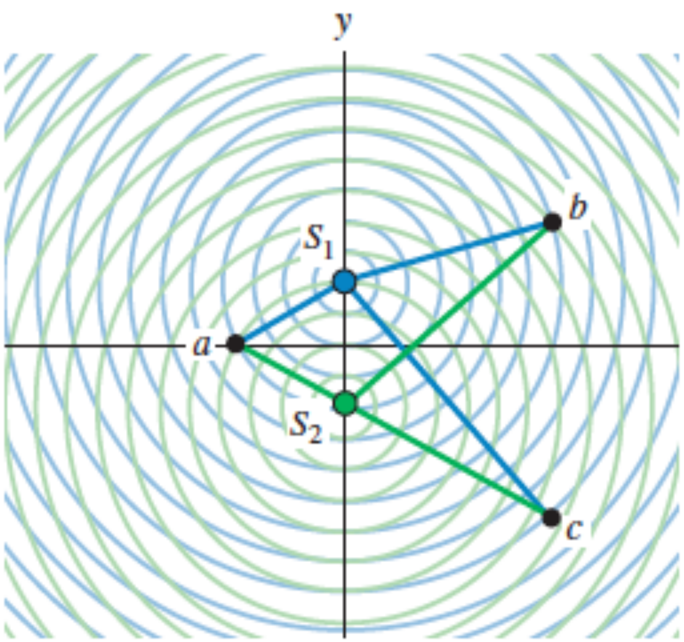
04. Fresnel's Ray Theory of Diffraction.

Buchwald (1989), Chap 5.

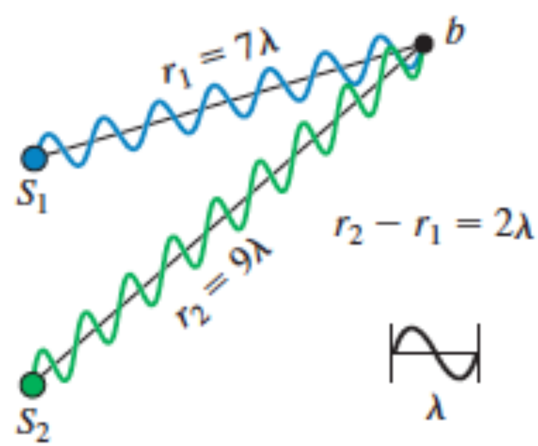
A. Interference: Contemporary View

- *Coherent sources* = Sources that produce waves with same amplitude and wavelength, and that have a constant phase relation.

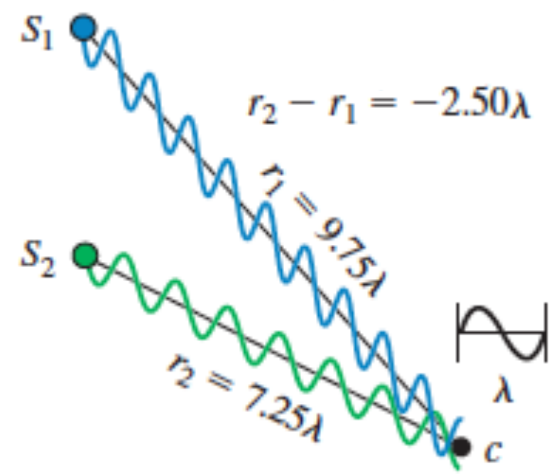
(a) Two coherant sources S_1, S_2 separated by distance 4λ.

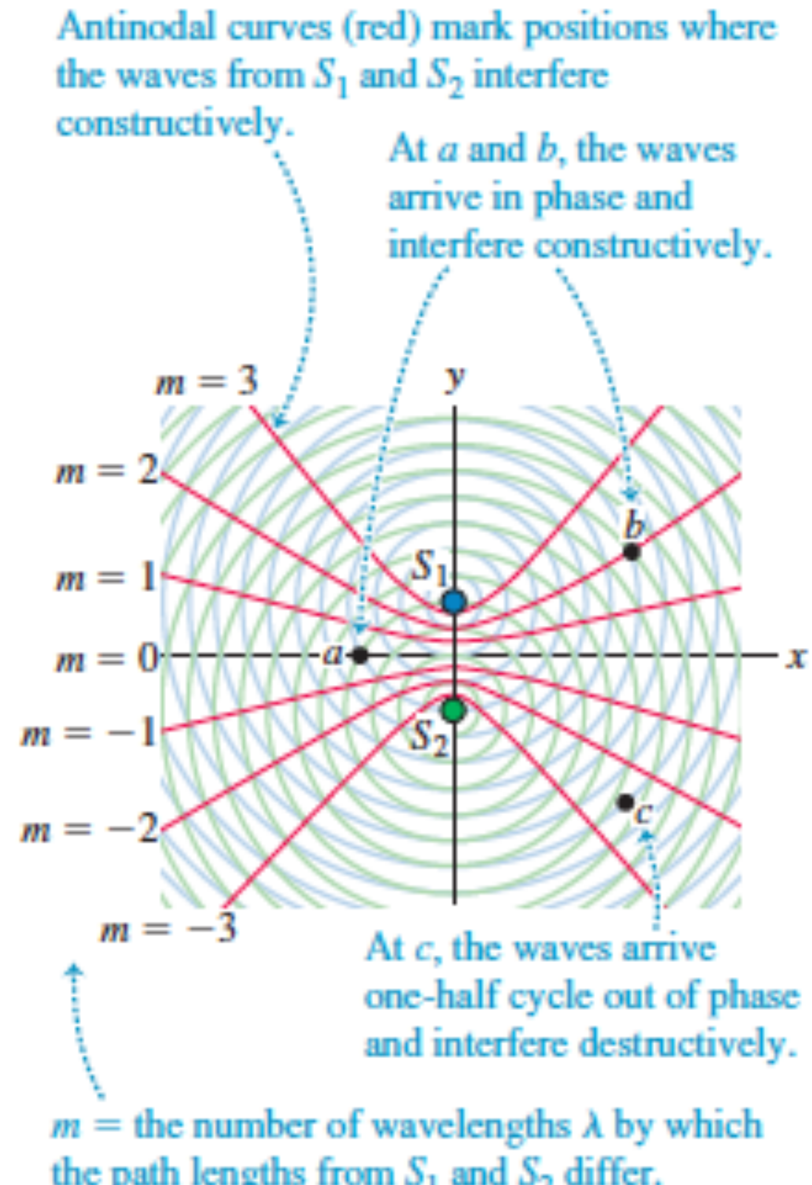


(b) Constructive interference: Waves interfere constructively if their path lengths differ by an integral number of wavelengths: $r_2 - r_1 = m\lambda$.



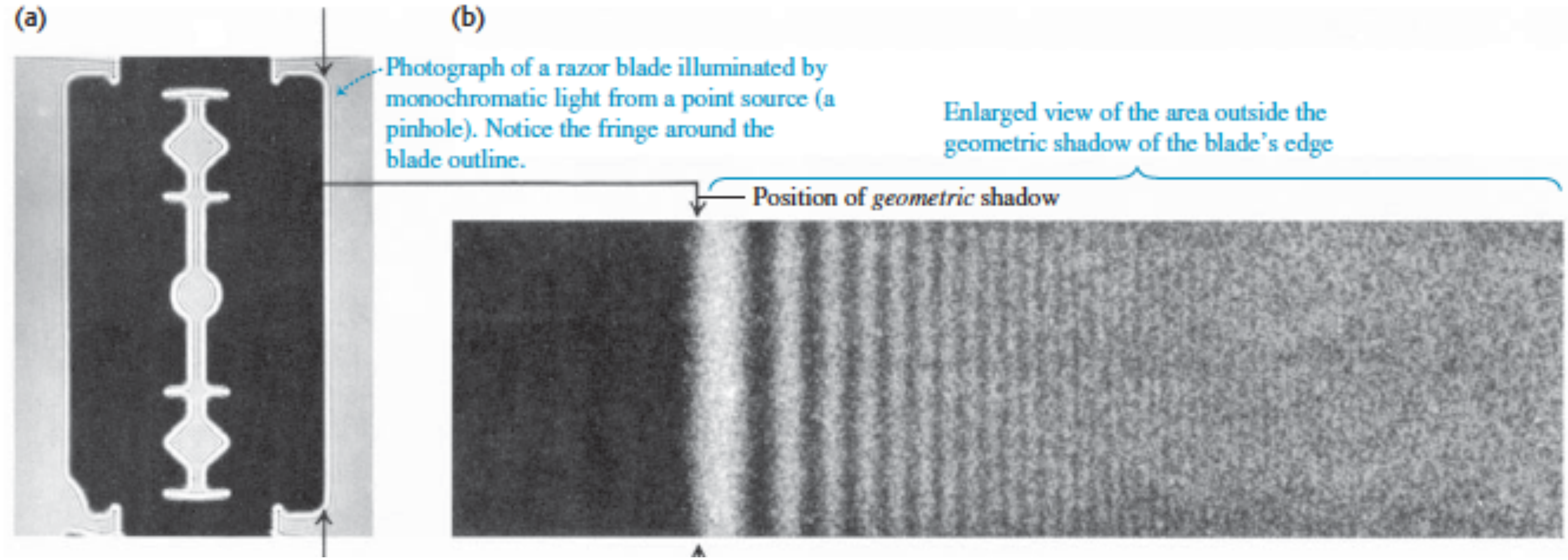
(c) Destructive interference: Waves interfere destructively if their path lengths differ by a half-integral number of wavelengths: $r_2 - r_1 = (m + ½)\lambda$.



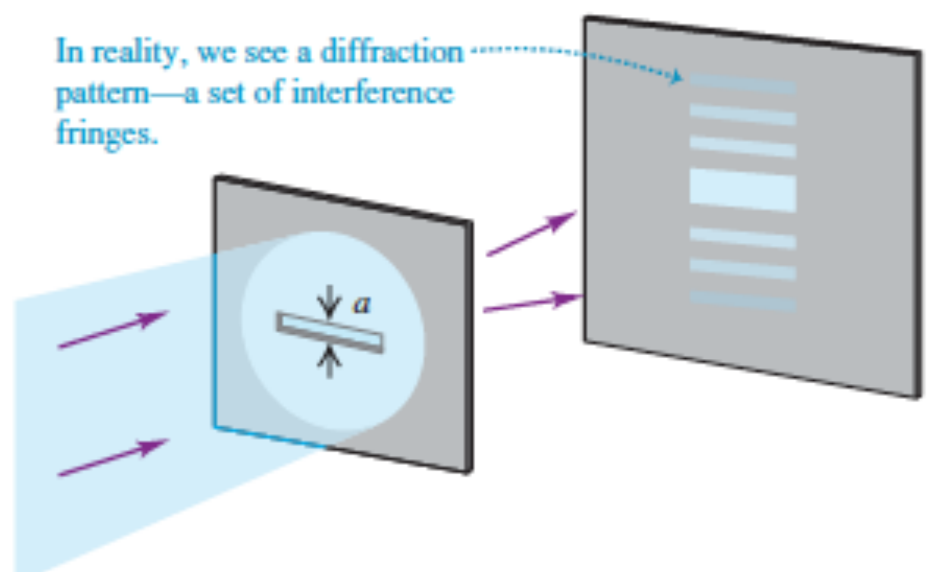

Antinodal curves (red) mark positions where the waves from S_1 and S_2 interfere constructively.
At a and b, the waves arrive in phase and interfere constructively.
$m = 3$
$m = 2$
$m = 1$
$m = 0$
$m = -1$
$m = -2$
$m = -3$
y
x
S_1
S_2
a
b
c
At c, the waves arrive one-half cycle out of phase and interfere destructively.
m = the number of wavelengths λ by which the path lengths from S_1 and S_2 differ.

B. Diffraction: Contemporary View

- Diffraction of light by an opaque object:

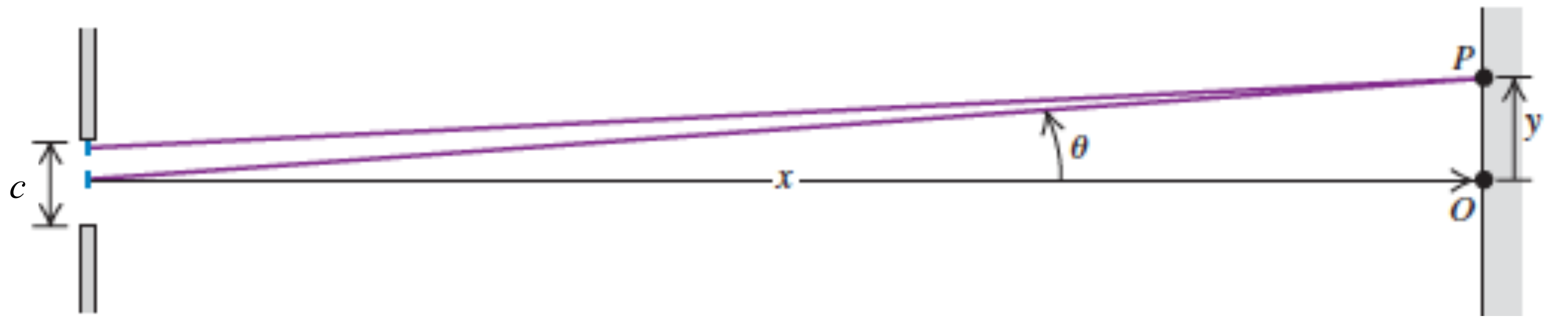


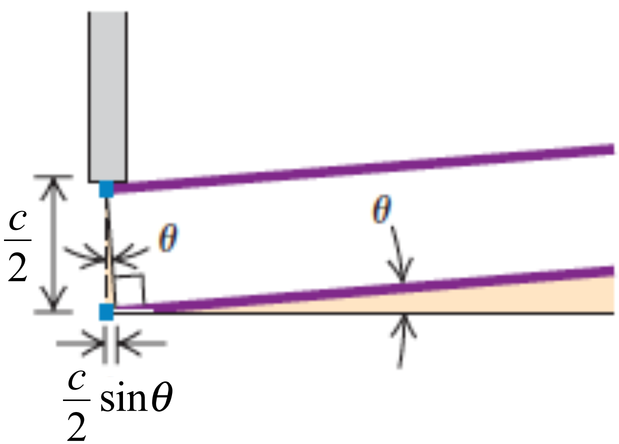
- Diffraction from a single slit:



How to locate dark fringes (minima) for single-slit diffraction:

- x = distance to screen; c = width of slit, λ = wavelength of light.
- *Assume*: $x \gg c \gg \lambda$ ("Fraunhoffer diffraction")





- Two ray paths to P differ by $(c/2)\sin\theta$.
- *Condition for destructive interference at P*:

$$(c/2)\sin\theta = (m + \tfrac{1}{2})\lambda \qquad (m = 0, \pm1, \pm2, ...)$$

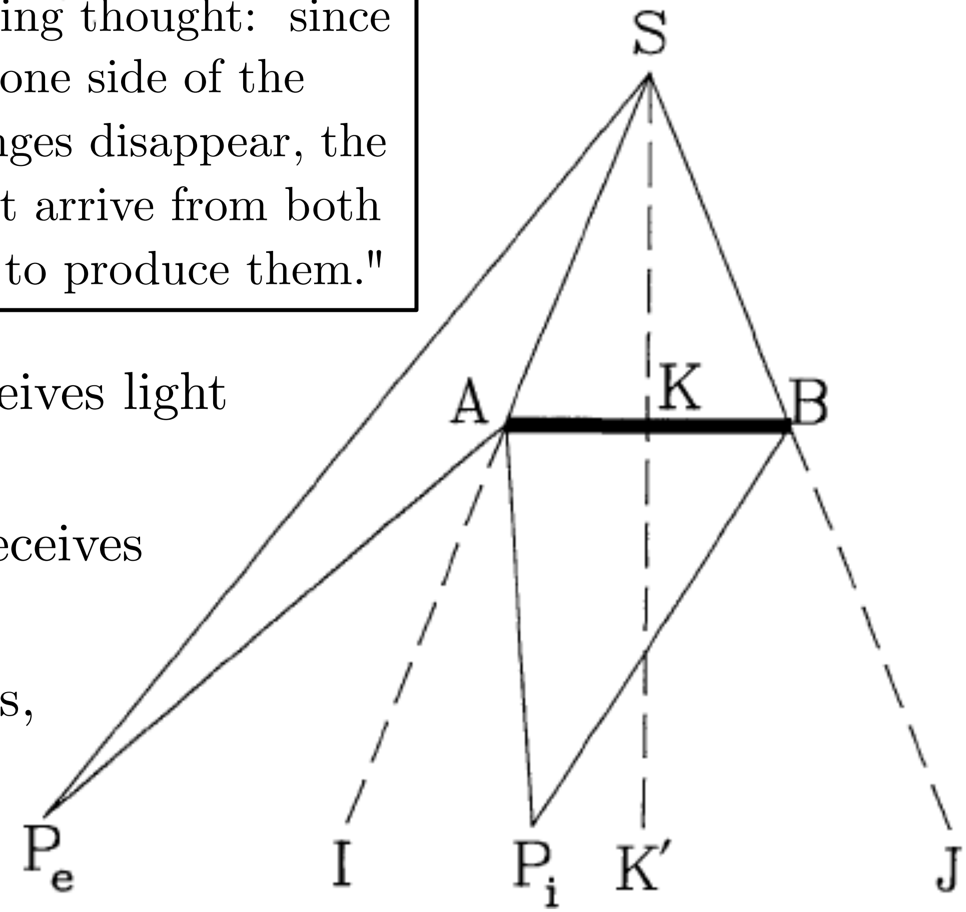
- *Or*: $\sin\theta = n\lambda/c$ $\qquad (n = \pm1, \pm3, ...)$
- *And*: *Location of dark fringe* $= y = x\tan\theta$.
- *Now*: For $c \gg \lambda$, $\sin\theta \approx \theta$, $\tan\theta \approx \theta$.
- *So*: $\theta = n\lambda/c$, and

$$y = x(n\lambda/c), \qquad n = \text{odd integer}$$

C. Fresnel's Binary Ray Theory of Diffraction

- 1815. Fresnel observes fringes formed inside and outside the geometric shadow of a wire.

"... I at once had the following thought: since intercepting the light from one side of the wire makes the internal fringes disappear, the concurrence of the rays that arrive from both sides is therefore necessary to produce them."



- Point P_i inside geometric shadow receives light from edges A, B of obstacle.
- Point P_e outside geometric shadow receives light from source S and edges A, B.
  - <u>*But*</u>: For sufficiently large obstacles, rays from B will not be significant.
- <u>*So*</u>: Only need consider two rays.
  - If path lengths are integral multiples of wavelength, then bright fringe results; if path lengths are half-integral multiples of wavelength, then dark fringe results.

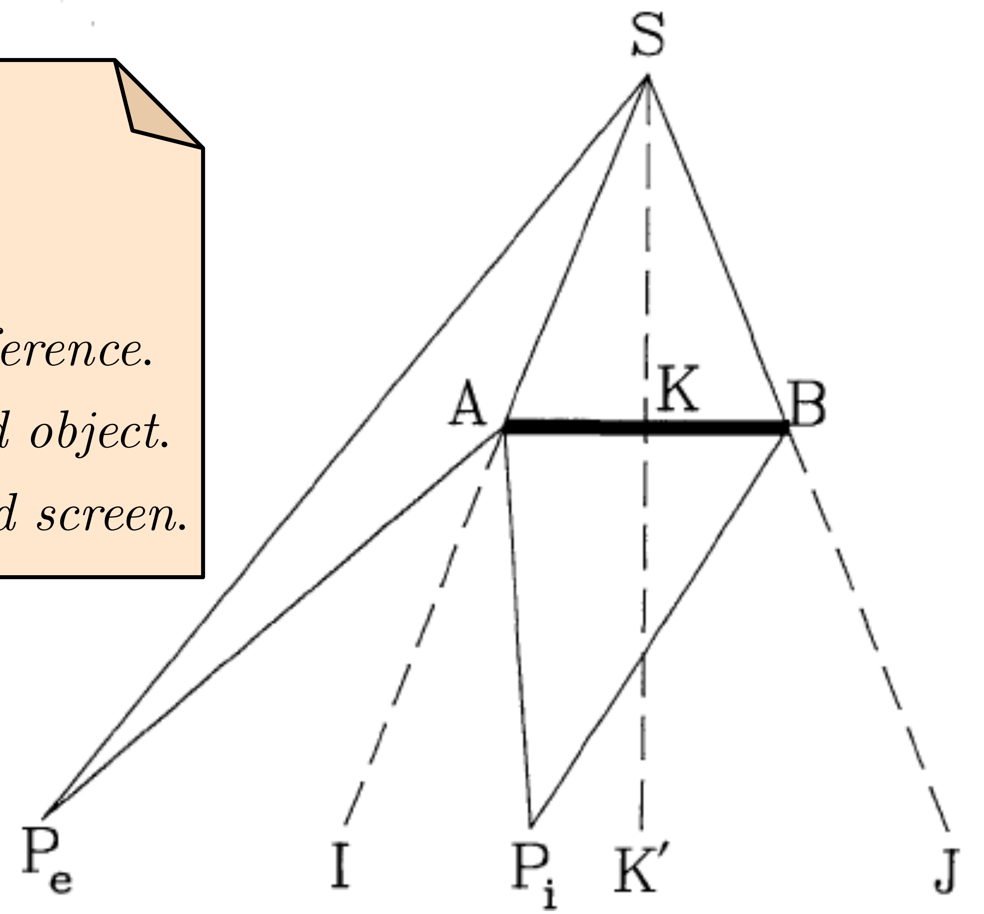
Location of External Fringe:

- Circles centered at S and A represent interfering waves.
- Intersection of circles approximates a hyperbola.
- Distance $y = P_e I$ from external fringe to edge of geometric shadow I can be approximated by:

External Fringe Formula:

$$y \approx \sqrt{\frac{2db(a+b)}{a}}$$

- $d = (SA + AP_e) - SP_e =$ *path difference.*
- $a = SK =$ *dist. between source and object.*
- $b = KK' =$ *dist. between object and screen.*



Location of Internal Fringe:

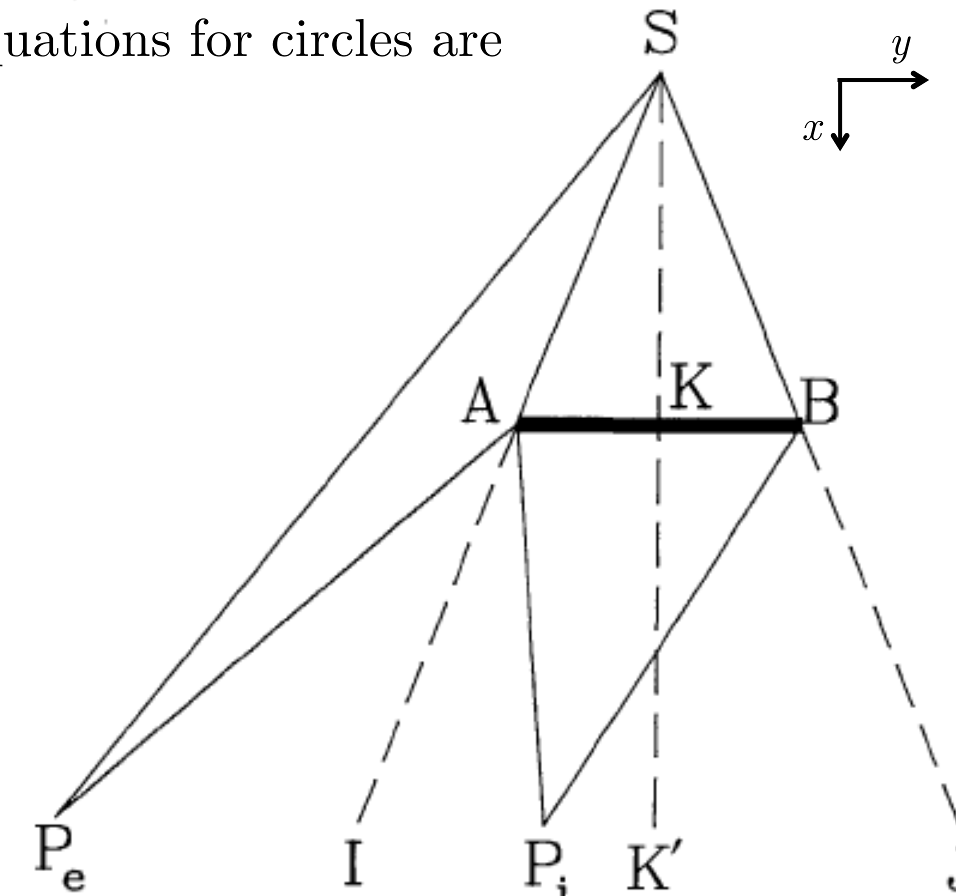
- If P_i is location of dark ("minimum") fringe, then it lies at intersection of circles centered at A and B with radii differing by $n/2$, n = odd integer.
- Let c = wire length, λ = wavelength of light, and consider (x, y)-coordinate system with origin at K.
- *Task*: Find coordinates of $P_i = (x, y)$, where $y = P_iK'$, $x = KK'$.
- $A = (0, -c/2)$, $B = (0, c/2)$, and equations for circles are

$$(y - c/2)^2 + x^2 = r^2$$
$$(y + c/2)^2 + x^2 = (r + n\lambda/2)^2$$

- *Or*: $y = n/(2c)(r\lambda + \lambda^2/4)$.
- *Or*: For small λ^2 and $r \approx b$:

Internal Fringe Formula:

$y = b(n\lambda/2c)$



- *Slight Problem*: Formula predicts 3rd ($n = 3$) dark fringe should be outside geometric shadow.
- *But*: Internal formula holds as long as contribution from far edge is not significant (*i.e.*, distinction between internal and external fringes doesn't refer directly to geometric shadow, but rather to effects from the three sources).

Fresnel's Principle: (Buchwald, pg. 132.)

A wave impinging on a material body sets its particles in vibration, with the result that each such particle becomes a source of secondary radiation that propagates both in the original medium (reflection) and in the refracting medium at appropriate speeds.

- *But*: "... the fronts proper are not yet for Fresnel the loci of secondary emission as in Huygens's Principle."

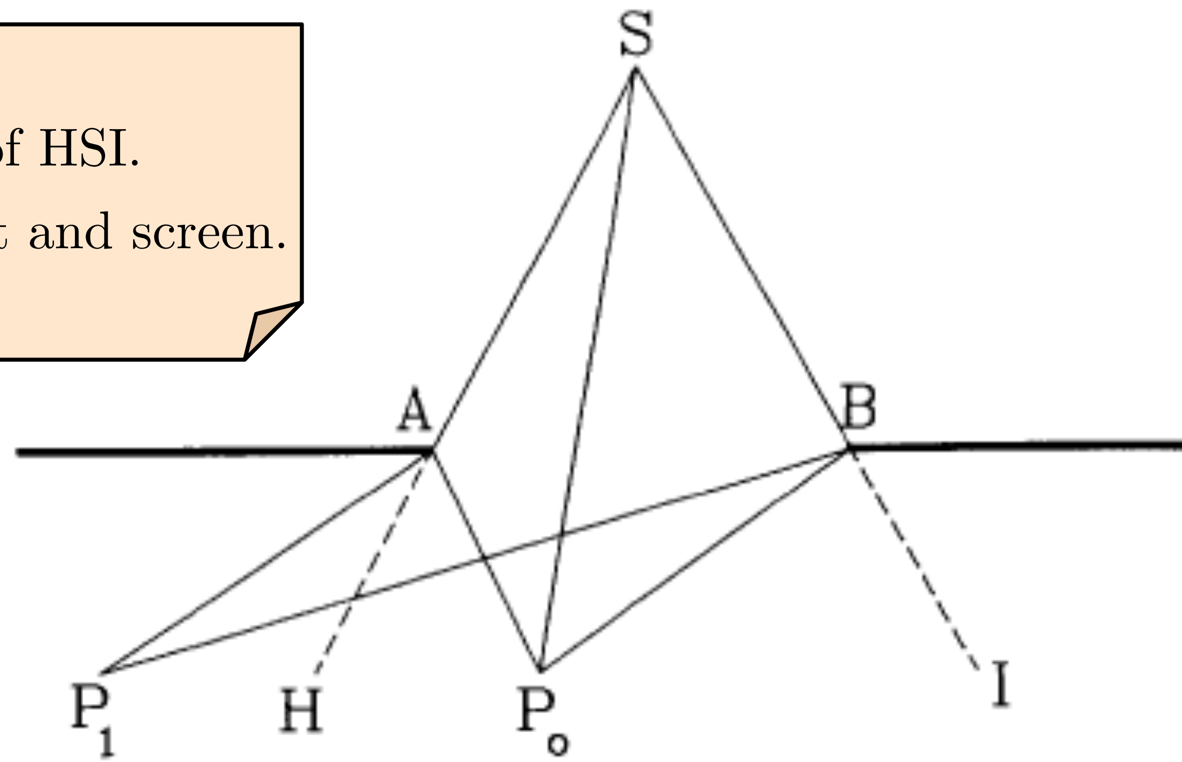
> "The most natural hypothesis [to explain diffraction] is that the molecules of the body set in vibration by the incident light become the centers of new undulations. Analogy leads one to suppose that in reflection the molecules that compose the surface of the reflecting body also become centers of new luminous undulations. How is it that these undulations are sensibly propagated only in a direction that makes with this surface an angle equal to the angle of incidence? That is easy to explain when one sees that, in every other direction, the vibrations of the reflected rays are mutually contrary and destroy one another."

- *In other words*: "Fresnel then thought that light propagates in right lines, not because Huygens's secondary waves conspire with one another to form succeeding fronts, but because a given element of a given front is prevented dynamically by its contiguous neighbors from dilating in any direction but the front normal." (Buchwald, pg. 132.)

D. The Binary Ray Theory Supplemented with the Efficacious Ray

- 1816. Fresnel experiments with single slit diffraction.
- Use internal fringe formula to compute fringe locations *outside* region *HSI*:

$y_n = [b\lambda(2n - 1)/2c]$
- y_n = distance to center of HSI.
- b = distance between slit and screen.
- c = width of slit.



- *Thus*: First fringe should be located at $b\lambda/2c$.
- *And*: Distance between fringes is twice distance from center to first fringe:

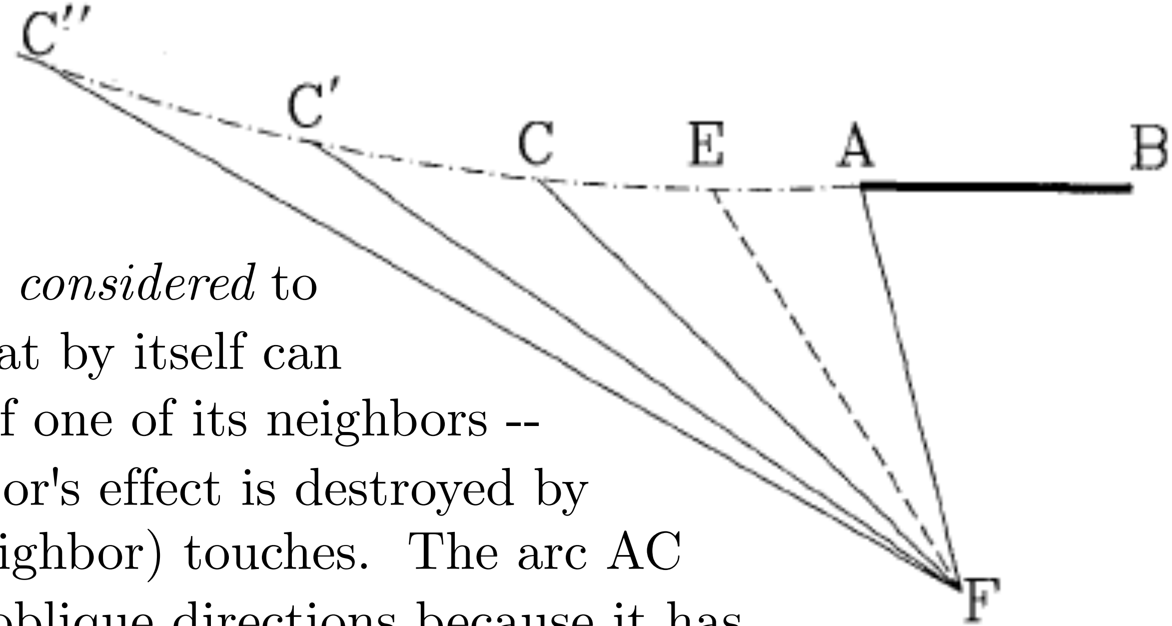
$$y_{n+1} - y_n = b\lambda/c = 2y_1$$

- *Problem*: Fresnel observes first fringe at $b\lambda/c$.
- *And*: Subsequent fringes are displaced by $b\lambda/2c$ (half the predicted fringe spacing) from their predicted locations.

"...some new reflections and observations made me doubt the exactness of a hypothesis from which I had calculated my formulas: that the center of undulation of the inflected light was always at the edge proper of the opaque body or, what is the same, that inflected light could come only from rays that have touched its surface."

- "... a cardinal assumption of Fresnel's physical theory at this time was that each point on the front acts *only* along the normal to the front; there was no notion at all of oblique radiation of any kind." (Buchwald, pg. 143.)
- Fresnel now shifts locations of ray emissions off material object and onto the near portion of the wavefront.
- *A radical change*: "... because it fundamentally altered the role of the diffractor from an active one (as a secondary emitter) to a passive one (as an interrupter of the front)." (Buchwald, pg. 143.)

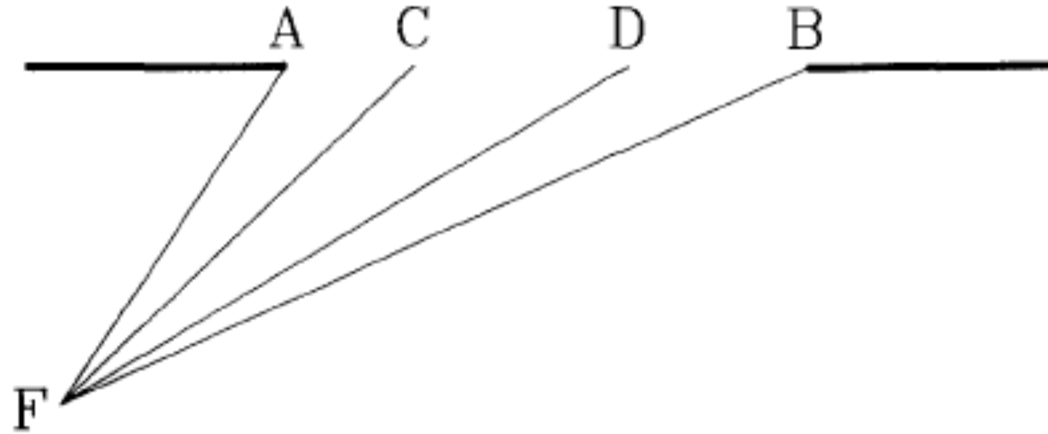
- Wavefront AC'' hits obstacle AB.
- Let FA, FC, FC', FC'', ... have lengths $AF + n\lambda/2$.



- Each arc on front "... *may be considered* to produce oblique radiation that by itself can destroy only half the effect of one of its neighbors -- the second half of the neighbor's effect is destroyed by the other arc that it (this neighbor) touches. The arc AC therefore has a net effect in oblique directions because it has only one neighbor..." (Buchwald, pg. 145.)
- Fresnel chooses the center E of the emitting arc AC as the "principal" point of emission.
- *Then*: EF represents the new ray (the "efficacious ray") emitted by the wavefront, and it is a quarter-wavelength longer than AF.

Fixes problem with single-slit diffraction

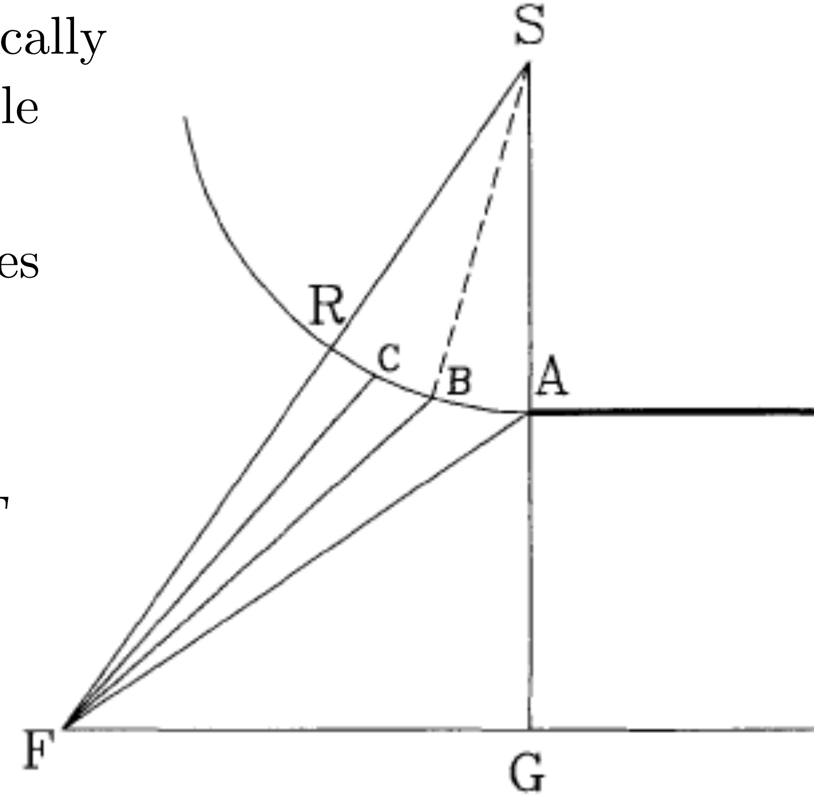
- *Recall*: Problem was that first dark fringe is observed at $b\lambda/c$, but formula predicts $b\lambda/2c$ (half as much).
- Let δ = difference between radii AF, BF of wavelets centered at edges A, B.
- Let $AF = r \approx b$, so $BF = b + \delta$.
- *Then*: Wavelets are given by



$$(y - c/2)^2 + x^2 = b^2$$
$$(y + c/2)^2 + x^2 = (b + \delta)^2$$

- In original theory, $\delta = \lambda/2$, which gives incorrect formula $y = b\lambda/2c$.
- *But*: Now efficacious rays are $CF = b + \lambda/4$ and $DF = b + \delta - \lambda/4$.
- *And*: First dark fringe produced when DF and CF differ by a half wavelength: $DF - CF = \delta - \lambda/2 = \lambda/2$.
- *Which means*: $\delta = \lambda$.
- *And*: This yields observationally correct formula $y = b\lambda/c$.

- *But*: New theory cannot produce the empirically correct formula for external fringes in obstacle diffraction.
- Efficacious ray $BF = AF - \lambda/4$ and interferes with direct ray SRF.
- Let $y = FG$, $a = FA$, $b = AG$.
- *Then*: Path difference $d = (SB + BF) - SF$ $= SB + AF - SF - \lambda/4$.
- *And*: This gives $y \approx \sqrt{2(d + \lambda/4)b\dfrac{(a+b)}{a}}$.



- *So*: First fringe with $d = \lambda/2$ should be located at $y_1 \approx \sqrt{\dfrac{3\lambda b(a+b)}{2a}}$.
- *But*: Not what is observed.
- "This failure on Fresnel's part to retrieve the original formula for external diffraction fringes shows almost conclusively that he did not permit oblique radiation to occur except from points very near the edge of the diffractor. For if he had, he would have been able very simply to solve this problem." (Buchwald, pg. 150.)