

15. Information and Maxwell's Demon.

I. Dilemma for Information-Theoretic Exorcisms.

- Two Options:

(S) (*Sound*). The combination of object system and demon forms a canonical thermal system.

(P) (*Profound*). The combination of object system and demon does not form a canonical thermal system.

- If (S), then the 2nd Law applies and no appeal to information is necessary.
- If (P), then one needs a new physical postulate to explain why the 2nd Law applies phrased in terms of information and entropy.

- Concerning (P):

"The issue is whether a valid principle concerning the entropy costs of information acquisition and processing can defeat demonic devices."

(Earman & Norton 1999)

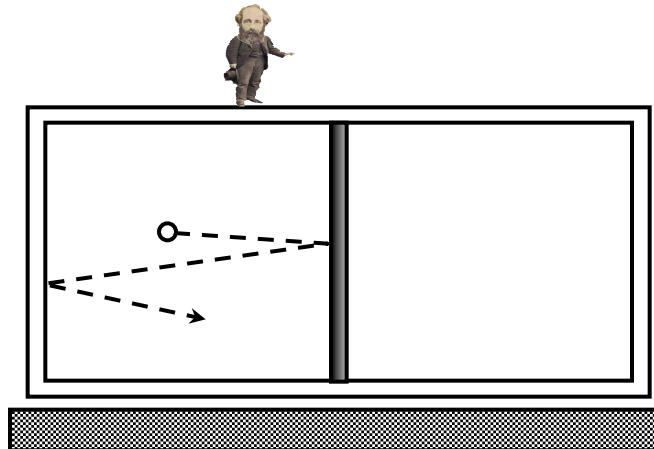
II. Two Approaches to Information-Theoretic Exorcisms.

- Szilard's Principle:

Gaining information that allows us to discern between n equally likely states is associated with a minimum cost in entropy of $k \log n$.



- Recall: In Szilard's one-molecule engine, to obtain information about which side the molecule is located requires a cost in entropy of $k \log 2$.



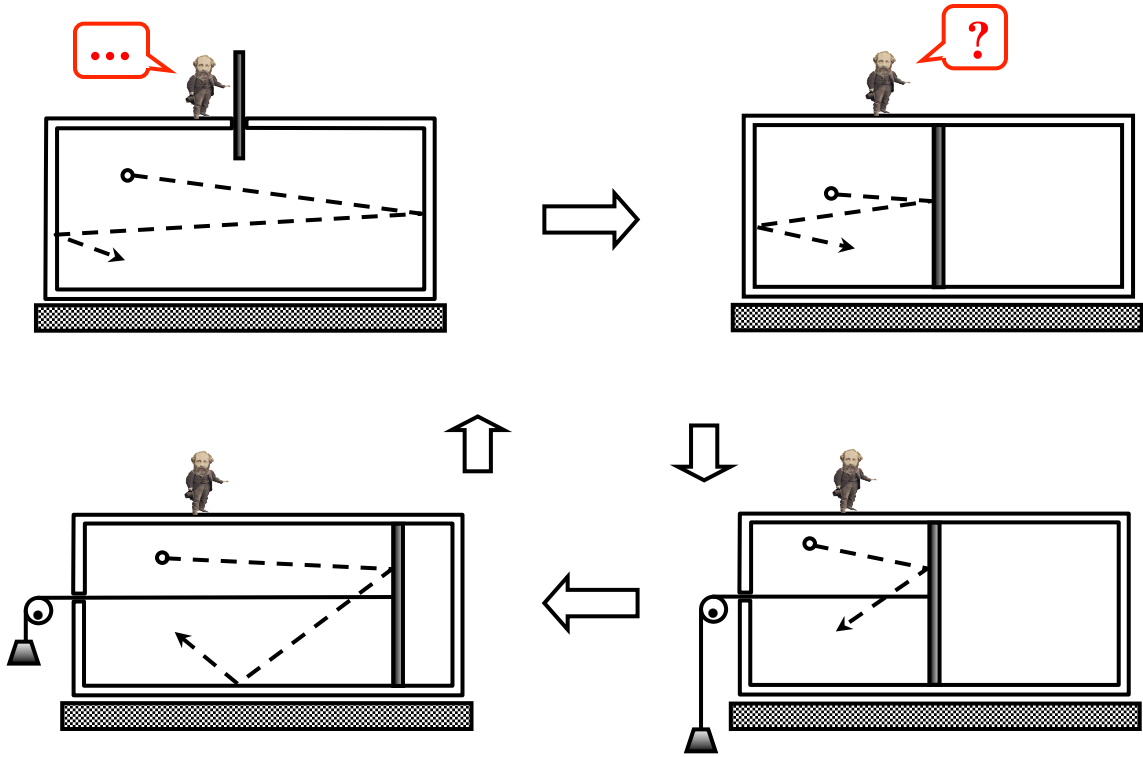
- Landauer's Principle (1961):

Erasing information that allows us to discern between n equally likely states is associated with a minimum cost in entropy of $k \log n$.



Rolf Landauer

- The demon works in a *cycle*.
- At some point, it must acquire information.
- At some later point, it must erase this information in order to return to its initial state.



1. Brillouin on Szilard's Principle (1953).

- Consider a thermodynamical system in a macrostate Γ_{D_i} corresponding to W ($= G(D_i)$) equiprobable microstates ("arrangements").
- Then: The Boltzmann entropy is given by $S = k \log W$.

Let the information I associated with a process that reduces the number of microstates from W_0 to W_1 be given by

$$I = k \log W_0 / W_1$$



Leon Brillouin

- Motivation: A reduction in the number of microstates corresponds to a positive value of I .

- Now: Suppose state 0 evolves to state 1 as a result of the use of I .
- Then: The entropy associated with the use of I is given by:

$$S_1 - S_0 = k \log W_1 / W_0 = -k \log W_0 / W_1 = -I$$

This transition is associated with a conversion of information I into "negentropy" (negative entropy)!

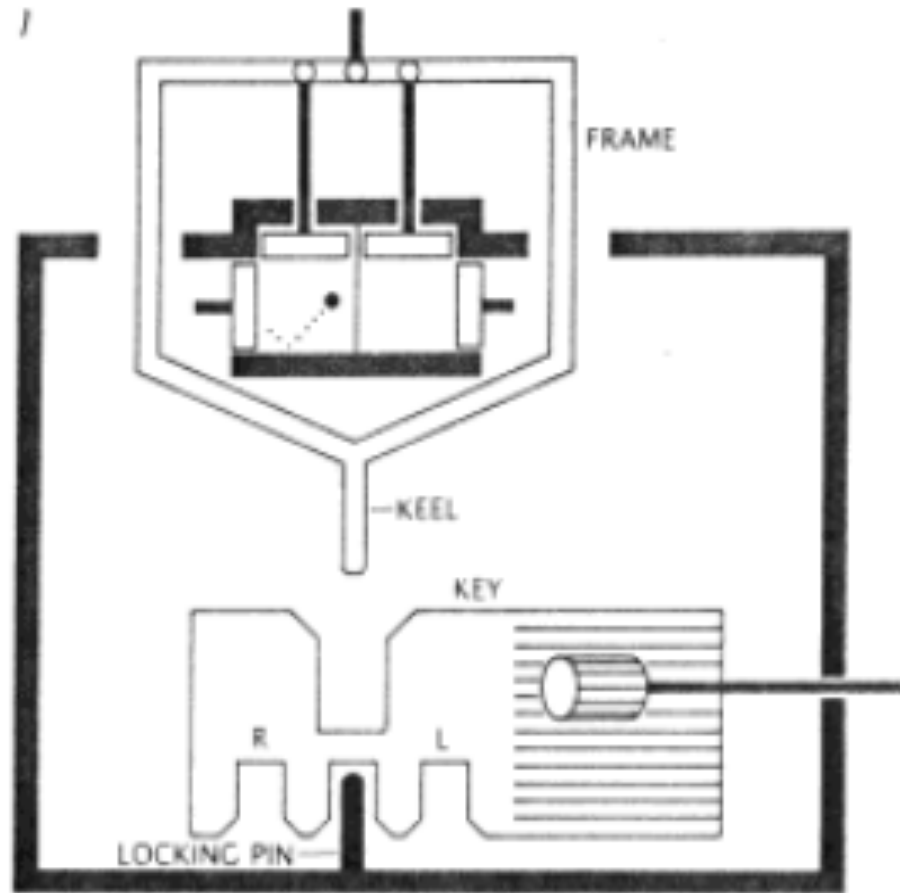


These remarks lead to an explanation of the problem of the Maxwell's Demon, which simply represents a device changing negentropy into information and back to negentropy...

- But: This takes option (S).
- So: There's no need for references to "information" or "negentropy".
- If demon and gas obey 2nd Law, then any increase in "negentropy" (decrease in entropy) associated with the demon will be compensated for by an increase in entropy somewhere else.

2. Challenge to Szilard's Principle.

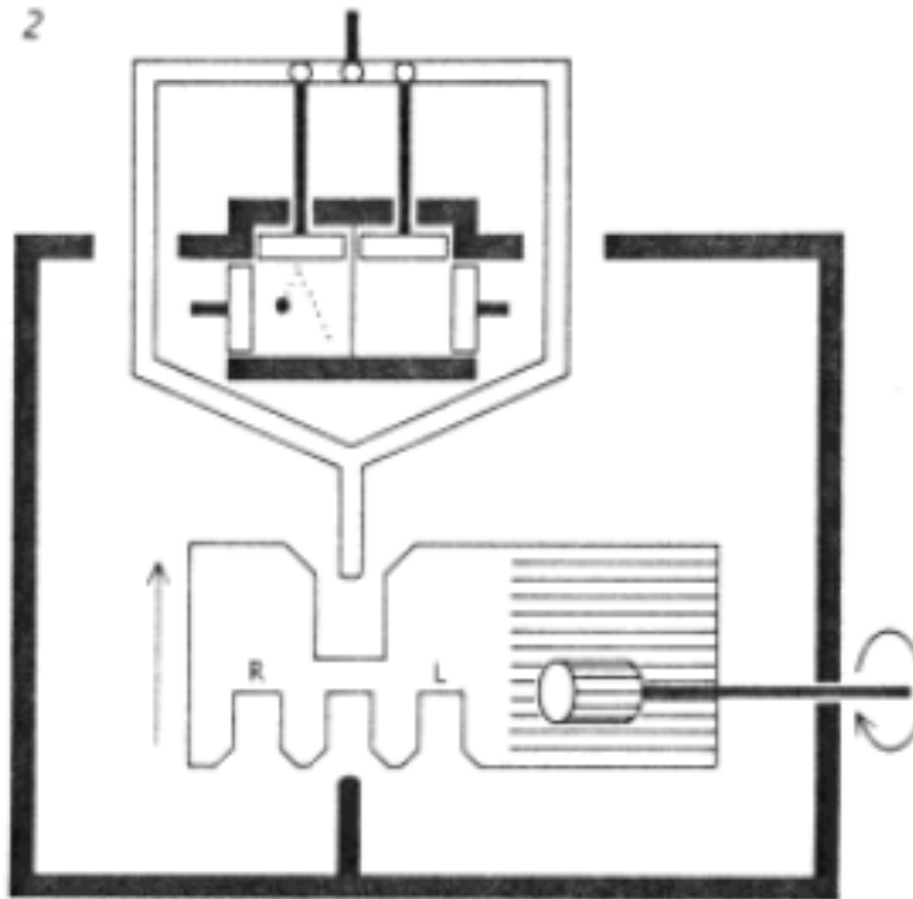
- Claim (Bennett 1987): Info acquisition can be achieved without entropy cost.



"A slightly modified Szilard engine sits near the top of the apparatus (1) within a boat-shaped frame; a second pair of pistons has replaced part of the cylinder wall. Below the frame is a key, whose position on a locking pin indicates the state of the machine's memory."

2. Challenge to Szilard's Principle.

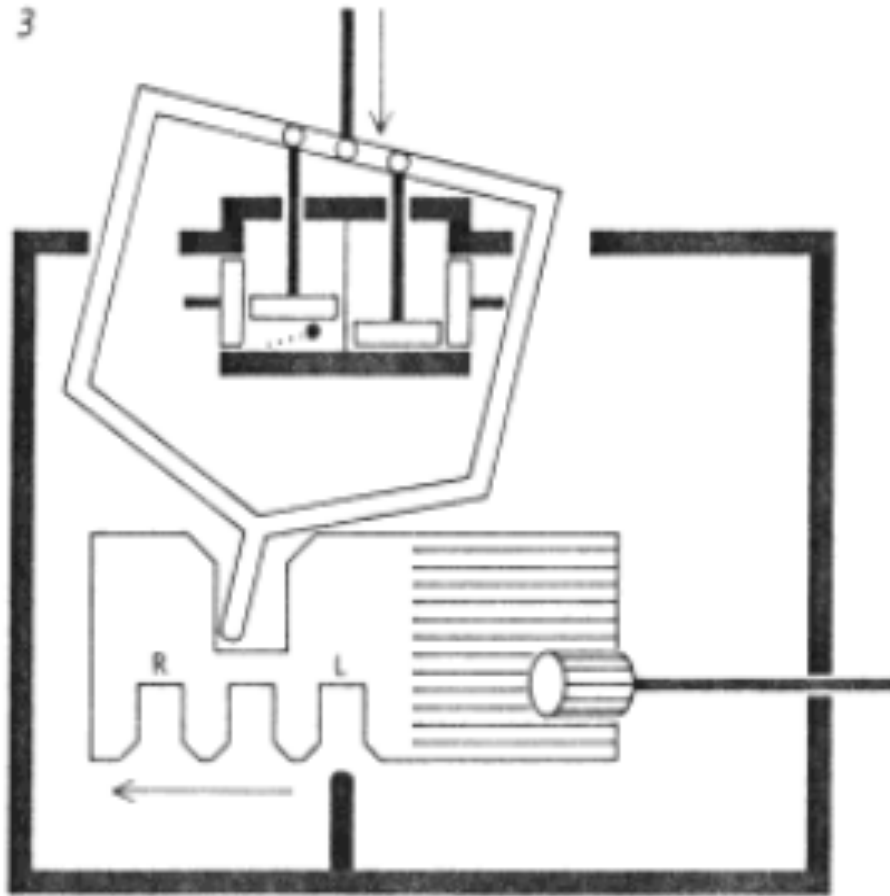
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"To begin the measurement (2) the key is moved up so that it disengages from the locking pin and engages a 'keel' at the bottom of the frame."

2. Challenge to Szilard's Principle.

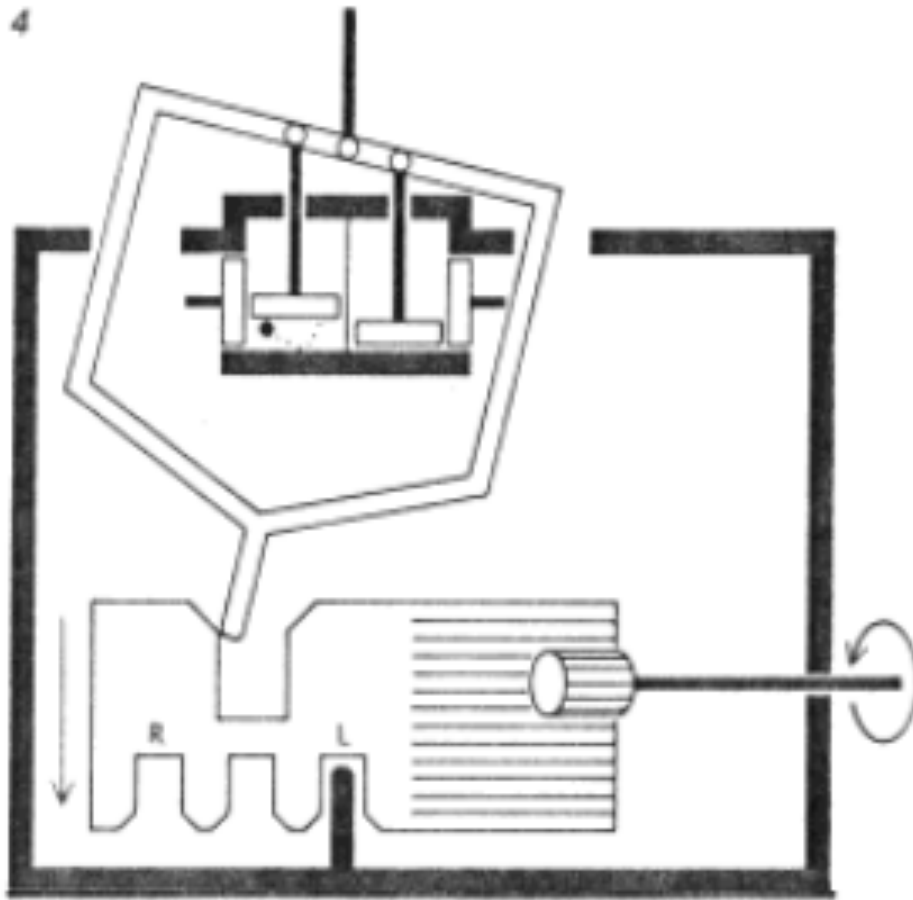
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"Then the frame is pressed down (3). The piston in the half of the cylinder containing no molecule is able to descend completely, but the piston in the other half cannot, because of the pressure of the molecule. As a result the frame tilts and the keel pushes the key to one side."

2. Challenge to Szilard's Principle.

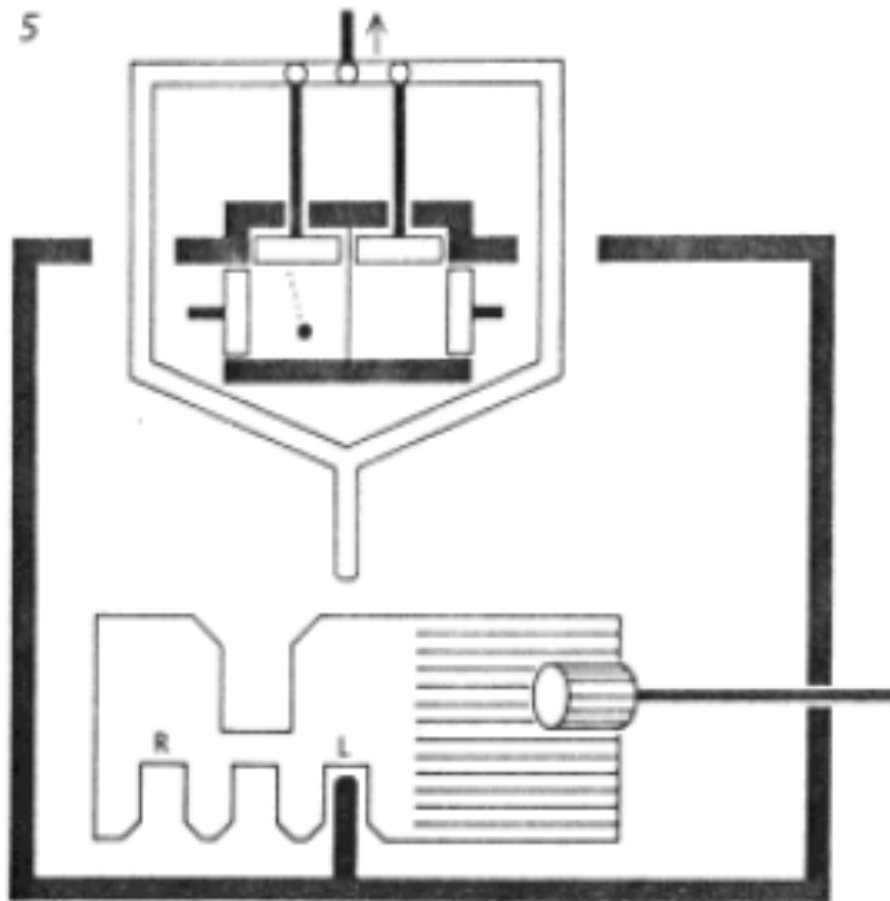
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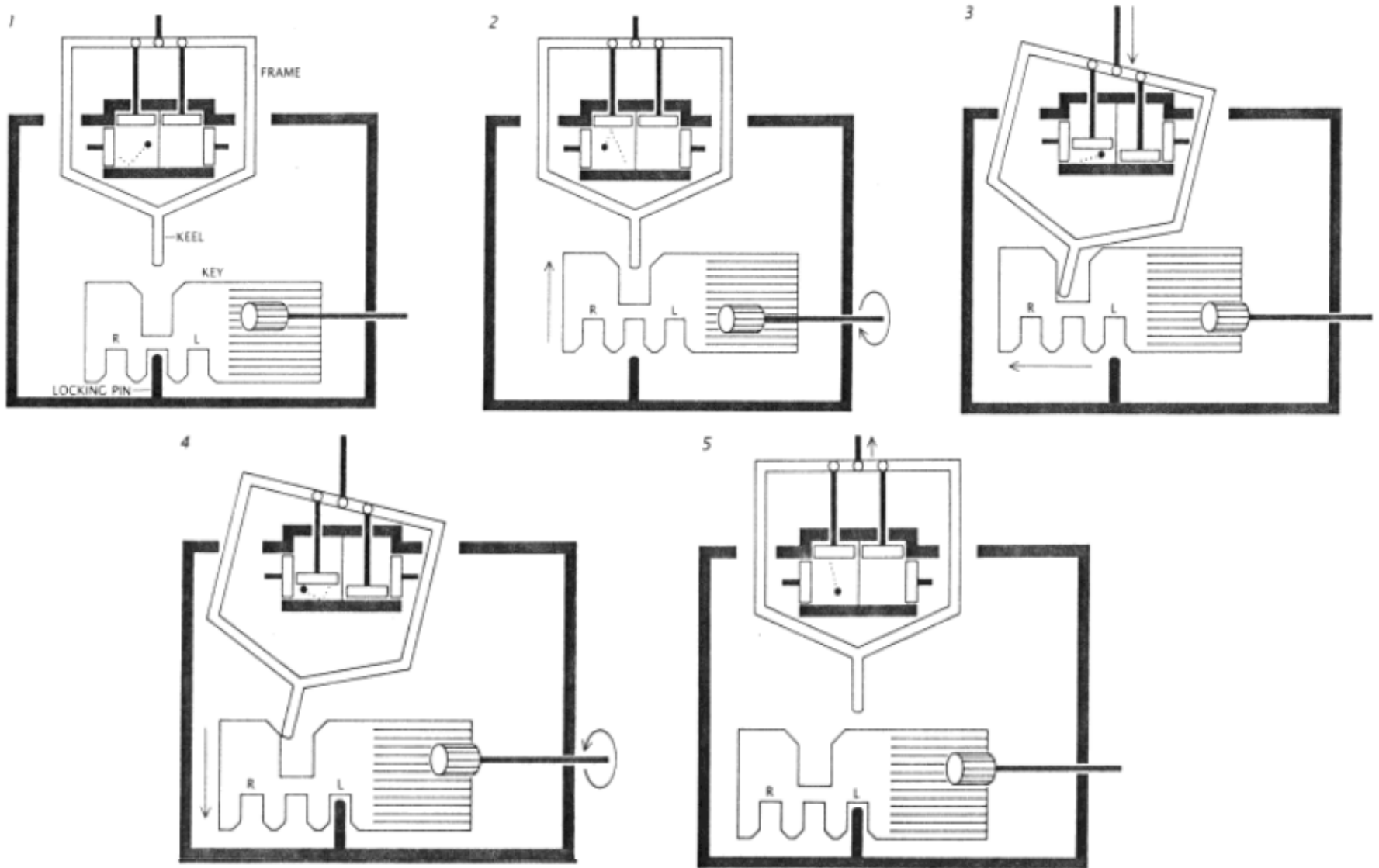
"The key, in its new position, is moved down to engage the locking pin (4), and the frame, is allowed to move back up (5)..."

2. Challenge to Szilard's Principle.

- Claim (Bennett 1987): Info aquisition can be achieved without entropy cost.



"The key, in its new position, is moved down to engage the locking pin (4), and the frame, is allowed to move back up (5), undoing any work that was done in compressing the molecule when the frame was pressed down."

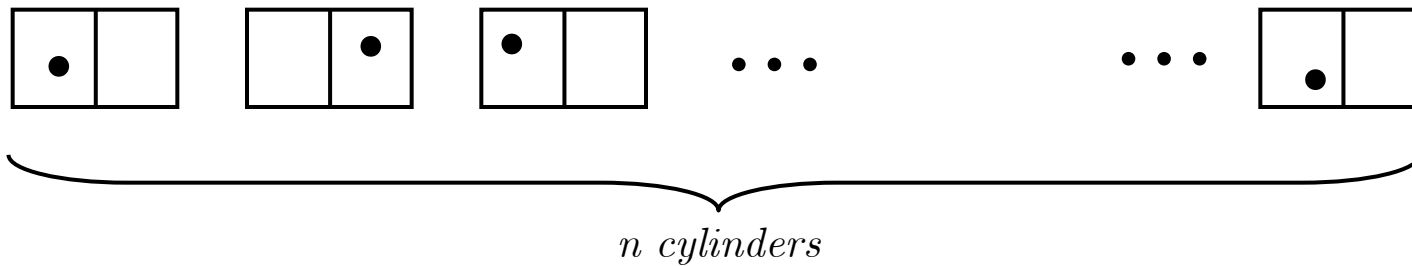


- Measurement without entropy cost?
- No: Any mechanical device will be subject to thermal fluctuations thus obliterating its measuring function (Earman and Norton 1999).

3. Landauer's Principle.

General Idea:

- *Logical states* of a computer must be represented by *physical states* of its hardware.
- Example: An n -bit memory register as an array of n two-chambered cylinders, each filled with a one-molecule gas.



Let molecule in left correspond to "0"; molecule in right correspond to "1".

- Each cylinder has 2 possible states; thus entire register has 2^n possible states.
- Now: Set register to zero (erase all bits).
- Before erasure: Register can be in any of 2^n states.
- After erasure: Register is in exactly one state.

Erase involves compressing many logical states into one; just like a piston!

- So: Erasure = compression of many physical states (high entropy) into exactly one (low entropy).

"Hence one cannot clear a memory register without generating heat and adding to the entropy of the environment. Clearing a memory is a thermodynamically irreversible operation."



Charles Bennett

- What's the Moral for Maxwell's Demon?
- E&N Problem #1:
Not all physical processes admit descriptions in terms of information erasure (recall Smoluchowski's one-way valve).
- E&N Problem #2:
Bennett claims Szilard's Principle *fails*, because we can *ignore* thermal fluctuations for measuring devices; while Landauer's Principle *succeeds*, because we *cannot ignore* thermal fluctuations for erasure devices (they are physical, thermal systems). Is this inconsistent?

- *E&N Problem #3*: Computerized demons don't need to erase information.
- Consider a binary state memory device with two states: "*L*" and "*R*".
- *Claim*: A routine in which the system is found to be in state *L*, and then switched to state *R*, is not an erasure routine. (Bennett agrees.)
 - *Why?*
 - It's logically reversible
 - It doesn't involve mapping many states to one.

- *Program for Szilard's One-Molecule Engine with No Erasure*:

1. Begin in memory register state *L*.
2. If molecule is in left side, do nothing to register.
3. If molecule is in right side, switch to state *R*.
4. Check register:
 - (i) If in state *L*, then do nothing. Commence expansion.
 - (ii) If in state *R*, then commence expansion and reset register to state *L*.

- *Result*: No erasure of memory states needed to return to start of cycle.

- So: Landauer's Principle in particular, and information-theoretic analyses in general, provide no sound basis for the 2nd Law.
- But: What if we restrict attention to thermal systems that explicitly model computational processes?
- In this particular context...

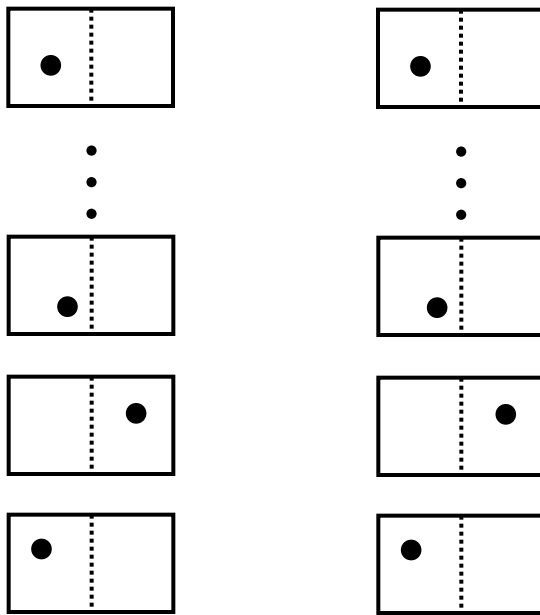
"The question at issue is at what stage of the information acquisition or information processing a computerized demon would fail as a perpetual motion machine, if we assume that the system is a canonical thermal system subject to the 2nd law." (Bub 2001)
- Relevant questions:
 - Does "*computational measurement*" cost entropy?
 - Does "*computational memory erasure*" cost entropy?

Computational Measurement

- Claim: No entropy cost for computational measurement.
- Why?

$$\left(\begin{array}{l} \textit{computational} \\ \textit{measurement} \end{array} \right) = \left(\begin{array}{l} \textit{correlation between the state} \\ \textit{of a measured system and the} \\ \textit{state of the memory register} \\ \textit{of a measurement device.} \end{array} \right) = \left(\begin{array}{l} \textit{copying} \\ \textit{operation} \end{array} \right)$$

- And: *Copying operations cost no entropy.*
- How So? (Feynman 1996)
- Consider two physical memory registers T_1 , T_2 initially in same state.
- Task: Reset both to zero state.



T_1

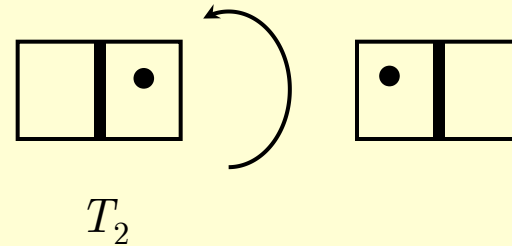
T_2

molecule in left = "0"

molecule in right = "1"

To reset T_2 using T_1 :

1. If first box of T_1 is "0", do nothing to first box of T_2 .
2. If first box of T_1 is "1", insert partition into first box of T_2 (trapping molecule on right) and then flip box:



3. Proceed to next box.

- Reset procedure of T_2 using T_1 is *reversible*: no entropy cost. (Entropy cost comes only in subsequently resetting T_1 with no help from another register.)
- Reverse operation, which likewise costs no entropy, is one in which T_1 is *copied* onto initially blank T_2 .
- Claim: This is a generic copying operation.
- So: Copying operations cost no entropy.

Computational Memory Erasure

- Recall: Earman & Norton's example of a "computerized" demon that operates a Szilard One-Molecule Engine with no information erasure.
- Bub: This is not a computer, but rather an automatic mechanism.

"In most instances, a computer pushes information around in a manner that is independent of the exact data which are being handled, and is only a function of the physical circuit connections."



- "The claim that information erasure, and not information acquisition or information processing, involves a minimum entropy cost depends on the observation that (i) measurement is essentially a copying operation with no entropy cost in principle, (ii) reversible computation is possible in principle, and (iii) erasure involves compressing the phase space of the physical system that functions as a memory register, which requires dumping heat into the environment." (Bub 2001).