The QECC Interpretation of AdS/CFT

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1. Mystery-Mongering
2. The Bulk-Locality Paradox
    - *AdS/CFT Dictionary*
    - *Bulk Reconstruction*
    - *Paradox?*
3. Resolution: The Boundary as a QECC
    - *Quantum Error Correction Codes*
    - *The QECC Interpretation*
4. Spacetime as a QECC?
    - *The Emergent Bulk*
    - *The HaPPY Code Realization*
    - *Common Core or Overarching Structure?*

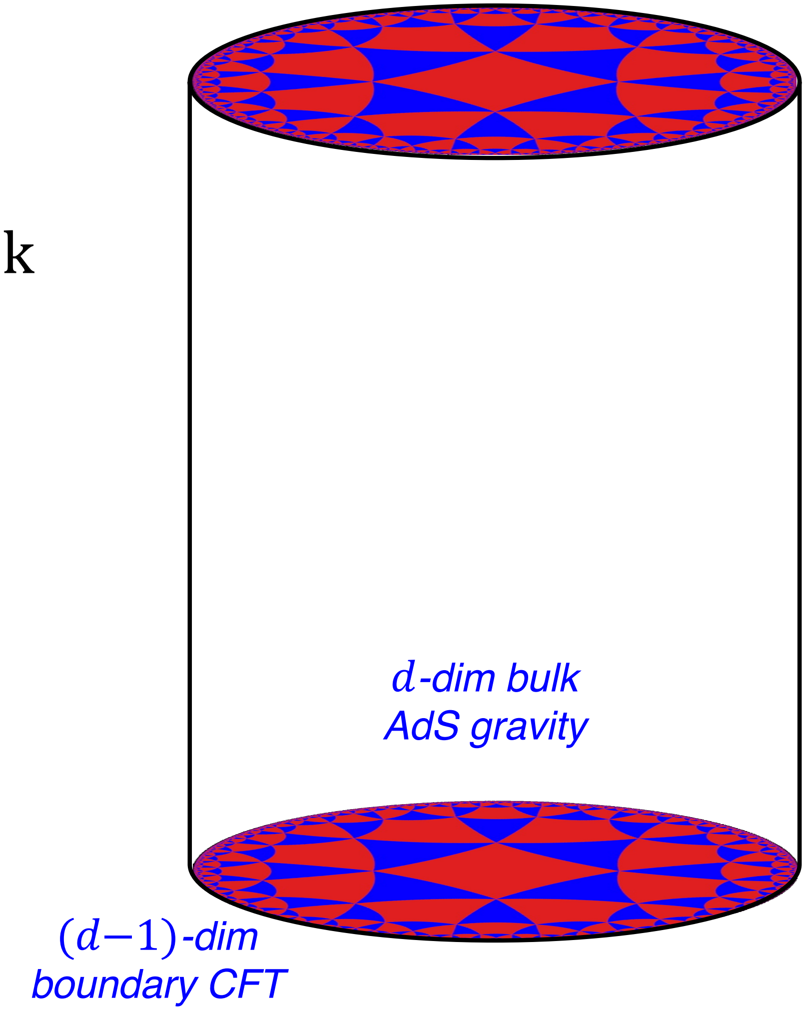
1. Mystery-Mongering

AdS/CFT Correspondence

- *Redundant aspect*: boundary degrees of freedom in $(d-1)$-dim are mapped to bulk degrees of freedom in d-dim.

- Suggests a quantum error-correcting code (QECC): A way to protect information *redundantly* against environmental degradation.

- Fodder for mystery-mongering...

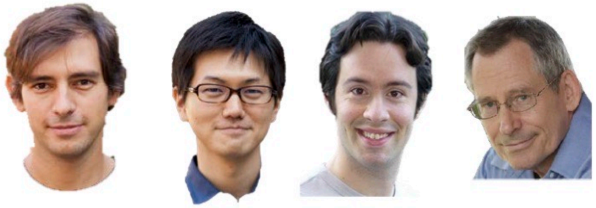


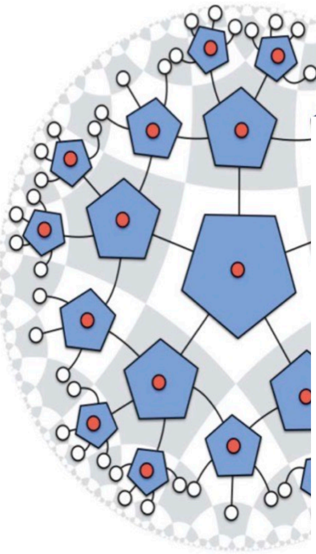
1. Mystery-Mongering

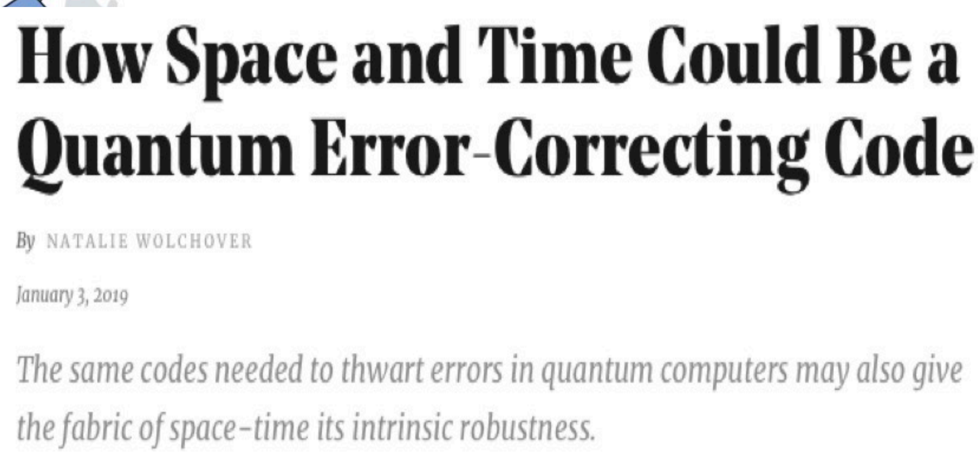
Is spacetime a quantum error-correcting code?

Fernando Pastawski, Beni Yoshida,
Daniel Harlow, John Preskill
= *HaPPY*

J. of High Energy Physics 06 (2015) 149







How Space and Time Could Be a Quantum Error-Correcting Code

By NATALIE WOLCHOVER

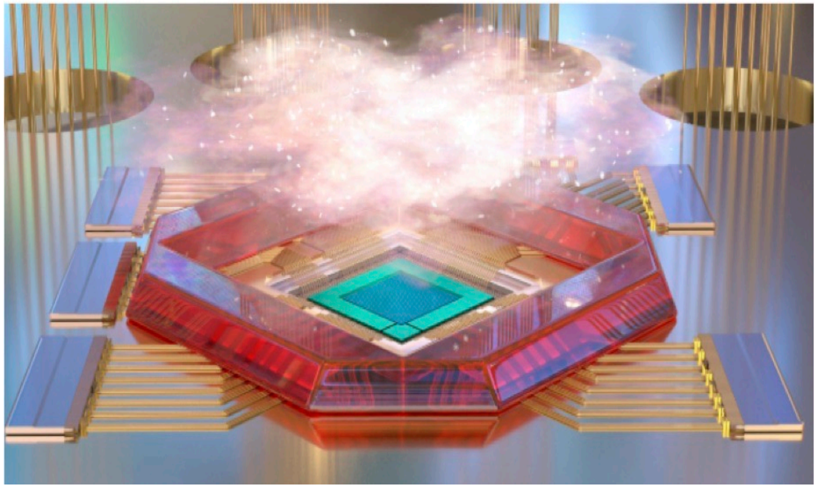
January 3, 2019

The same codes needed to thwart errors in quantum computers may also give the fabric of space-time its intrinsic robustness.

INSTITUTE FOR QUANTUM INFORMATION AND MATTER

GORDON AND BETTY MOORE FOUNDATION

John Preskill
STOC in Montreal
20 June 2017



1. Mystery-Mongering

Towards De-Mystification

- *"Bulk locality paradox"*: Under reasonable assumptions about locality, a standard way of representing a local bulk field on the boundary entails it is trivial.

- *Analogous condition in a QECC*: Local operators that detect and correct errors act like the identity on the codespace.

- *QECC interpretation of AdS/CFT*: A certain collection of bulk states is dual to a subspace of boundary states that forms the codespace for a QECC.

But what does this have to do with spacetime?

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2. The Bulk Locality Paradox

Harlow (2018)

The AdS/CFT Dictionary: 2 Versions

Differentiate version

Gubser *et al.* (1998)
Witten (1998)

$$Z_{\text{bulk}}[\phi_0] = Z_{\text{CFT}}[\phi_0]$$

Boundary value of bulk field $\phi(r, x)$

Source that couples to local boundary operator $\mathcal{O}(X)$

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle_{\text{CFT}} = \left[\frac{\delta}{\delta\phi_0(x_1)} \cdots \frac{\delta}{\delta\phi_0(x_n)} Z_{\text{bulk}}[\phi_0] \right]_{\phi_0=0}$$

Boundary correlation function as functional derivative of bulk partition function

Extrapolate version

Banks *et al.* (1998)

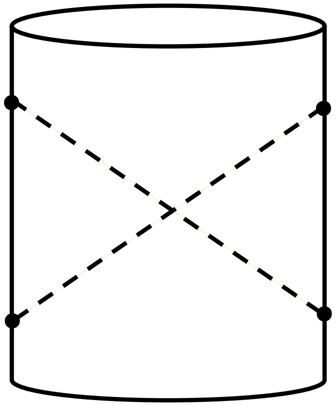
$$\lim_{r\to\infty} r^{\Delta}\phi(r,x) = \mathcal{O}(x)$$

"Extrapolating" a local bulk field to the boundary...

...to get dual local boundary operator

$$\langle \mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle_{\text{CFT}} = \lim_{r\to\infty} r^{n\Delta}\langle\phi(r,x_1)\cdots\phi(r,x_n)\rangle_{\text{bulk}}$$

Boundary correlation function as "extrapolation" of bulk correlation function



<u>*Analogous to LSZ formula*</u>:

Sources for boundary operators = interpolating bulk fields

Differentiate version

$$Z_{\text{bulk}}[\phi_0] = Z_{\text{CFT}}[\phi_0]$$

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle_{\text{CFT}} = \left[\frac{\delta}{\delta\phi_0(x_1)} \cdots \frac{\delta}{\delta\phi_0(x_n)} Z_{\text{bulk}}[\phi_0] \right]_{\phi_0 = 0}$$

Extrapolate version

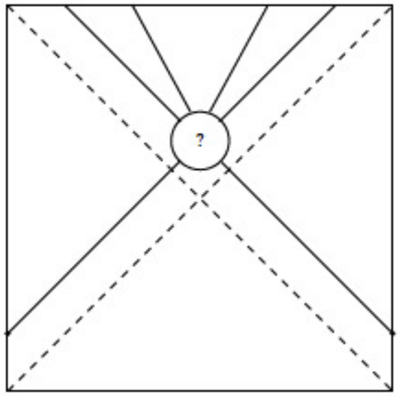
$$\lim_{r\to\infty} r^{\Delta} \phi(r, x) = \mathcal{O}(x)$$

$$\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle_{\text{CFT}} = \lim_{r\to\infty} r^{n\Delta} \langle \phi(r, x_1) \cdots \phi(r, x_n) \rangle_{\text{bulk}}$$

Equivalent!

Harlow & Stanford (2011)

- What about local bulk fields far from the boundary?
- What about observables lurking behind bulk horizons?



"We'd like to back off of the extrapolate dictionary" (Harlow 2018)

Bulk Reconstruction

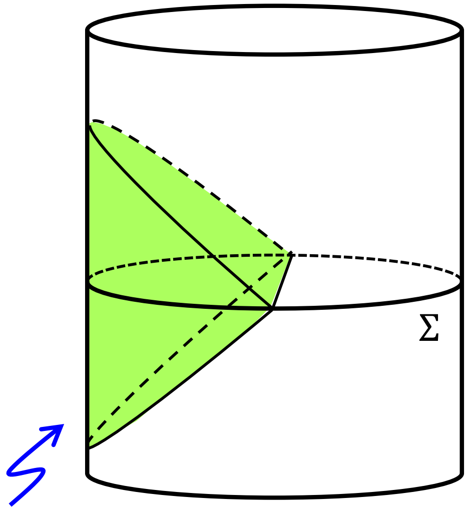
General Idea

1. Quantize bulk field $\phi(x)$: express in terms of creation/annihilation operators $a^\dagger$, a on a Fock space.
2. Use extrapolate dictionary to express $\mathcal{O}(X)$ in terms of $a^\dagger$, a.
3. Take Fourier transform of #2 to re-express $\phi(x)$ in terms of $\mathcal{O}(X)$.

$$\phi(x) = \int_\Omega K(x;X)\mathcal{O}(X)dX$$

Smearing function

- Step 1 can be done in two ways:
  - *Global coordinates*: Ω = set of boundary points spacelike separated from x.
  - *Rindler coordinates*: Ω = intersection of boundary with bulk AdS-Rindler wedge.



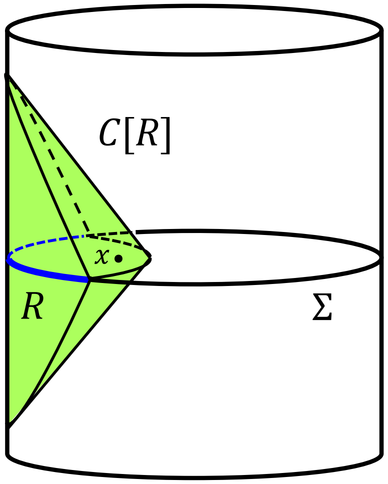
AdS-Rindler wedge

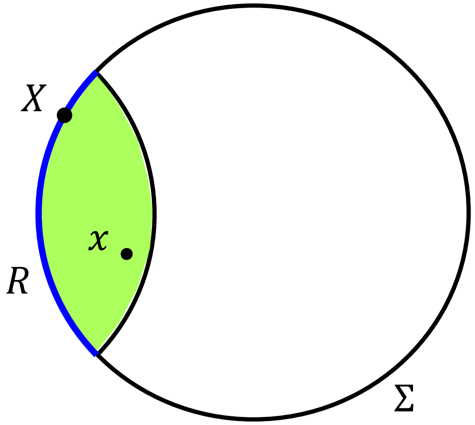
Now generalize to get...

Causal wedge

$$C[R] \equiv J^{+}_{\text{bulk}}[D_{\text{bnd}}[R]] \cap J^{-}_{\text{bulk}}[D_{\text{bnd}}[R]]$$

All bulk points that are causally accessible from the boundary region causally determined by a spatial boundary region R.





AdS-Rindler representation of bulk field

$$\phi(x)|_{x\in C[R]} = \int_{D_{\text{bnd}}[R]} K(x;X)\mathcal{O}(X)dX \equiv \mathcal{O}_{(\phi;R)}$$

Bulk field at a point as a boundary operator on a region

Causal wedge reconstruction conjecture

For any boundary spatial region R, any bulk field in $C[R]$ can be represented by a boundary operator on R.

A powerful entry in the AdS/CFT dictionary?

- non-local!

- trivial?

Paradox?

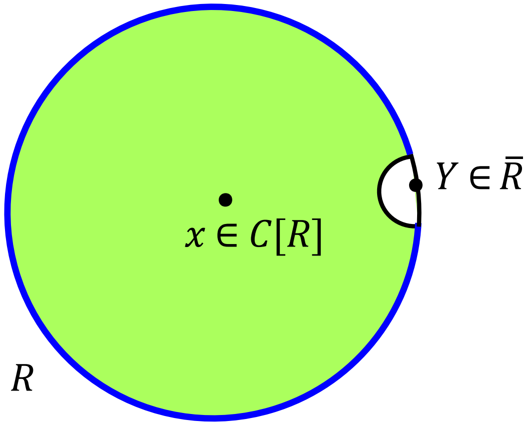
Claim. The AdS-Rindler representation $\mathcal{O}_{(\phi;R)}$ is trivial.

Almheiri *et al*. (2015)
Pastawski *et al*. (2015)
Harlow (2018)

Proof sketch.

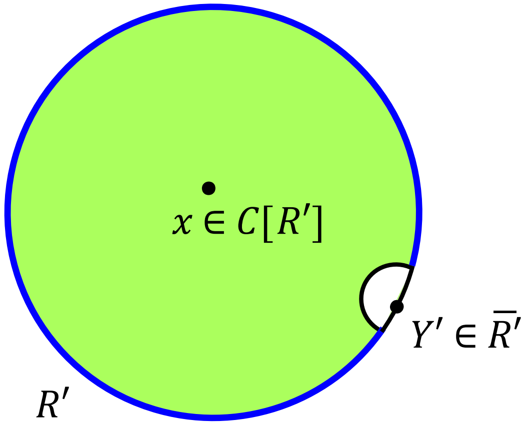
(i) For any $\phi(x)$ and any local $\mathcal{O}(Y)$ on the same timeslice, there is an $\mathcal{O}_{(\phi;R)}$ such that $\mathcal{O}(Y)$ lies in $\bar{R}$ and hence $[\mathcal{O}_{(\phi;R)}, \mathcal{O}(Y)] = 0$ (*local commutativity*).

Implication: A bulk field admits multiple AdS-Rindler reps, each commuting with some arbitrary boundary operator on the same timeslice.



$$\mathcal{O}_{(\phi;R)} = \mathcal{O}_{(\phi;R')}$$

(ii) *Uniqueness*: For a given $\phi(x)$,
$\mathcal{O}_{(\phi;R)} = \mathcal{O}_{(\phi;R')} = \cdots \equiv \mathcal{O}_{(\phi)}$



(iii) By the timeslice axiom, $\mathcal{O}_{(\phi)}$ must be a multiple of the identity.

Timeslice Axiom (Primitive Causality)

Let Σ be a Cauchy surface in a spacetime $\mathcal{M}$, and let U be any open neighborhood of Σ. Then the algebra of observables $\mathfrak{R}(U)$ generated by local operators with support in U coincides with the algebra of observables $\mathfrak{R}(\mathcal{M})$ on $\mathcal{M}$ (i.e., $\mathfrak{R}(U)$ acts irreducibly on the corresponding state space).

Intuition: Dynamical laws (hyperbolic differential equations) can be formulated that govern $\mathfrak{R}(\mathcal{M})$.

Schur's Lemma: Any operator that commutes with all irreducible representations of an algebra is a multiple of the identity.

Timeslice Axiom + Schur's Lemma

Let Σ be a Cauchy surface in a spacetime $\mathcal{M}$, and let U be any open neighborhood of Σ. Then any operator which commutes with all local operators with support in U must be a multiple of the identity.

Paradox?

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Almheiri *et al*. (2015)
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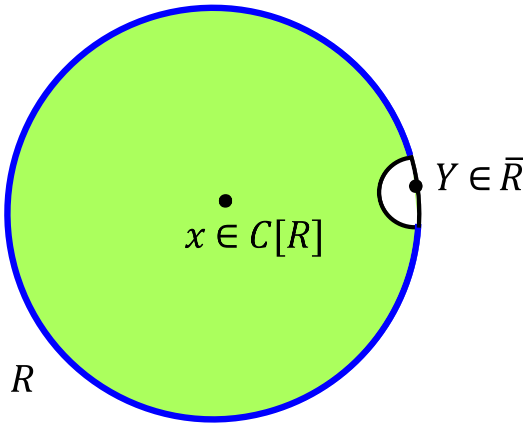
Implication: A bulk field admits multiple AdS-Rindler reps, each commuting with some arbitrary boundary operator on the same timeslice.

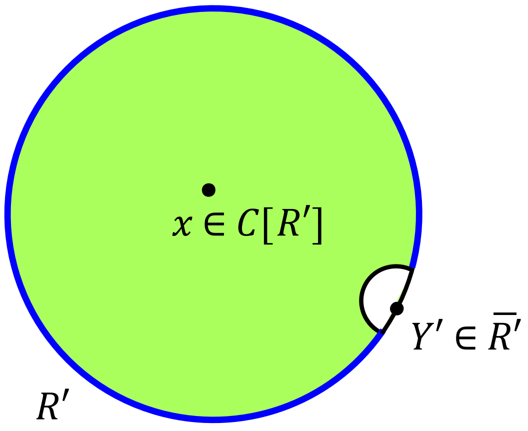
(ii) *Uniqueness*: For a given $\phi(x)$, $\mathcal{O}_{(\phi;R)} = \mathcal{O}_{(\phi;R')} = \cdots \equiv \mathcal{O}_{(\phi)}$

How might this Fail?

(iii) By the timeslice axiom, $\mathcal{O}_{(\phi)}$ must be a multiple of the identity.



$$\mathcal{O}_{(\phi;R)} = \mathcal{O}_{(\phi;R')}$$



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3. Resolution: The Boundary as a QECC

Almheiri *et al.* (2015)
Harlow (2018)

Erasure Protection Quantum Error-Correction Code

(1) n-qudit *physical* Hilbert space $\mathcal{H}^{(n)} = \mathcal{H}_R^{(m)} \otimes \mathcal{H}_{\bar{R}}^{(n-m)}$, where $\mathcal{H}_R^{(m)}$ and $\mathcal{H}_{\bar{R}}^{(n-m)}$ consist of m- and $(n-m)$-qudit states with support in R and $\bar{R}$.

(2) Map that encodes k *logical* qudits in n *encoded logical* qudits of a *codespace* $\mathcal{H}_C \subset \mathcal{H}^{(n)}$ in such a way that the former can be recovered if access is limited to some set R of $m < n$ of the latter.

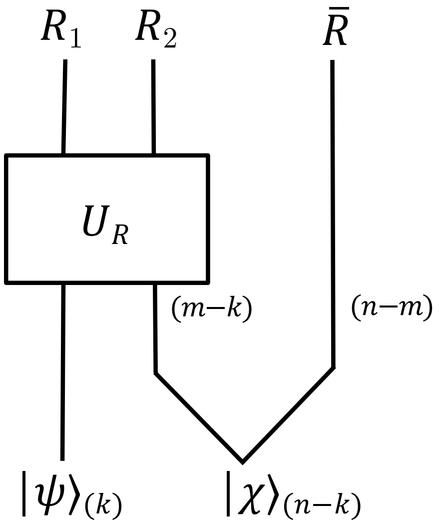
Claim. An encoding map exists *if and only if* R can be decomposed into $R_1 \cup R_2$ with k and $(m-k)$ qudits, *resp*., such that the basis states for $\mathcal{H}_C$ can be expressed by

$$|\tilde{i}\rangle = (U_R \otimes I_{\bar{R}})\left(|i\rangle_{R_1} \otimes |\chi\rangle_{R_2\bar{R}}\right)$$

unitary operator on R

k-qudit state in R_1

$(n-k)$-qudit state in $R_2 \cup \bar{R}$



Example: 3 qutrit code ($k = 1, n = 3, m = 2$)

$k = 1$ logical qutrit: $|\psi\rangle = \sum_{i=0}^{2} C_i|i\rangle = C_0|0\rangle + C_1|1\rangle + C_2|2\rangle$

Encoded in $n = 3$ physical qutrits:

$$|\tilde{\psi}\rangle = \sum_{i=0}^{2} C_i|\tilde{\imath}\rangle = C_0|\tilde{0}\rangle + C_1|\tilde{1}\rangle + C_2|\tilde{2}\rangle$$

$$|\tilde{0}\rangle = \frac{1}{\sqrt{3}}(|000\rangle + |111\rangle + |222\rangle)$$

$$|\tilde{1}\rangle = \frac{1}{\sqrt{3}}(|012\rangle + |120\rangle + |201\rangle)$$

$$|\tilde{2}\rangle = \frac{1}{\sqrt{3}}(|021\rangle + |102\rangle + |210\rangle)$$

Encoding map

$$|\tilde{\imath}\rangle = (U_{12} \otimes I_3)(|i\rangle_1 \otimes |\chi\rangle_{23})$$

$$|\tilde{\psi}\rangle = (U_{12} \otimes I_3)(|\psi\rangle_1 \otimes |\chi\rangle_{23})$$

where:

$$|\chi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$$

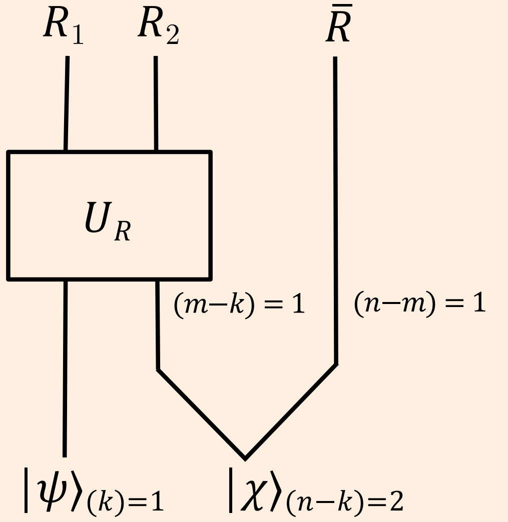
U: $|00\rangle \rightarrow |00\rangle$ $|11\rangle \rightarrow |20\rangle$ $|22\rangle \rightarrow |10\rangle$

$|01\rangle \rightarrow |11\rangle$ $|12\rangle \rightarrow |01\rangle$ $|20\rangle \rightarrow |21\rangle$

$|02\rangle \rightarrow |22\rangle$ $|10\rangle \rightarrow |12\rangle$ $|21\rangle \rightarrow |02\rangle$

Recovery for access limited to $m = 2$ qutrits:

Lost	*Recovery*
3	$(U^{\dagger}_{12} \otimes I_3)|\tilde{\psi}\rangle = |\psi\rangle_1 \otimes |\chi\rangle_{23}$
2	$(U^{\dagger}_{31} \otimes I_2)|\tilde{\psi}\rangle = |\psi\rangle_3 \otimes |\chi\rangle_{12}$
1	$(U^{\dagger}_{23} \otimes I_1)|\tilde{\psi}\rangle = |\psi\rangle_2 \otimes |\chi\rangle_{13}$



Equivalent necessary and sufficient conditions for encoding map:

(a) *QECC Condition*: Any $(n-m)$-qudit operator $O_{\bar{R}}$ on $\bar{R}$ acts like a multiple of the identity on $\mathcal{H}_C$:

$$\langle\tilde{\imath}|(O_{\bar{R}}\otimes I_R)|\tilde{\jmath}\rangle = c\delta_{ij}$$

Bulk locality paradox?

(b) *Erasure-Protection Condition*: Any n-qudit operator $\tilde{O}$ on $\mathcal{H}_C$ can be expressed as an m-qudit operator $O_R = U_R(O_{R_1}\otimes I_{R_2})U_R^\dagger$ with support on R, such that, for all $|\tilde{\psi}\rangle \in \mathcal{H}_C$:

$$(O_R\otimes I_{\bar{R}})|\tilde{\psi}\rangle = \tilde{O}|\tilde{\psi}\rangle, \quad (O^\dagger{}_R\otimes I_{\bar{R}})|\tilde{\psi}\rangle = \tilde{O}^\dagger|\tilde{\psi}\rangle$$

Causal wedge reconstruction conjecture?

(c) Within $\mathcal{H}_C$, any n-qudit operator $\tilde{O}$ commutes with all $(n-m)$-qudit operators $O_{\bar{R}}$ with support on $\bar{R}$:

$$\langle\tilde{\imath}|[\tilde{O}, (O_{\bar{R}}\otimes I_R)]|\tilde{\jmath}\rangle = 0$$

Bulk locality paradox?

Moreover...

Harlow (2017)

Claim. Let $\mathcal{H}$ be a finite-dim Hilbert space which factorizes into $\mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$, let $\mathcal{H}_C$ be a subspace of $\mathcal{H}$, and let M be a von Neumann algebra on $\mathcal{H}_C$. Then the following are equivalent:

(i) *Erasure-Protection Condition* (b):

Any $\tilde{O} \in M$ can be expressed as an O_R on $\mathcal{H}_R$:

$$(O_R \otimes I_{\bar{R}})|\tilde{\psi}\rangle = \tilde{O}|\tilde{\psi}\rangle, \quad (O^{\dagger}{}_R \otimes I_{\bar{R}})|\tilde{\psi}\rangle = \tilde{O}^{\dagger}|\tilde{\psi}\rangle, \quad \forall|\tilde{\psi}\rangle \in \mathcal{H}_C$$

(ii) *Finite-dim RT Formula*:

There is an operator $\mathcal{L}_R \in M$ such that for any state $\tilde{\rho}$ on M,

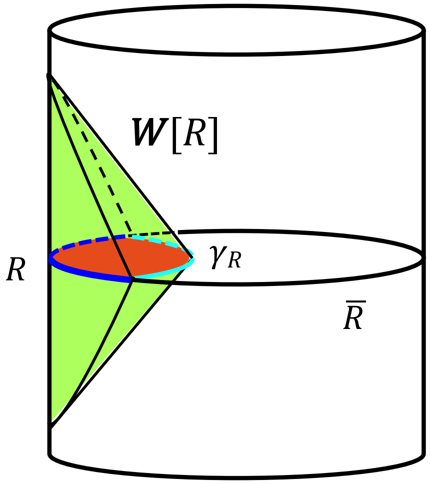
$$S(\tilde{\rho}_R) = \mathrm{Tr}[\tilde{\rho}\mathcal{L}_R] + S(\tilde{\rho})$$

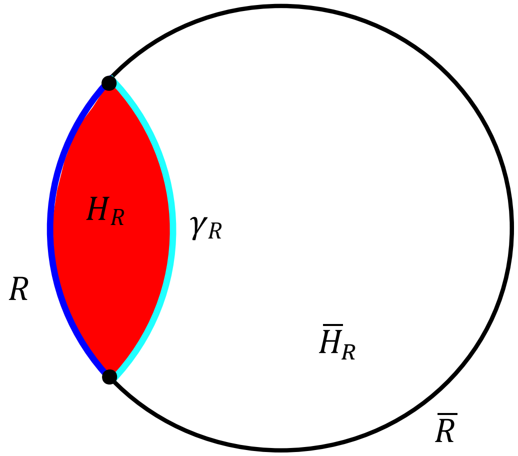
Entanglement entropy of reduced state $\tilde{\rho}_R$

v.N. entropy of $\tilde{\rho}$

To link with AdS/CFT...

For boundary spatial subregion R, let γ_R be the minimal area bulk surface with the same boundary as R, and let H_R be the bulk surface bounded by R and γ_R.





Ryu & Takayanagi (2006)

Ryu-Takayanagi (RT) Formula

$$S(\rho_R) = \text{Area}(\gamma_R)/4G + S(\rho_{H_R})$$

Entanglement entropy of boundary subsystem localized in R

Entanglement entropy of bulk subsystem localized in H_R

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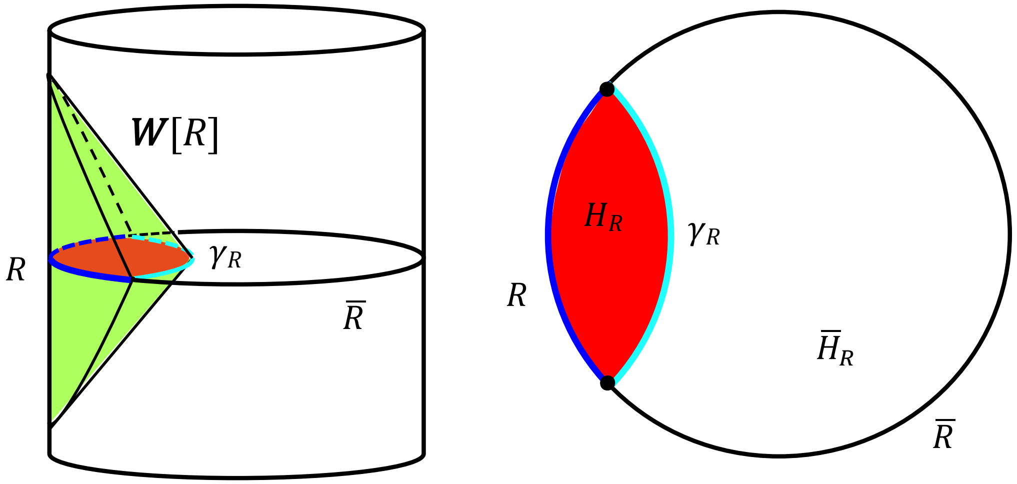

Ryu & Takayanagi (2006)

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Entanglement wedge

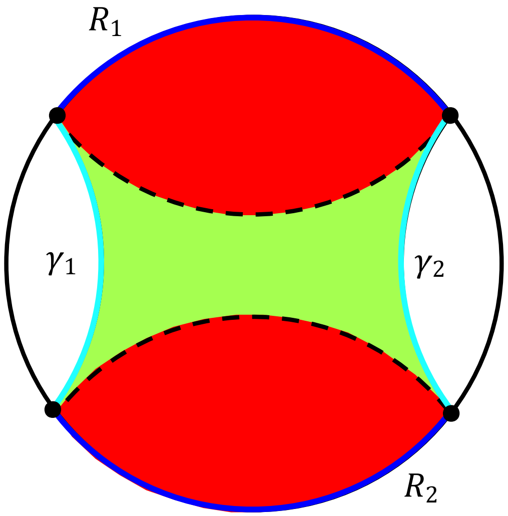
$$W[R] \equiv D_{\mathrm{bulk}}[H_R]$$

Headrick *et al.* (2014)

Recall causal wedge of R:

$$C[R] \equiv J^{+}_{\mathrm{bulk}}[D_{\mathrm{bnd}}[R]] \cap J^{-}_{\mathrm{bulk}}[D_{\mathrm{bnd}}[R]]$$

$W[R]$ and $C[R]$ can differ when R is union of disjoint regions...



$R = R_1 \cup R_2$
$\gamma_R = \gamma_1 \cup \gamma_2$
$H_R = red \cup green$

$W[R]$ contains red and green regions whereas $C[R]$ just contains red regions.

Harlow (2017)

Claim. Let $\mathcal{H}$ be a finite-dim Hilbert space which factorizes into $\mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$, let $\mathcal{H}_C$ be a subspace of $\mathcal{H}$, and let M be a von Neumann algebra on $\mathcal{H}_C$. Then the following are equivalent:

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There is an operator $\mathcal{L}_R \in M$ such that for any state $\tilde{\rho}$ on M,

$$S(\tilde{\rho}_R) = \mathrm{Tr}[\tilde{\rho}\mathcal{L}_R)] + S(\tilde{\rho})$$

Connection with AdS/CFT

- $\mathcal{H}$ = CFT Hilbert space
- $\mathcal{H}_R, \mathcal{H}_{\bar{R}}$ = CFT degrees of freedom on boundary spatial regions $R, \bar{R}$
- $\mathcal{L}_R = \mathrm{Area}(\gamma_R)/4G$
- M = algebra of bulk operators in $W[R]$
- $\mathcal{H}_C$ = subspace of CFT states that represent bulk states in $W[R]$

The QECC Interpretation of AdS/CFT

QECC	**AdS/CFT**
Physical qudit Hilbert space $\mathcal{H}^{(n)} = \mathcal{H}_R^{(m)} \otimes \mathcal{H}_{\bar{R}}^{(n-m)}$	CFT Hilbert space $\mathcal{H} = \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$, for boundary subregion R.
Code supspace $\mathcal{H}_C \subset \mathcal{H}^{(n)}$ (*encoded logical qudits*)	Subspace $\mathcal{H}_C \subset \mathcal{H}$ of CFT states that represent bulk states in $W[R]$.
Erasure-Protection Condition: Any n-qudit operator on $\mathcal{H}_C$ can be expressed as an m-qudit operator on R.	*Entanglement Wedge Reconstruction*: Any bulk field on $W[R]$ can be expressed as a CFT operator on R.
QECC Condition: Any $(n-m)$-qudit operator on $\bar{R}$ acts as a multiple of the identity on $\mathcal{H}_C$.	Any local CFT operator on $\bar{R}$ acts as a multiple of the identity on bulk states in $W[R]$.
Finite-dim RT Formula: For any state $\tilde{\rho}$ on $\mathcal{H}_C$, $S(\tilde{\rho}_R) = \mathrm{Tr}[\tilde{\rho}\mathcal{L}_R)] + S(\tilde{\rho})$.	*RT Formula*: $S(\rho_R) = \mathrm{Area}(\gamma_R)/4G + S(\rho_{H_R})$

- *Redundancy*: Info associated with a CFT operator on R is protected against erasure of $\bar{R}$ by encoding it redundantly (non-uniquely) in an AdS-Rindler rep of a bulk field in $W[R]$.
    - *Interpretation*: Codespace is space of low-energy CFT states; distinct encoded logical CFT operators act in same way on low-energy states, but differ on high-energy states.

Pastawski *et al.* (2015)

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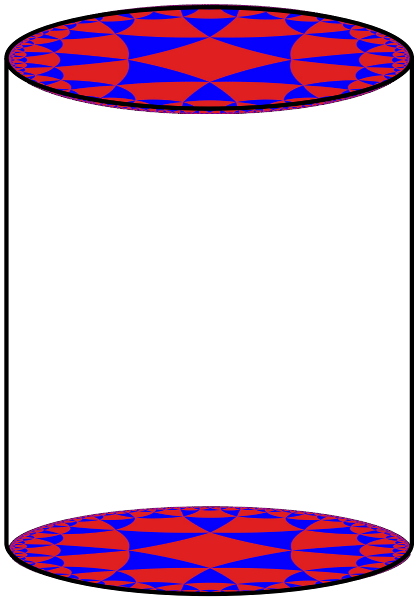
4. Spacetime as a QECC?

The Emergent Bulk



Bulk theory is a way of protecting boundary degrees of freedom against erasure.

What does this have to do with spacetime?

- A QECC isn't the sort of thing associated with a spacetime.
- A QECC can be *realized* by physical systems, which can possess spatiotemporal properties.

Suggestion: To the extent that the bulk emerges from the boundary, and the boundary has the structure of a QECC, perhaps (bulk) spacetime emerges from a QECC.

- *But*: Bulk/boundary duality is *symmetrical* (*i.e.*, exact), and emergence is *asymmetrical*.

Teh (2013)
Dieks *et al.* (2015)
de Haro *et al.* (2016)
de Haro (2017)

What might underwrite the asymmetry needed for emergence?

Proposal

(a) Realize erasure-protection QECC as a discrete lattice system of qudits (HaPPY code).

(b) Take continuum limit to get AdS/CFT.

(c) Interpret (b) as emergence of spacetime.

The HaPPY Code Realization

Pastawski, Yoshida, Harlow, Preskill (2015)

Perfect tensor representation of erasure-protection QECC:

A 2ℓ-indexed linear map from one qudit to $2\ell-1$ qudits that protects against the erasure of any $\ell-1$ of the latter.

- *Ex. 1*. Three qutrit code ($\ell = 2$): Encodes 1 qutrit in 3 qutrits against the erasure of any 1 of the latter.

$$T_{i_1 i_2 i_3, j} = \sqrt{3}\langle i_1 i_2 i_3 | \tilde{j} \rangle$$

$$|\tilde{j}\rangle = (U_{12} \otimes I_3)(|j\rangle_1 \otimes |\chi\rangle_{23})$$

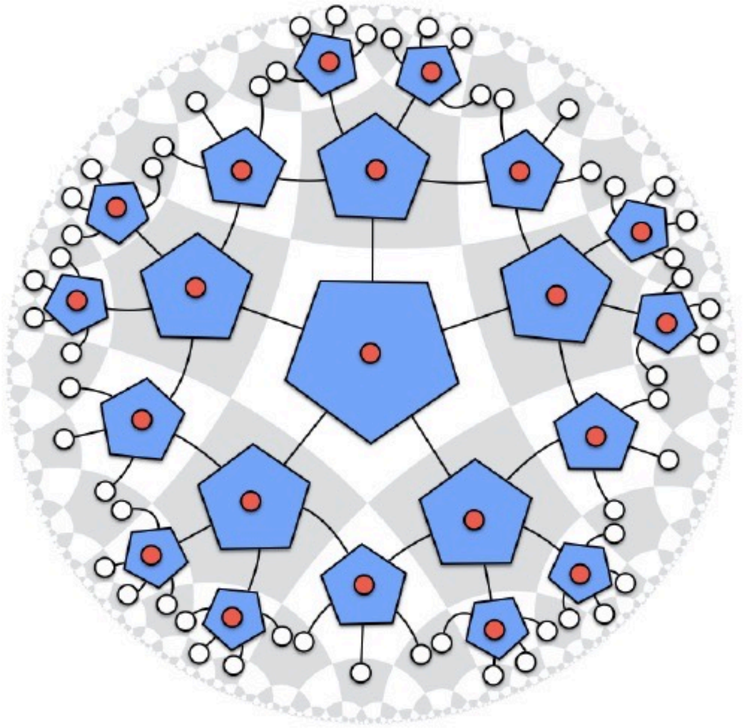
"Perfect tensor" = even-indexed tensor such that any bipartition of indices defines a unitary transformation

- *Ex. 2*. Five qubit code ($\ell = 3$): Encodes 1 qubit in 5 qubits against the erasure of any 2 of the latter.

$$T_{i_1 i_2 i_3 i_4 i_5, j} = 2\langle i_1 i_2 i_3 i_4 i_5 | \tilde{j} \rangle$$

HaPPY code

- (5,4) tiling of hyperbolic plane, one 6-index perfect tensor per lattice face.
- 1 free index per face (logical qudit); remaining 5 indices contracted with adjacent tensors.
- Boundary consists of some number of uncontracted indices.



Claim 1. HaPPY code forms a tensor network equivalent to a perfect tensor $T_{i_1 \ldots i_n, j_1 \ldots j_k} = \langle i_1 \ldots i_n | \widetilde{j_1 \ldots j_k} \rangle$ that encodes k bulk logical qudits in n boundary physical qudits against the erasure of some number of the latter.

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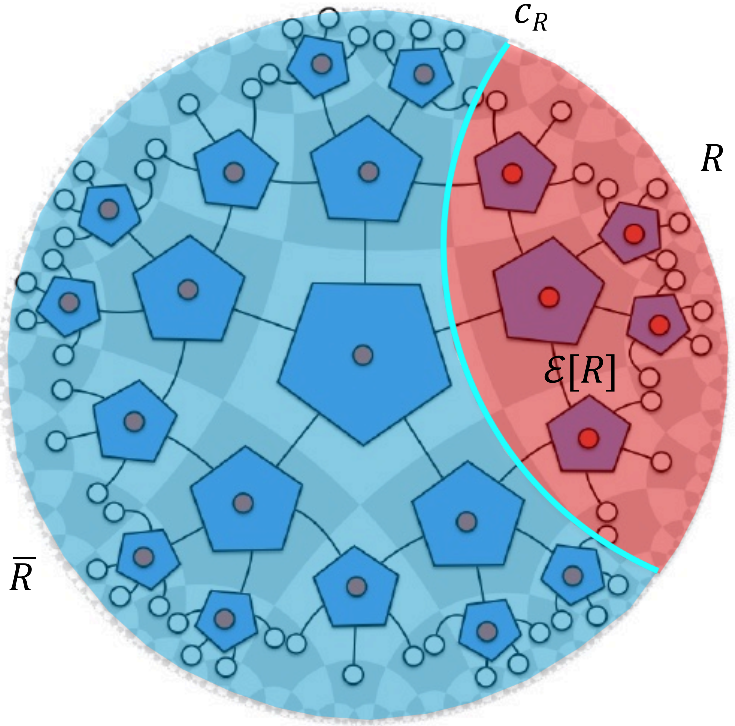

Claim 2. Lattice versions of $W[R]$ and RT formula can be constructed: Let $R =$ subset of uncontracted boundary indices, and let c_R be the "minimum length" cut with same boundaries as R that divides network into two perfect tensors P, Q, with P defined by the tensors in the region containing R, and Q by those in the region containing $\overline{R}$.

- *Lattice entanglement wedge* $\mathcal{E}[R]$ = set of tensors that define P.
- *Lattice RT formula*: Entanglement entropy of R is given by the "length" $|c_R|$ of c_R (number of indices that it cuts).

The QECC and HaPPY code Interpretations of AdS/CFT

QECC	**HaPPY code**	**AdS/CFT**
Physical qudit Hilbert space $\mathcal{H}^{(n)} = \mathcal{H}_R^{(m)} \otimes \mathcal{H}_{\bar{R}}^{(n-m)}$	$\mathcal{H}_{\text{HC}} = \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$, for boundary subregion R.	$\mathcal{H}_{\text{CFT}} = \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$, for boundary subregion R.
Code supspace $\mathcal{H}_C \subset \mathcal{H}^{(n)}$ (*encoded logical qudits*)	Subspace $\mathcal{H}_C \subset \mathcal{H}_{\text{HC}}$ of states that represent bulk states in $\mathcal{E}[R]$.	Subspace $\mathcal{H}_C \subset \mathcal{H}_{\text{CFT}}$ of states that represent bulk states in $W[R]$.
Erasure-Protection Condition: Any n-qudit operator on $\mathcal{H}_C$ can be expres-sed as an m-qudit operator on R.	Any operator on $\mathcal{E}[R]$ can be pushed out to an operator on R.	*$W[R]$ Reconstruction*: Any bulk field on $W[R]$ can be expressed as a CFT operator on R.
QECC Condition: Any $(n-m)$-qudit operator on $\bar{R}$ acts as a multiple of the identity on $\mathcal{H}_C$.	Any operator on $\bar{R}$ acts as a multiple of the identity on bulk states in $\mathcal{E}[R]$.	Any CFT operator on $\bar{R}$ acts as a multiple of the identity on bulk states in $W[R]$.
Finite-dim RT Formula: For any state $\tilde{\rho}$ on $\mathcal{H}_C$, $S(\tilde{\rho}_R) = \text{Tr}[\tilde{\rho}\mathcal{L}_R)] + S(\tilde{\rho})$.	*Lattice RT Formula*: $S(\rho_R) = \lvert c_R \rvert$	*RT Formula*: $S(\rho_R) = \text{Area}(\gamma_R)/4G + S(\rho_{H_R})$

HaPPY code

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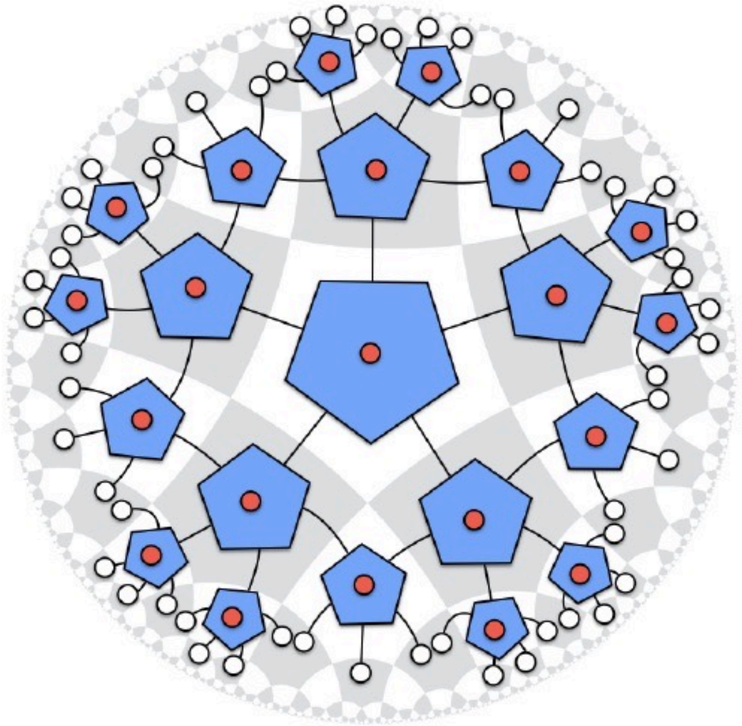


Continuum limit: $(\#vertices) \rightarrow \infty$, $(\#vertices)/area = \text{const.}$

- Expect to recover AdS/CFT correspondence!

Suggests: Both CFT and AdS gravity emerge in continuum limit of Happy Code discrete lattice system.

CFT ⟷ *duality* ⟷ *AdS*

continuum limit ↑

boundary qudits

↑

physical qudits

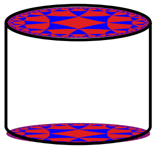
continuum limit ↑

bulk qudits

↑

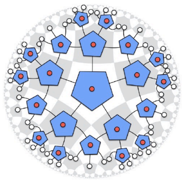
encoded logical qudits

AdS/CFT



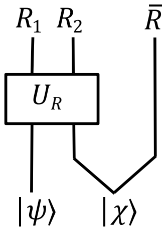
emergence of spacetime ↑

HaPPY code



realization ↑

QECC structure



Suggests: Both CFT and AdS gravity emerge in continuum limit of Happy Code discrete lattice system.

Realist Interpretive Options

Le Bihan & Read (2018)

- *Discrimination*: Only one of two dual theories describes an actual world candidate.

Perhaps for QECC Interpretation, but not for emergentist HaPPY code interpretation...

- *Common Core*: Dual theories share a common mathematical core.

Vistarini (2017),
de Haro (2020), de Haro &
Butterfield (2018; 2021)

$$(\mathcal{S}, \mathcal{Q}, \mathcal{D})$$

Informed by differentiate dictionary?

states

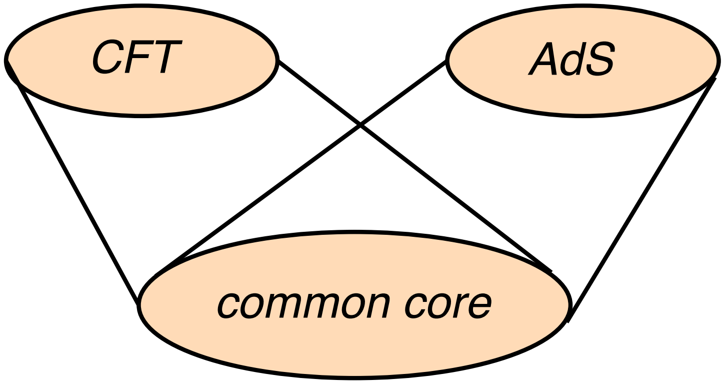
quantities

dynamics

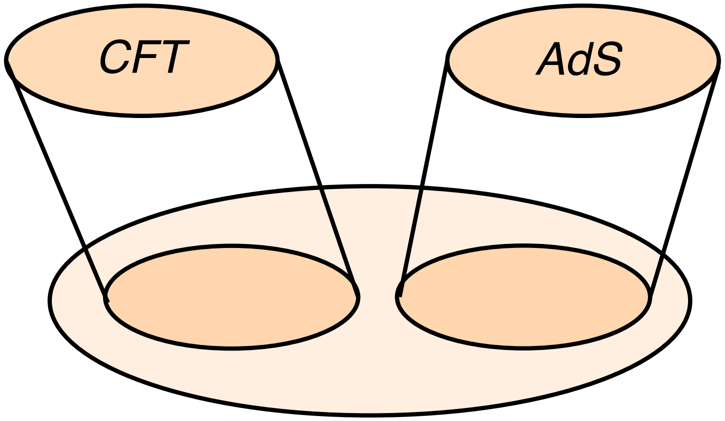
- *Should we "back off"?*
- *Should we allow purely kinematical "core"?*

- *Overarching Theory*: Dual theories can be embedded in an overarching theory.

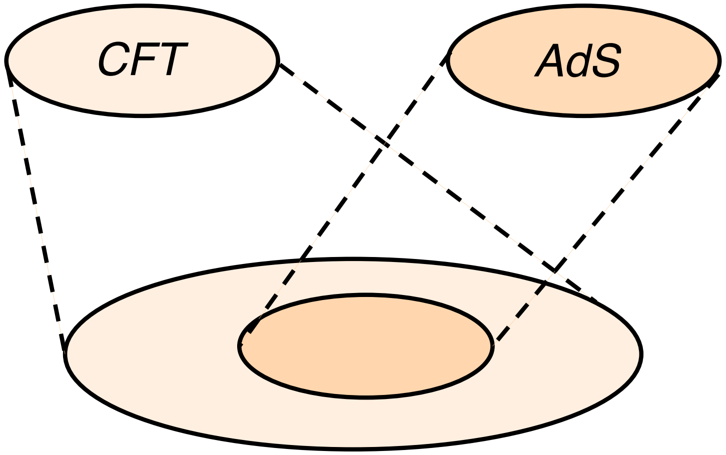
- *Is a QECC a "theory"?*
- *Is the emergence relation an embedding relation?*



Common core



Overarching "structure"



HaPPY Code Interpretation

Overarching structure is QECC, but CFT and AdS gravity are not embedded in it; rather, they emerge as different aspects of a realization of it.

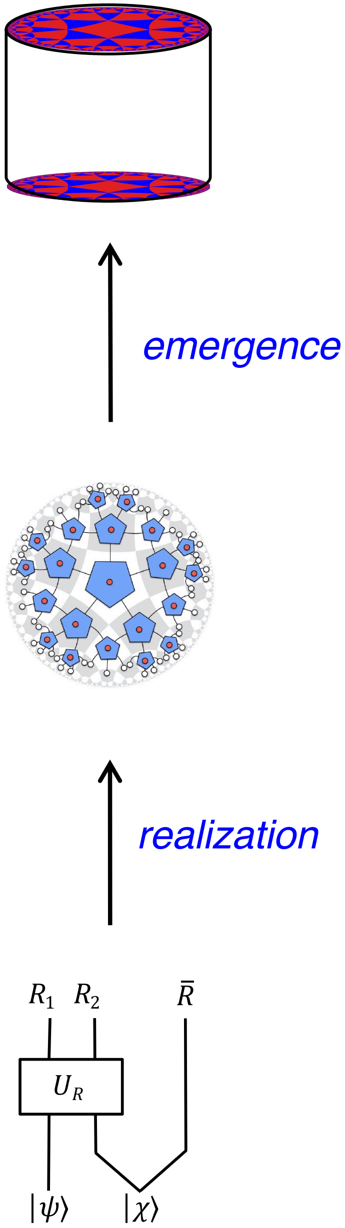
Conclusion

- Is Spacetime a QECC?

  - No!

- Does spacetime emerge in a continuum limit from a discrete physical system on a lattice whose kinematically possible states realize a QECC?

  - Possibly!



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