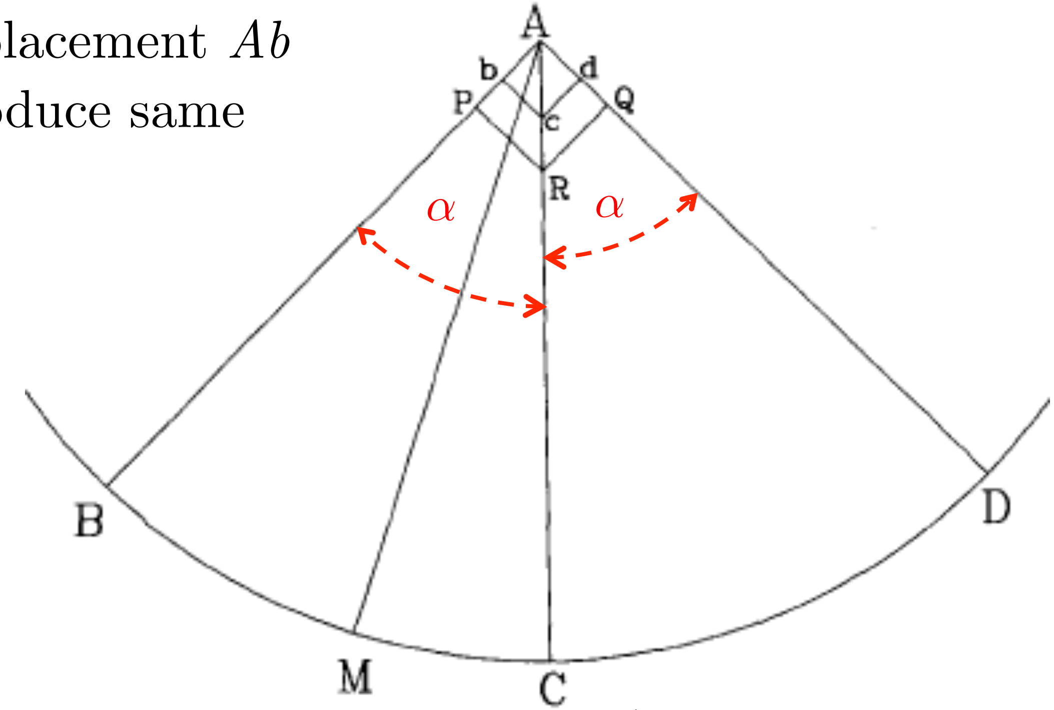
- *Or*: $AP = \dfrac{AR}{2\cos\alpha}$

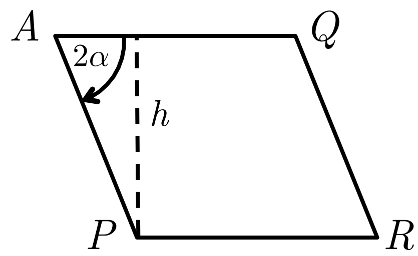
- *Similarly*: $AQ = \dfrac{AR}{2\cos\alpha}$

- *Fresnel's Interpretation*:

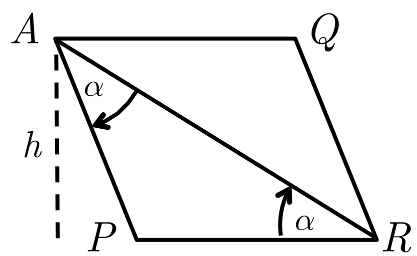
$$(\textit{effect at } B \textit{ due to } Ab) = \frac{1}{2\cos\alpha}(\textit{effect at } C \textit{ due to } Ac)$$

$$(\textit{effect at } D \textit{ due to } Ad) = \frac{1}{2\cos\alpha}(\textit{effect at } C \textit{ due to } Ac)$$





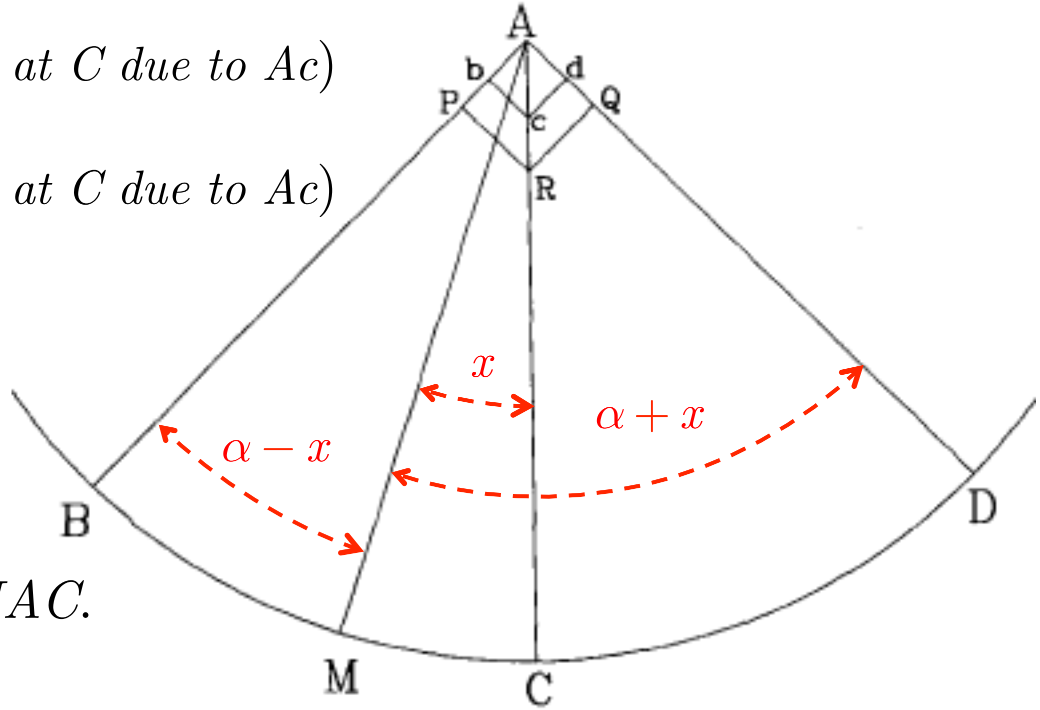
$AP\sin 2\alpha = h$



$AR\sin\alpha = h$

$$(\mathit{effect\ at\ B\ due\ to\ Ab}) = \frac{1}{2\cos\alpha}\,(\mathit{effect\ at\ C\ due\ to\ Ac})$$

$$(\mathit{effect\ at\ D\ due\ to\ Ad}) = \frac{1}{2\cos\alpha}\,(\mathit{effect\ at\ C\ due\ to\ Ac})$$



- <u>*Now assume*</u>:

  $(\mathit{effect\ at\ M\ due\ to\ Ac})$

  $= \psi(x) \times (\mathit{effect\ at\ C\ due\ to\ Ac})$

  for inclination factor $\psi(x)$, $x = \measuredangle MAC$.

- <u>*Then*</u>:

  $(\mathit{effect\ at\ M\ due\ to\ Ac}) = (\mathit{effect\ at\ M\ due\ to\ Ab}) + (\mathit{effect\ at\ M\ due\ to\ Ad})$

  $$= \psi(\alpha - x) \times (\mathit{effect\ at\ B\ due\ to\ Ab}) + \psi(\alpha + x) \times (\mathit{effect\ at\ D\ due\ to\ Ad})$$

  $$= \frac{\psi(\alpha - x)}{2\cos\alpha} \times (\mathit{effect\ at\ C\ due\ to\ Ac}) \ + \ \frac{\psi(\alpha + x)}{2\cos\alpha} \times (\mathit{effect\ at\ C\ due\ to\ Ac})$$

- <u>*So*</u>: $\psi(x) = \psi(\alpha - x)/2\cos\alpha + \psi(\alpha + x)/2\cos\alpha$
- <u>*And*</u>: $\psi(0) = 1 = \psi(\alpha)/\cos\alpha$.
- <u>*Thus*</u>: $\psi(\alpha) = \cos\alpha$.