16. Black Hole Thermodynamics. Part 1.

1. Gravitational Collapse.

Gravitational Collapse in Newtonian Physics

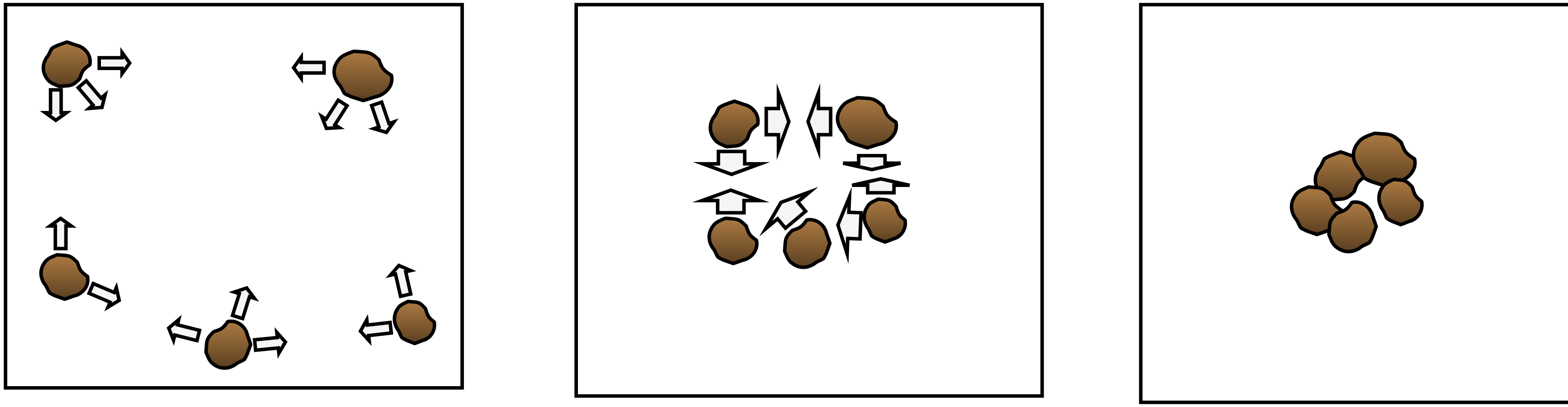
- Michell (1784) "On the Means of discovering the Distance, Magnitude, *etc.* of the Fixed Stars...", *Phil. Trans. Roy. Soc. London*, lxxiv, 35-57.
- Escape velocity of sun = 497 times smaller than velocity of light.
- *So*: Any object with same density as sun and radius 497 times larger will trap all light that falls on it:

John Michell
(1724-1793)

> "... if the semi-diameter of a sphere of the same density with the sun were to exceed that of the sun in the proportion of 500 to 1, a body falling from an infinite height towards it would have acquired at its surface a greater velocity than that of light, and consequently, supposing light to be attracted by the same force in proportion to its *vis inertiae*, with other bodies, all light emitted from such a body would be made to return towards it, by its own proper gravity."

- How might such objects originate?

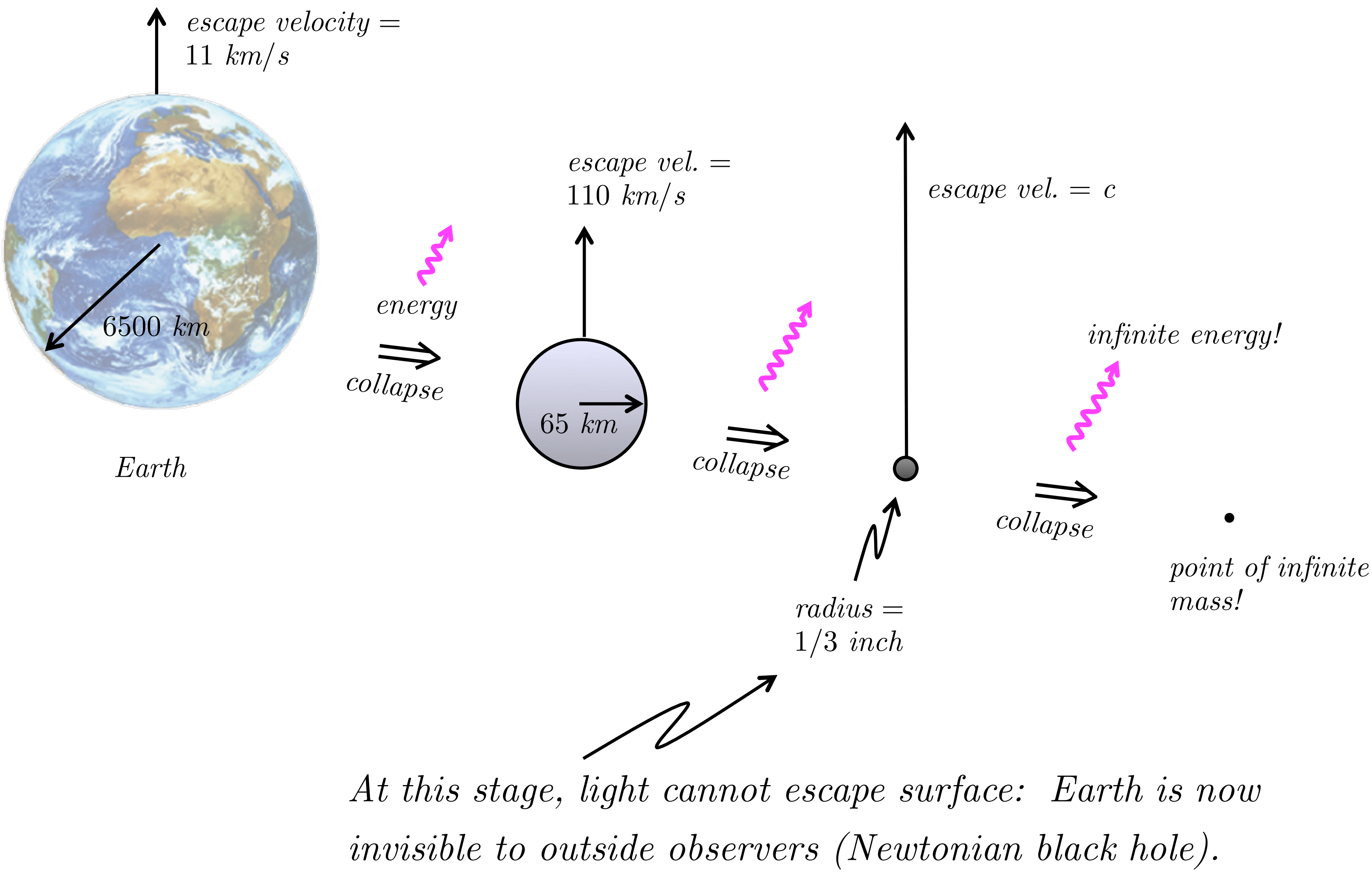
- Consider Newtonian gravitational collapse: $F_G \propto 1/r^2$.
  - Decreasing the distance by half increases the force by 4.



Matter widely dispersed in space : Grav. force weak. $\Longrightarrow$ *Grav. force increases as distance decreases.* $\Longrightarrow$ *Collapse accelerates.*

- Is this an entropy-decreasing process?
  - For *interaction-free* gas, increased "clumping" is associated with decrease in entropy.
  - *But*: For *self-gravitating* gas, decrease in entropy due to clumping is typically overcome by increase in entropy due to *release of energy*.

- In Newtonian gravitational collapse, if nothing intervenes, matter collapses to a point with *infinite* energy release.



Newtonian Picture: Complete Gravitational Collapse

$$\begin{bmatrix} \textit{Unrestrained} \\ \textit{gravitational} \\ \textit{collapse} \end{bmatrix} \Longrightarrow \begin{bmatrix} \textit{infinite} \\ \textit{energy} \\ \textit{release} \end{bmatrix} + \begin{bmatrix} \textit{singular point} \\ \textit{of infinite} \\ \textit{matter density} \end{bmatrix} + \begin{bmatrix} \textit{background} \\ \textit{spacetime} \\ \textit{unaffected} \end{bmatrix}$$

In General Relativity (Einstein 1916):

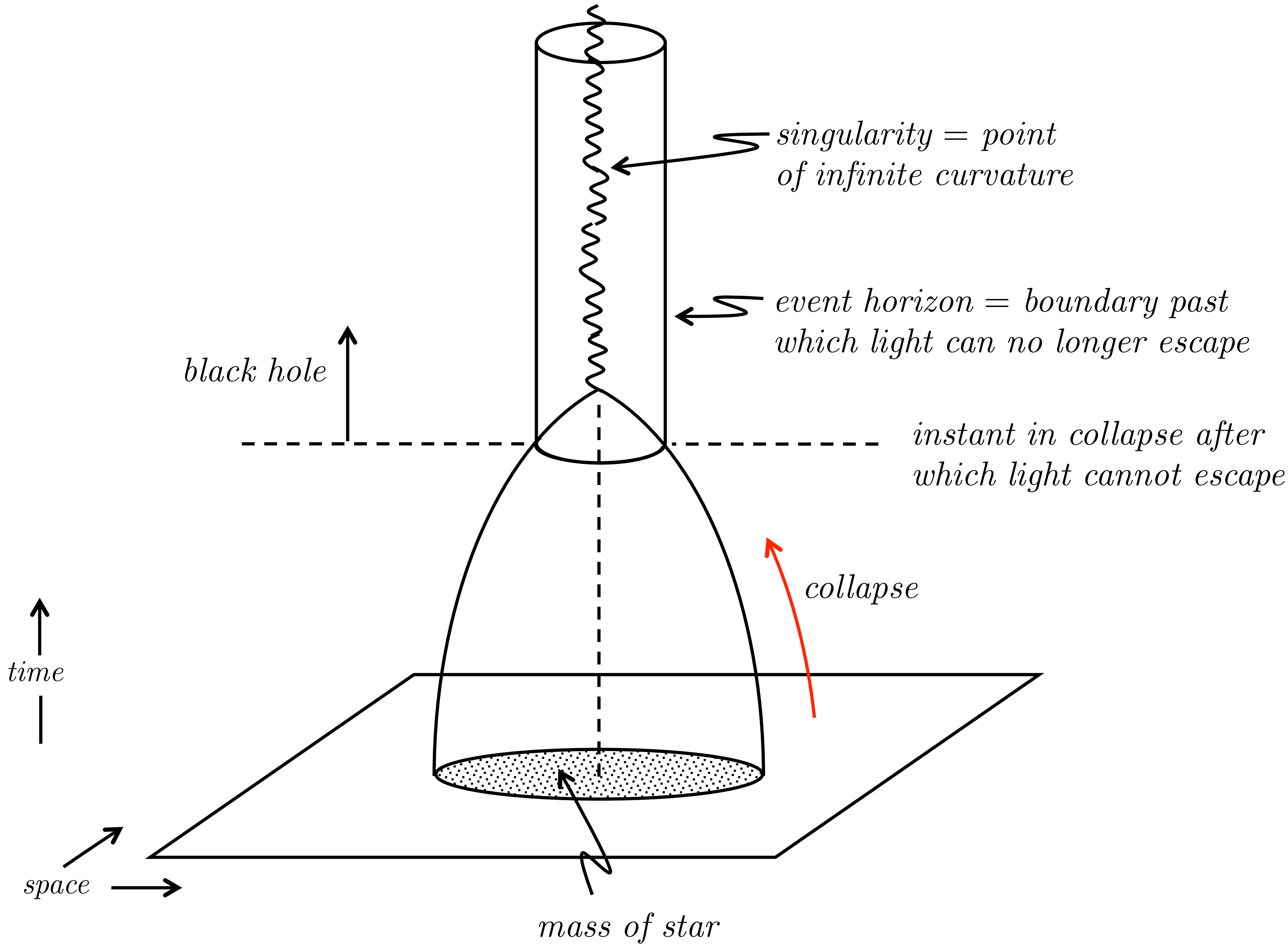
- Equivalence of mass and energy.
- Curvature of spacetime is determined by mass-energy density.

Relativistic Picture: Complete Gravitational Collapse

$$\begin{bmatrix} \textit{Unrestrained} \\ \textit{gravitational} \\ \textit{collapse} \end{bmatrix} \Longrightarrow \begin{bmatrix} \textit{finite energy} \\ \textit{release} \end{bmatrix} + \begin{bmatrix} \textit{singular point of infinite} \\ \textit{spacetime curvature} \end{bmatrix}$$

2. Relativistic Black Holes.

Ex 1: Schwarzschild black hole: no charge or rotation (Schwarzschild 1916).



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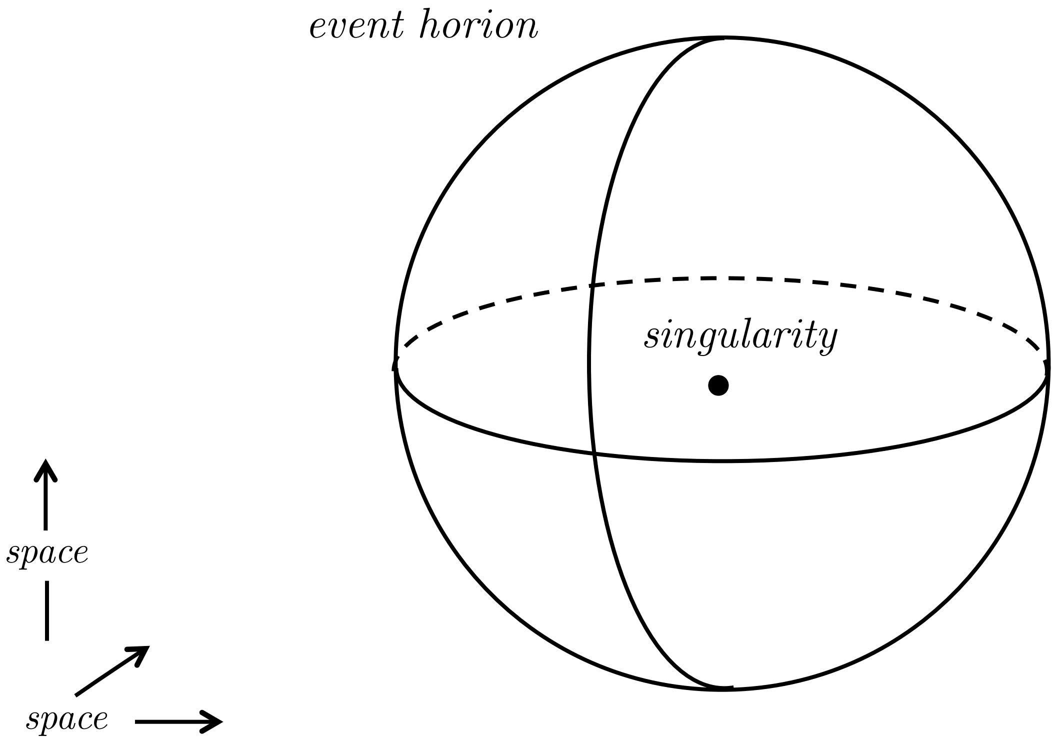
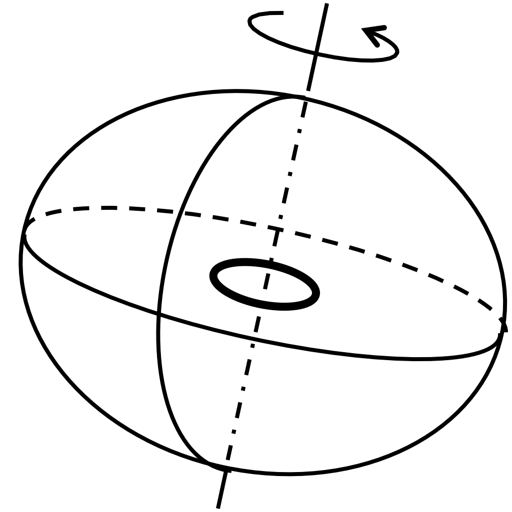


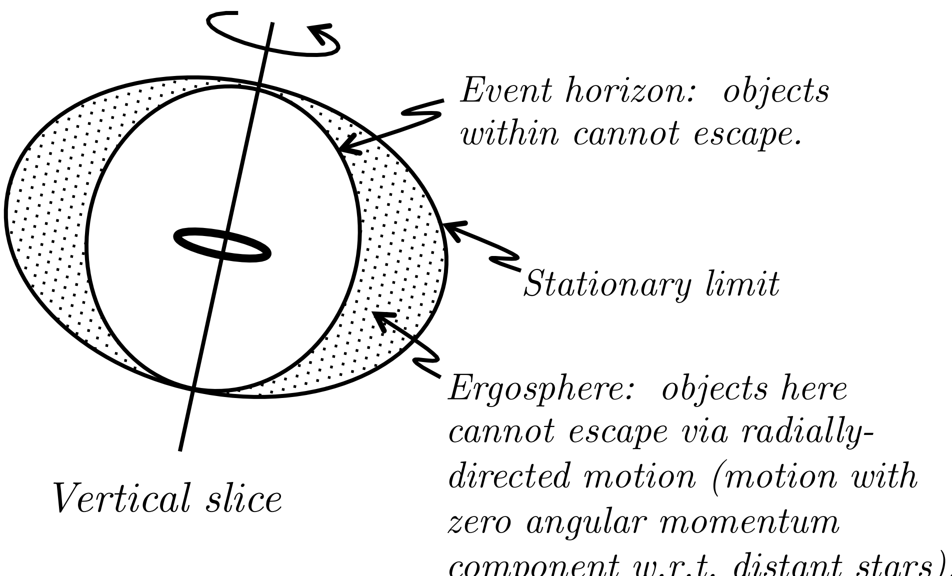
Ex 2: Rotating black hole. (Kerr 1963).



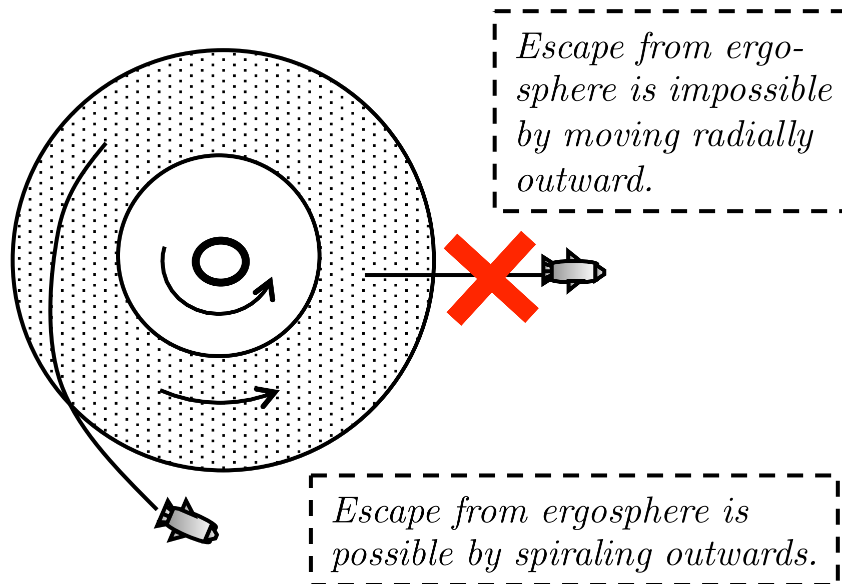
- ***Consequences of rotation***:
  - Singularity is a ring, not a point.
  - Inertial frames near singularity are dragged.

- ***Stationary limit*** = distance from singularity at which objects can no longer have zero angular momentum with respect to distant stars.

- ***Ergosphere*** = Region between event horizon and stationary limit.



Vertical slice

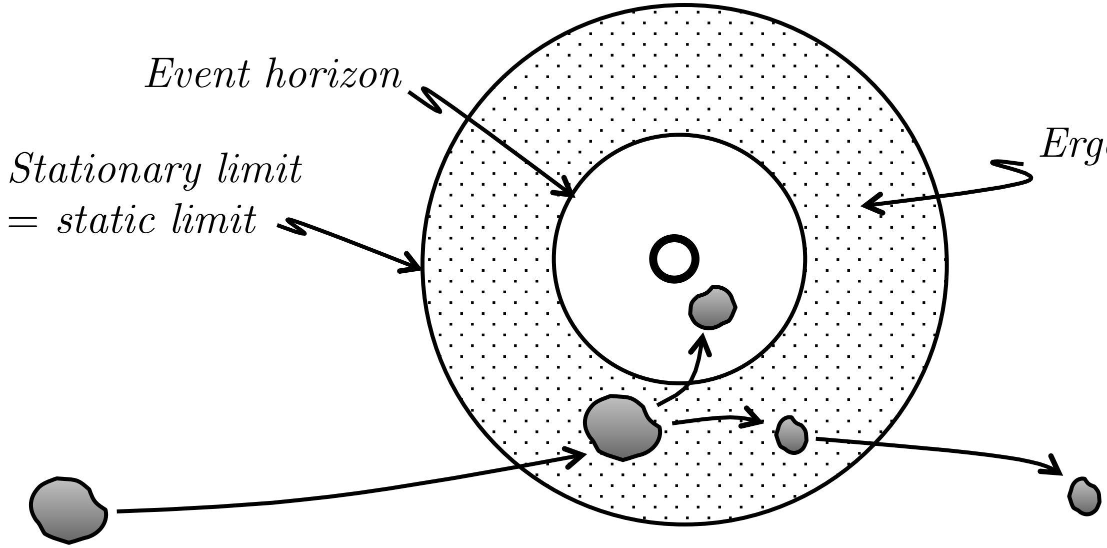


Horizontal slice

- <u>*Static limit*</u> = distance from singularity at which objects can no longer escape *via* radially directed motion.
- Inside static limit, the "Killing" vector field ξ^a that encodes time-translation symmetries is *spacelike*: $|\xi^a| > 0$.
- <u>*So:*</u> Inside static limit, objects with *negative energy*, $E = -\xi^a p_a$, can exist.

- Allows the following procedure for extracting energy (Penrose 1969):

Negative energy object = object that requires more energy than its rest mass in order to move it to "infinity".



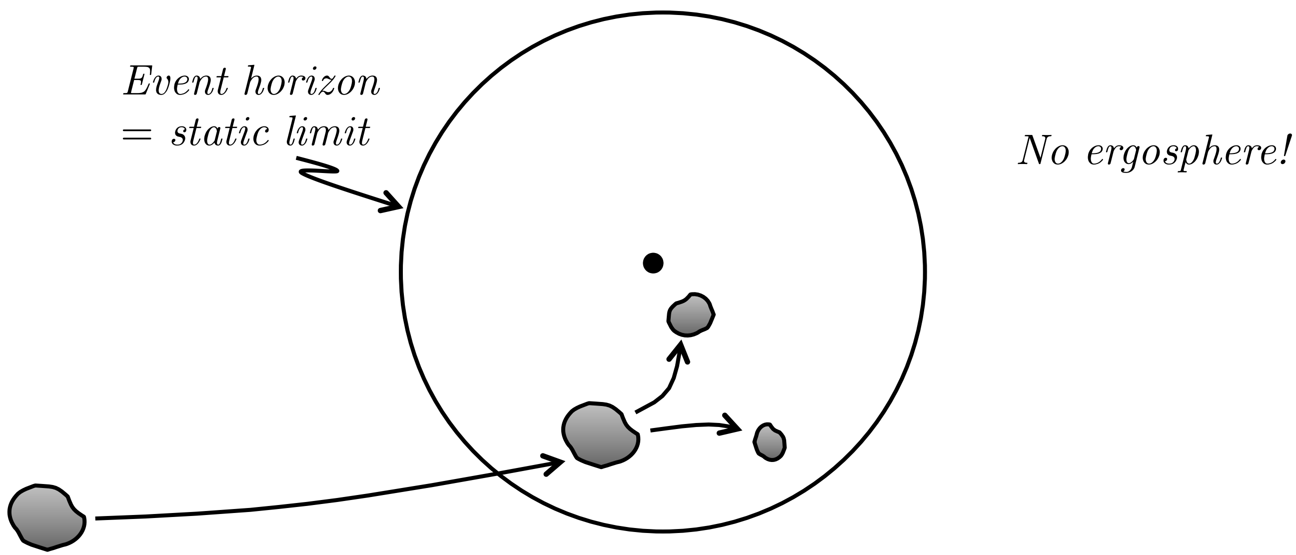
Lump of matter with energy E shot into ergosphere.

Fragments into two chunks with energies E_1, E_2 such that
(a) $E_1 + E_2 = E$.
(b) $E_1 < 0$, $E_2 > 0$.

Chunk with neg energy falls into singularity, chunk with positive energy emerges with greater energy than initial lump!

Why won't this work for a Schwarzschild black hole?

- Schrwarschild black hole: static limit = event horizon.
- *So*: Negative energy states are possible inside event horizon.
- *But*: To guarantee your rock is split into negative energy and positive energy pieces, you need to be inside the region of negative energy states when you split it!



Lump of matter with energy E shot into event horizon.

Fragments into two chunks with energies E_1, E_2 such that
(a) $E_1 + E_2 = E$.
(b) $E_1 < 0$, $E_2 > 0$.

Both chunks can't escape!

General Properties of Relativistic Black Holes

- ***No Hair Conjecture:*** A black hole is completely characterized by its mass M, charge Q, and angular momentum J.

Four types of black hole:

	nonrotating $(J = 0)$	*rotating* $(J \neq 0)$
uncharged $(Q = 0)$	Schwarzschild	Kerr
charged $(Q \neq 0)$	Reissner-Nordström	Kerr-Newman

- *Radius of event horizon*: $R_h = M + [M^2 - Q^2 - (J/M)^2]^{1/2}$
- *Area of event horizon*: $A = 4\pi[R_h{}^2 + (J/M)^2]$
  - *So:* A small change in mass δM will correspond to small changes in area δA, charge δQ, and angular momentum δJ.

- ***Hawking (1971) Area Theorem:*** $\delta A \geq 0$ in any process.
  - *Ex:* Suppose two black holes with areas A_1, A_2 collide to form black hole with area A_3. Then $A_3 \geq A_1 + A_2$.

Stephen Hawking

3. The Laws of Black Hole Mechanics. (Bardeen, Carter, Hawking 1973)

	Black Hole Mechanics	*Thermodynamics*
0th Law	Surface gravity κ is constant over the event horizon of a stationary black hole. *κ is acceleration needed to keep an object at event horizon.*	Temperature T is constant throughout a body in thermal equilibrium.
1st Law	$\delta M = (1/8\pi)\kappa\delta A + \Phi\delta Q + \Omega\delta J$ *Φ is electrostatic potential, Ω is rotational velocity.*	$dE = TdS + pdV + \Omega dJ$
2nd Law	$\delta A \geq 0$ in any process.	$\delta S \geq 0$ in any process.
3rd Law	$\kappa = 0$ is not achievable by any process.	$T = 0$ is not achievable by any process.

- Formally identical if $A/4 = S$ and $(1/2\pi)\kappa = T$.
- Is this merely a *formal equivalence*, or does it have a *physical basis*?