A Duality Between Topology and Entanglement? ER=EPR, Part 2

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1. Entanglement Entropy vs. Minimal Area Cross-Section
   - *Conservation Under "Local Operations"*
   - *Monogamy*
2. "No-Go" Theorems
   - *No-Signaling vs. Non-traversability*
   - *No Cloning vs. No Topology Change*
   - *Non-detectability*
3. A Solution to the Firewall Problem?
4. A Causal Explanation of Entanglement Correlations?

1. Entanglement Entropy vs. Minimal Area Cross-Section

Two Claims:

- Both measures are conserved under "local operations".
- Both measures satisfy a monogamy constraint.

Implication: Do they measure the same thing?

Entanglement entropy as a measure of entanglement:

For a bipartite system AB characterized by a state ρ_{AB}, the **entanglement entropy** S_A of subsystem A is given by:

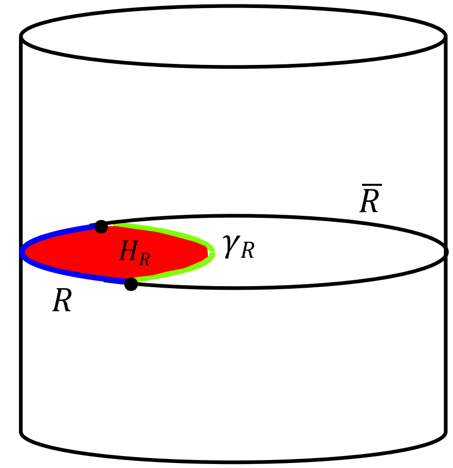
$$S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$$

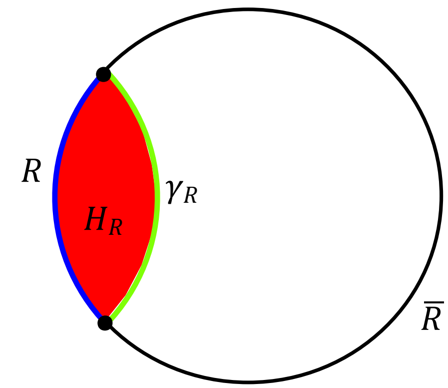
where $\rho_A = \mathrm{Tr}_B \rho_{AB}$

Minimal area cross-section as a measure for wormholes:

RT Formula: $S_R = \text{Area}(\gamma_R)/4G$

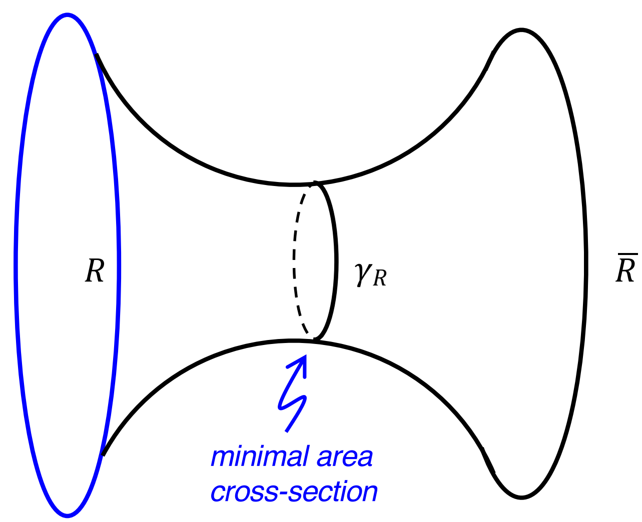
- R = subregion of boundary spatial slice.
- γ_R = bulk surface such that:
  - (a) γ_R has same boundary as R.
  - (b) γ_R and R are homologous: $\partial H_R = \gamma_R \cup R$ for some spacelike bulk surface H_R.
  - (c) γ_R extremizes area functional.





Note: When the bulk is a wormhole spacetime, and R is one of its boundaries, γ_R is given by the minimal area cross-section of the wormhole's throat.

Boundary of bulk = $R \cup \bar{R}$



- *To uphold (b), γ_R must separate left and right boundaries; hence γ_R = cross-section of wormhole (spatial slice).*
- *To uphold (c), γ_R must be cross-section with minimal area.*

Conservation Under "Local Operations"

Entanglement entropy:

Claim 1a (*Conservation of entanglement entropy*). Let AB be a bipartite system in a pure state ρ_{AB} with Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$. Then the entanglement entropy S_A of subsystem A is invariant under transformations of the form $U_A \otimes U_B$, where U_A and U_B are unitary operators that act on $\mathcal{H}_A$ and $\mathcal{H}_B$.

- $U_A \otimes U_B$ is "local" in the sense that its action on AB consists of *separate* actions on subsystems A and B.
  - A *separability* understanding of locality.
  - Distinct from a *localizability* understanding of locality.

$U_A \otimes U_B$ can act separably on A and B even when A and B are not localized in contractible regions of space.

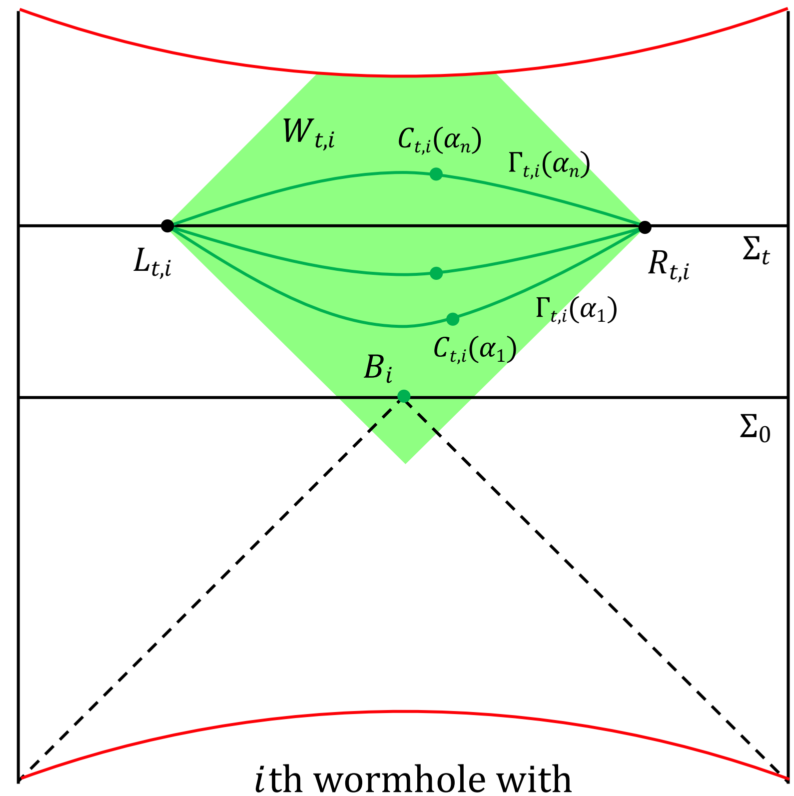
Minimal area cross-section for multiple dynamic wormholes:

Set-Up

- Multi-wormhole spacetime M:
  - Asymptotically AdS, globally hyperbolic, satisfies null curvature condition.
  - Contains some number of wormholes, each joining a left L_i and right R_i horizon, with $L = \bigcup_i L_i$, $R = \bigcup_i R_i$.
  - "Past initialized" = all wormholes far apart for $t \leq 0$.

- $\{\Sigma_t\}$ = spacelike foliation of M.
  - $L_t = L \cap \Sigma_t$, $R_t = R \cap \Sigma_t$.
  - W_t = causal diamond anchored at L_t and R_t.
  - $\{\Gamma_t(\alpha)\}$ = spacelike slicing of W_t with minimal area cross-section $C_t(\alpha)$.
  - C_t = the maximal $C_t(\alpha)$ over all $\Gamma_t(\alpha)$.

C_t is a generalization of minimal area cross-section for single static wormhole to multiple dynamic wormholes



ith wormhole with bifurcation surface B_i

Claim 1b (*Conservation of minimal area cross-section*). For the family of spacetime regions W_t defined as the causal diamonds anchored on the piecewise-connected outermost apparent horizons L_t and R_t for an arbitrary set of dynamical, past-initialized wormholes and black holes satisfying the null curvature condition, the corresponding maximin surface C_t dividing the left and right collections of wormholes has an area independent of t, equaling the sum of the areas of the initial bifurcation surfaces for the wormholes linking the left and right sets of horizons.

Proof sketch:

- Show that the area of C_t is upper bounded by the sum of the areas of the initial bifurcation surfaces $\sum_i area[B_i]$. (Requires null curvature condition.)
- Show that the area of C_t is lower bounded by $\sum_i area[B_i]$.

Claim 1b (*Conservation of minimal area cross-section*). For the family of spacetime regions W_t defined as the causal diamonds anchored on the piecewise-connected outermost apparent horizons L_t and R_t for an arbitrary set of dynamical, past-initialized wormholes and black holes satisfying the null curvature condition, the corresponding maximin surface C_t dividing the left and right collections of wormholes has an area independent of t, equaling the sum of the areas of the initial bifurcation surfaces for the wormholes linking the left and right sets of horizons.

"Thus, the maximin surface dividing one collection of wormhole mouths from another has an area that is conserved under arbitrary spacetime evolution and horizon mergers as well as arbitrary addition of matter satisfying the null energy condition."

This is a statement in general relativity "...that is a precise analogue of the statement...of conservation of entanglement."

"[Claim 1b] is the gravitational dual of entanglement conservation and thus constitutes an explicit characterization of the ER=EPR duality in the classical limit."

Two Claims:

(i) The time independence of C_t is an invariance under classical time evolution.

Reasonable: Given global hyperbolicity, W_t encodes all possible dynamics that may govern time evolution of matter fields off of any of the $\Gamma_t(\alpha)$.

(ii) Local time evolution on the ER side is the dual of local unitary evolution on the EPR side.

- *Concern #1*: Unitary evolution is more general than time evolution.
  - Hence Claim 1a is broader than Claim 1b, and hence the latter is not the dual to the former.
- *Concern #2*:
  - "Local operations" in Claim 1a are local in a *separability* sense.
  - "Local operations" in Claim 1b (local time evolution) are local in a *localization* sense.

Dynamical evolution of a localized system from one initial data slice to another.

Verdict: *Suggestive, but not conclusive, of duality.*

Monogamy

Entanglement monogamy: For a tripartite system ABC,

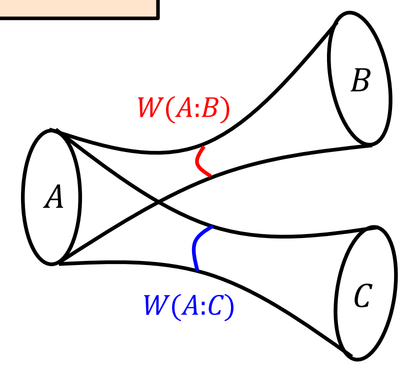
$$E(A:B) + E(A:C) \leq E(A:BC)$$

where E is an entanglement measure. So for fixed $E(A:BC)$:

(EM1) The greater the entanglement between A and B, the less between A and C.

(EM2) If A and B are maximally entangled, then A and C cannot be entangled.

Assumedly, the ER=EPR dual would be:



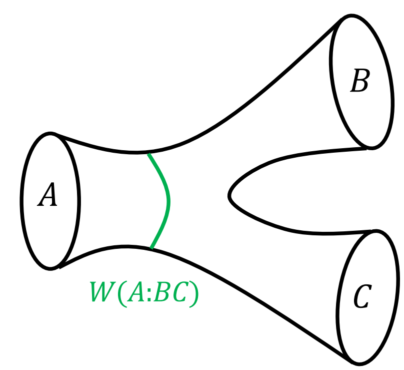
Wormhole monogamy: For spatiotemporal regions A, B, C,

$$W(A:B) + W(A:C) \leq W(A:BC)$$

where W is a measure of wormhole connectivity. So for fixed $W(A:BC)$:

(WM1) The greater the wormhole connectivity between A and B, the less between A and C.

(WM2) If A and B are connected by a maximal wormhole, then A and C cannot be connected by a wormhole.



(1) Is there an expression for monogamy involving *entanglement entropy*?

(2) Does this expression hold for the *minimal area cross-section* of a wormhole?

Monogamy of mutual information:

Def. The **mutual information** of a bipartite system AB is given by:

$$I(A:B) \equiv S_A + S_B - S_{AB}$$

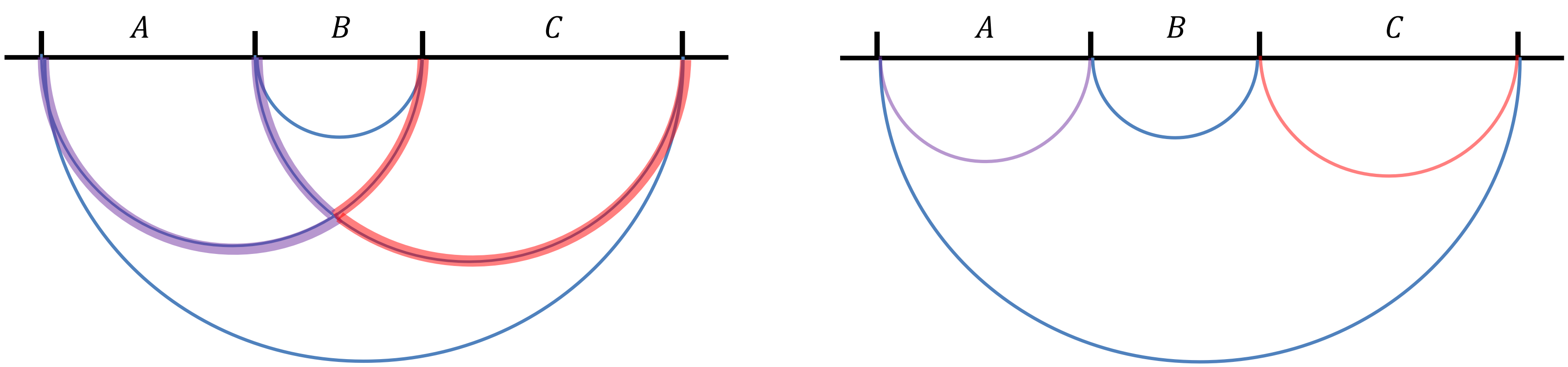
Difference between amount of entanglement of subsystems, and amount of composite system.

Monogamy constraint for $I(A:B)$ entails:

$$S_A + S_B + S_C + S_{ABC} \leq S_{AB} + S_{BC} + S_{AC}$$

Minimal area surfaces in RT formula satisfy this constraint!

Hayden, Headrick, Maloney (2013)



$$a(\gamma_{AB}) + a(\gamma_{BC}) + a(\gamma_{AC}) = a(\tilde{\gamma}_A) + a(\tilde{\gamma}_C) + a(\gamma_{ABC} + \gamma_B)$$

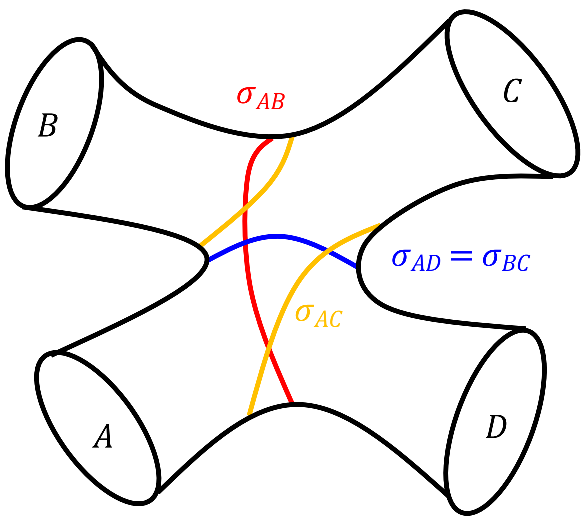
$$\geq a(\gamma_A) + a(\gamma_B) + a(\gamma_C) + a(\gamma_{ABC}) \quad \text{since } a(\tilde{\gamma}_A) \geq a(\gamma_A),\ a(\tilde{\gamma}_C) \geq a(\gamma_C)$$

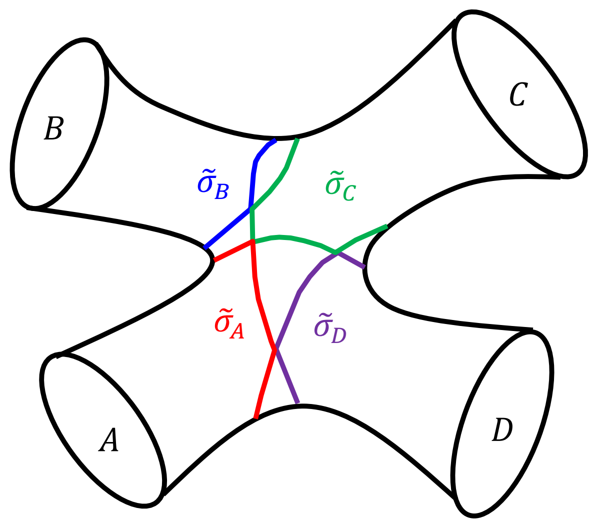
And so do minimal area cross-sections of wormholes...

Monogamy of mutual information for wormholes:

Claim 2. For a multi-throat wormhole that connects regions A, B, C with each other and with any other region D:

$$a(\sigma_A) + a(\sigma_B) + a(\sigma_C) + a(\sigma_D) \leq a(\sigma_{AB}) + a(\sigma_{BC}) + a(\sigma_{AC})$$





$$\tilde{\sigma}_A = \sigma_{AB}|_{ac \cap ad} \cup \sigma_{AC}|_{ad \cap ab} \cup \sigma_{AD}|_{ab \cap ac}$$
$$\tilde{\sigma}_B = \sigma_{BA}|_{bc \cap bd} \cup \sigma_{BC}|_{bd \cap ba} \cup \sigma_{BD}|_{ba \cap bc}$$
$$\tilde{\sigma}_C = \sigma_{CA}|_{cb \cap cd} \cup \sigma_{CB}|_{cd \cap ca} \cup \sigma_{CD}|_{ca \cap cb}$$
$$\tilde{\sigma}_D = \sigma_{DA}|_{db \cap dc} \cup \sigma_{DB}|_{dc \cap da} \cup \sigma_{DC}|_{da \cap db}$$

(i) $a(\tilde{\sigma}_A) + a(\tilde{\sigma}_B) + a(\tilde{\sigma}_C) + a(\tilde{\sigma}_D) \geq a(\sigma_A) + a(\sigma_B) + a(\sigma_C) + a(\sigma_D)$

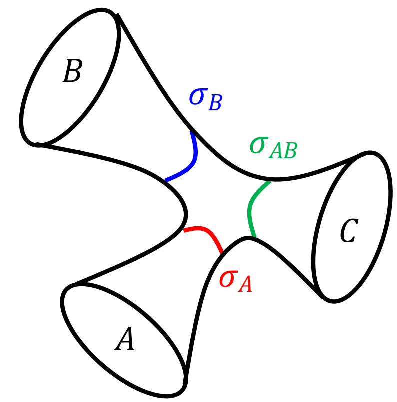
(ii) $a(\tilde{\sigma}_A) + a(\tilde{\sigma}_B) + a(\tilde{\sigma}_C) + a(\tilde{\sigma}_D) \leq a(\sigma_{AB}) + a(\sigma_{BC}) + a(\sigma_{AC})$

Concerns

- Is monogamy an essential characteristic of entanglement?
  - *Not all measures of entanglement are monogamous.*

- What is mutual information for wormholes?

$$I_W(A:B) \equiv a(\sigma_A) + a(\sigma_B) - a(\sigma_{AB})$$



  - *Difference between the sum of the sizes of the two individual throats, A, B, and the size of the "joint" throat AB.*
  - *Not indicative of connectivity...*

- Should the ER=EPR duality be restricted to pure bi-partite entangled states and "bi-throated" wormholes?
  - *For a pure composite state* ρ_{AB}

$$S_{AB} = 0 \quad \Leftrightarrow \quad a(\sigma_{AB}) = 0 \quad \Leftrightarrow$$ A and B are connected by a bi-throated wormhole

***Verdict**: Suggestive, but not conclusive, of duality.*

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No-Signaling vs. Non-traversability

Maldacena & Susskind (2013)

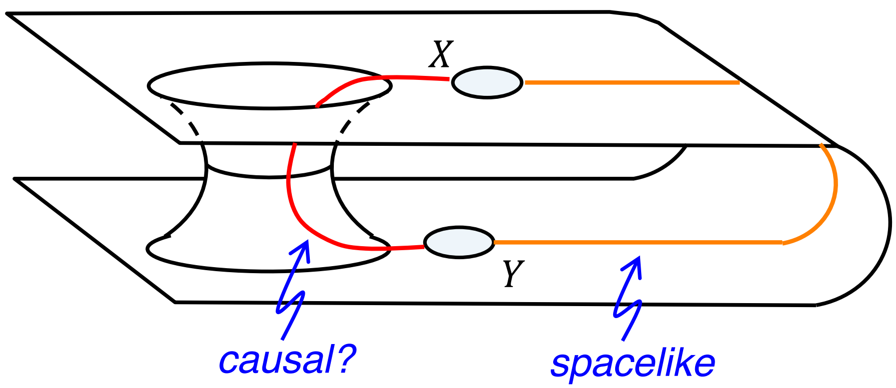
Claim 3a (*No-Signaling*). For a bipartite system AB in a pure state ρ_{AB} associated with Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$, the reduced density operator ρ_A for subsystem A is invariant under transformations of the form $I_A \otimes U_B$, where I_A is the identity on $\mathcal{H}_A$, and U_B is a unitary operator that acts on $\mathcal{H}_B$.

The statistics that govern measurement outcomes on A are encoded in ρ_A, and local operations on B do not affect it.

Consequence: If a superluminal signal is a signal that exceeds the speed of light with respect to an inertial frame, then entanglement correlations cannot be used to send superluminal signals.

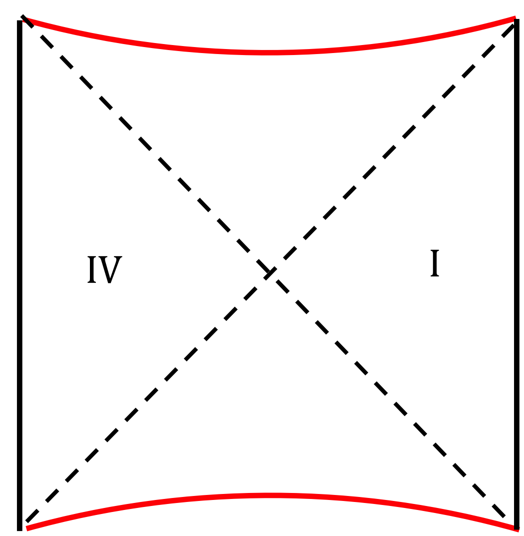
Can a wormhole be used to send superluminal signals?

Case of interest:



However...

Claim 3b (*Non-traversability*). Any future-directed curve that passes through a wormhole must become spacelike along some of its extent.



Disanalogies

- A wormhole can be made traversible; an entanglement correlation cannot be made to send superluminal signals.

Necessary condition for wormhole traversability:
A violation of the Averaged Null Energy Condition (ANEC)

Morris *et al.* (1988)

$$\int_0^\infty T_{ab} n^a n^b d\lambda \geq 0$$

"There is more positive than negative energy on any null curve n^a."

- *No-Signaling* and *Conservation of Entanglement Entropy* are both consequences of:

> Under an arbitrary separable operation $U_A \otimes U_B$ on a pure state of a bipartite system, the reduced density operator ρ_A transforms as $\rho_A \to \rho'_A = U_A \rho_A U^\dagger_A$.

No analogous claim underwriting *Non-traversability* and *Conservation of Minimal Area Cross-Section*.

No Cloning vs. No Topology Change

Bao *et al.* (2015a)

- "The no-cloning theorem and the topology-conservation theorem... are in fact dual no-go theorems under ER=EPR."

No Cloning theorem. There is no linear unitary operator U such that $U|v\rangle|0\rangle = |v\rangle|v\rangle$, for unknown qubit $|v\rangle$.

Tipler (1979), Borde (1994)

No Topology Change theorem. Let M be a compact region of a spacetime that satisfies the Einstein equations with spacelike boundaries Σ_1 and Σ_2. If the average null energy condition (ANEC) and the generic condition (GC) hold in M, then Σ_1 and Σ_2 are diffeomorphic and $M = \Sigma_1 \times [0, 1]$.

(ANEC & GC hold in M) $\Rightarrow$ ~(topology change in M)

(GC) Every causal geodesic contains at least one point p at which $k^a k^b k_{[c} R_{d]ab[e} k_{f]} \neq 0$, where k^a is the tangent vector to the geodesic at p.

"The tidal force is non-zero at p."

Strategy: Demonstrate

(i) A violation of No Cloning for certain entangled states entails superluminal signaling.

(ii) A violation of No Topology Change for a wormhole spacetime entails superluminal signaling.

Claim 4a (*Superluminal signaling via entangled states and cloning*). If two subsystems A and B comprise a bipartite system in an entangled state, and B is repeatedly cloned such that

(a) each B-clone remains entangled with A and unentangled with the other clones; and

(b) if the entanglement between A any B-clone is broken, the entanglement between A and the others remains intact;

then a superluminal signal can be sent from A to B.

- <u>*Task for superluminal signaling*</u>: To determine if a measurement outcome at B was due to a measurement outcome at A.
- If Bob at B has clones, he can measure them sequentially: The same result indicates a measurement at A.
  - Condition (a) guarantees a measurement at A influences all clones in the same way.
  - Condition (b) guarantees a same result at B is not due to a measurement of a prior B-clone.

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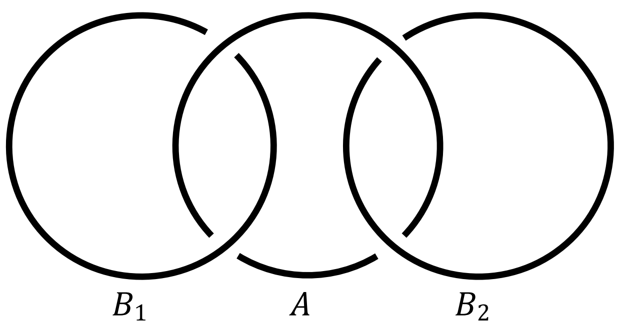
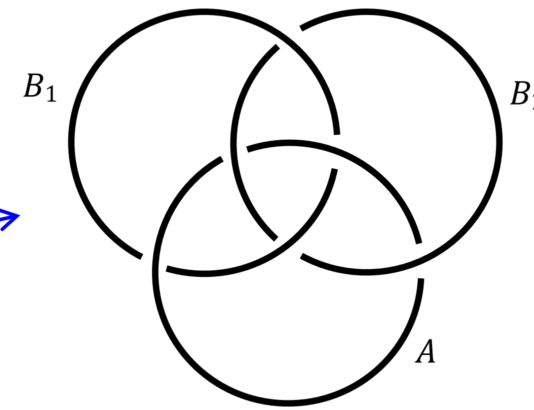


Satisfies (a) & (b)

Doesn't satisfy (a) & (b)



$$|\phi\rangle = \sqrt{½}\{|0\rangle_A|0_x\rangle_{B_1}|0_x\rangle_{B_2} + |1\rangle_A|1_x\rangle_{B_1}|1_x\rangle_{B_2}\}$$
$$= \sqrt{½}|0\rangle_{B_1}\{\sqrt{½}(|0\rangle_A|0_x\rangle_{B_2} + |1\rangle_A|1_x\rangle_{B_2})\}$$
$$+ \sqrt{½}|1\rangle_{B_1}\{\sqrt{½}(|0\rangle_A|0_x\rangle_{B_2} - |1\rangle_A|1_x\rangle_{B_2})\}$$
$$= \sqrt{½}|0\rangle_{B_2}\{\sqrt{½}(|0\rangle_A|0_x\rangle_{B_1} + |1\rangle_A|1_x\rangle_{B_1})\}$$
$$+ \sqrt{½}|1\rangle_{B_2}\{\sqrt{½}(|0\rangle_A|0_x\rangle_{B_1} - |1\rangle_A|1_x\rangle_{B_1})\}$$

$$|\psi\rangle = \sqrt{½}\{|0\rangle_A|0\rangle_{B_1}|0\rangle_{B_2} + |1\rangle_A|1\rangle_{B_1}|1\rangle_{B_2}\}$$

$$|0_x\rangle = \sqrt{½}\{|0\rangle + |1\rangle\}$$
$$|1_x\rangle = \sqrt{½}\{|0\rangle - |1\rangle\}$$

Claim 4b (*Superluminal signaling via wormholes and topology change*). Let M be a compact region that has spacelike boundaries, satisfies GC, and contains two disjoint regions A, B connected by a wormhole. If M undergoes topology change, then the wormhole can be made traversible and used to send superluminal signals.

Consequence of No Topology Change theorem in the form:

$$(\text{topology change in } M) \Rightarrow \sim(\text{ANEC \& GC hold in } M)$$

<u>*Concerns*</u>:

- Not a duality between cloning and topology change in general.
- Claim 4b is more general than Claim 4a: Entanglement in general is not sufficient for superluminal signaling via cloning; whereas a wormhole in the presence of topology change is.
- No Cloning is underwritten by linearity of observables, whereas No Topology Change is dependent on contingent features (ANEC, GC) of a spacetime.

Non-detectability

Bao *et al.* (2015b)

- Non-detectability of entanglement is dual to non-detectability of wormholes.

Claim 5a (*Non-detectability of entanglement*).

(i) No observable can determine if a system is in an unknown entangled state; but,

(ii) there is an observable that can determine if a system is in a known entangled state.

(i) Suppose there is an observable P_E that projects any vector in a multi-partite Hilbert space $\mathcal{H}$ onto the set of entangled vectors in $\mathcal{H}$.

- Linearity requires that if P_E answers "yes" to two vectors separately, it answers "yes" to their sum.
- *But*: The sum of two entangled vectors may not be an entangled vector.

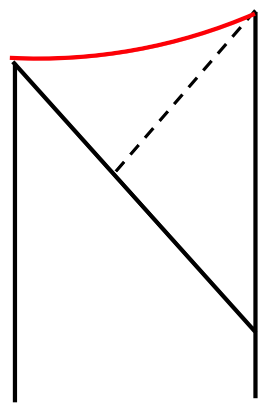
(ii) Suppose $|\phi\rangle$ is a known entangled vector. Then $P_{|\phi\rangle} \equiv |\phi\rangle\langle\phi|$ projects onto span$\{|\phi\rangle\}$.

Claim 5b (*Non-detectability of wormholes*).

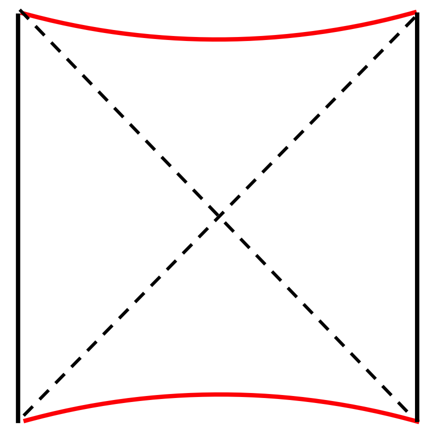
(i) No local observable can determine if a black hole is in an unknown wormhole state (i.e., one-sided or two-sided), but

(ii) there is a local observable that can determine if a black hole is in a known wormhole state (i.e., if it is of a particular two-sided type).

Bao *et al.* (2015b)



One-sided black hole



Two-sided black hole

(i) External region in vicinity of event horizon is identical in a two-sided and a one-sided black hole spacetime

(ii) Two observers outside spatially distant black holes jump in:

- If they meet, they can determine the *type* of wormhole.

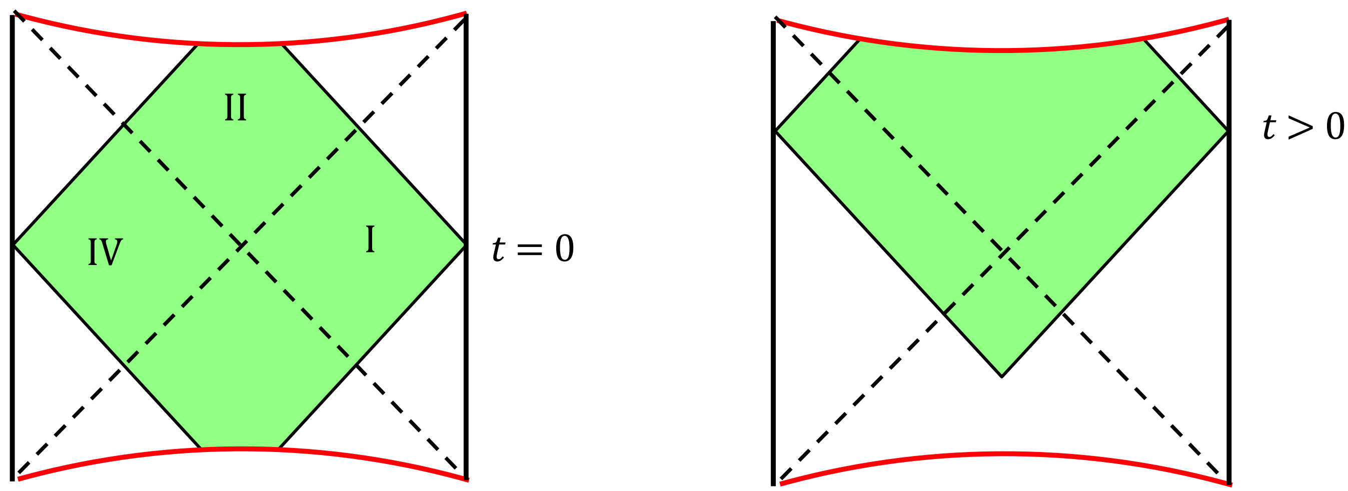
analogous to projecting onto a particular entangled state

<u>*Classifying wormhole type*</u>:

$$\sum_n e^{-\beta E_n/2} e^{-iE_n t} |n_1^* n_2\rangle \propto |\Psi(t)\rangle \equiv |\Psi_\alpha\rangle$$

time-dependent TFD state

One parameter family of different entangled states.

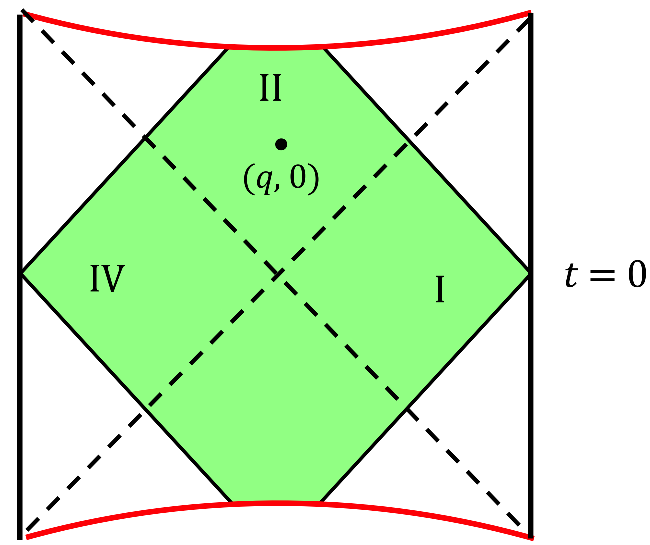


<u>*Proposal*</u>: The wormhole at $t = t_0$ dual to $|\Psi(t_0)\rangle$ is given by the bulk causal diamond anchored at t.

Maldacena & Susskind (2013)

Detecting wormhole type:

Claim: Can determine α by measuring tidal force at intersection in region II of timelike geodesics that originate in regions I and IV.



Proof sketch

1. In region II, Kruskal coordinates satisfy
$$T^2 - X^2 = e^{f'(r_H)r_*},$$
where $f(r) = 1 - r_S/r + r^2/a^2$ and $r_*(r) = \int dr/f(r)$.

2. Let point of intersection be $(T, X) = (q, 0)$, or
$$(q - \alpha)^2 = e^{f'(r_H)r*}$$
Idea: Parameter α measures "shift" in T-direction: $T \to T - \alpha$.

3. For a local Lorentz frame $(\hat{t}, \hat{r}, \hat{\theta})$
$$C_{\hat{r}\hat{\theta}\hat{r}\hat{\theta}} = -1/a^2 - GM/r^3$$
Now solve for r and hence α.

Idea: Weyl tensor encodes tidal effects in vacuum spacetimes via equation of geodesic deviation.

<u>*Disanalogy*</u>:

- Entanglement non-detectability is due to non-linearity: The sum of entangled states need not be an entangled state.

A consequence of the relativity of entanglement to system decomposition?

- Wormhole non-detectability is due to the local nature of curvature (single observer), or to a particular way of classifying wormholes.

No similar relativity? Can a spacetime admit a wormhole under one decomposition into subregions but not another?

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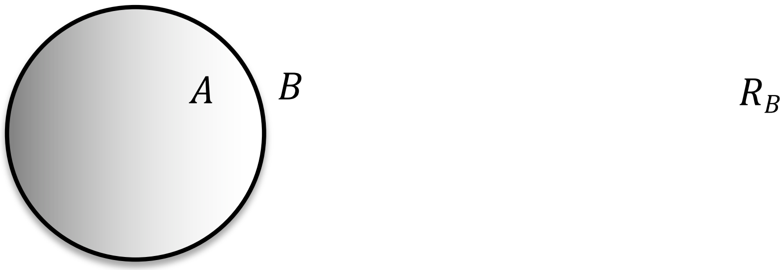


Wallace (2020), Harlow (2016)

(a) For any late stage radiation mode B just ouside the event horizon, there is an early stage mode R_B far from the horizon that is maximally entangled with it.

(b) For any late stage mode B just outside the horizon, there is an interior mode A maximally entangled with it.

(c) (*Monogamy*) A system cannot be maximally entangled with two other systems.

Statistical mechanical analysis. Page (1993)

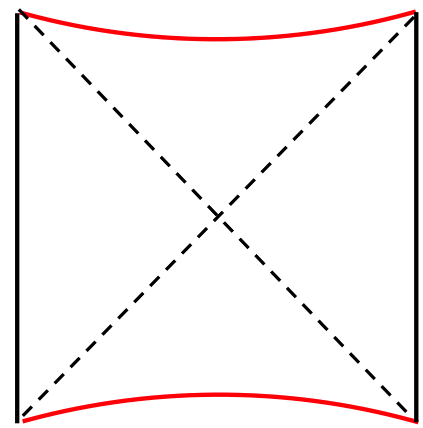
Quantum field theoretic analysis of modes on either side of a horizon.

AMPS (2013): Deny (b): Claim that every late stage mode B just outside horizon corresponds to a high-energy interior mode A, all of which form a "firewall".

Almheiri *et al.* (2013)

<u>Problem</u>: GR entails a smooth horizon (no "drama").

Firewall motivation?



$\langle T_{ab}\rangle_{\Omega_W}$ diverges at horizon

Expectation value of stress-energy tensor of scalar field with respect to "wedge" vacuum $|\Omega_W\rangle = |0\rangle_L \otimes |0\rangle_R$.

Concern:

- Derived by expressing T_{ab} in field modes that belong to "global" representation of local wedge algebra of observables.
- Similar divergence in global total number operator N with respect to wedge vacuum:

$\langle N\rangle_{\Omega_W}$ diverges at horizon

"an artifact of the peculiarity of disjoint representations, not a guide to the particle contents of the Minkowski vacuum..." (Arageorgis *et al*. 2003).

$\langle T_{ab}\rangle_{\Omega_W}$ diverges at horizon

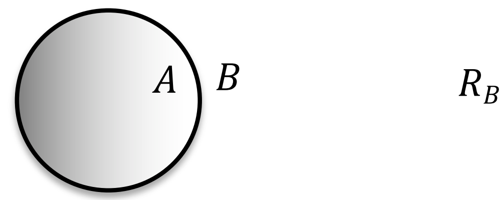
An artifact of the peculiarity of disjoint representations, not a guide to what happens if one jumps into a black hole?

Suppose we take firewalls seriously...

(a) For any late stage radiation mode B just ouside the event horizon, there is an early stage mode R_B far from the horizon that is maximally entangled with it.

(b) For any late stage mode B just outside the horizon, there is an interior mode A maximally entangled with it.

(c) (*Monogamy*) A system cannot be maximally entangled with two other systems.



ER=EPR Resolution I: "Disruption"

Maldacena & Susskind (2013)

- Deny (b) for particular B that corresponds to an R_B that has been distilled by an infalling observer:
  - *Distillation "disrupts" entanglement between B and A via a wormhole connecting R_B and A, but leaves all other entangled exterior/interior modes intact.*

"...in our view the particle [A] was created at the other end of the ER bridge when Alice distilled R_B."

- *Thus*: For infalling observer who has distilled R_B, there is an exterior mode B and a single high-energy interior mode A such that

  (i) B is maximally entangled with R_B; and

  (ii) R_B and A are connected by a wormhole.

Doesn't this entail R_B and A are entangled?

Does R_B now violate monogamy?

(a) For any late stage radiation mode B just ouside the event horizon, there is an early stage mode R_B far from the horizon that is maximally entangled with it.

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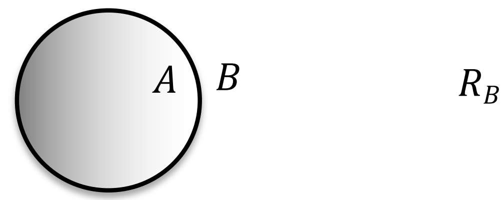
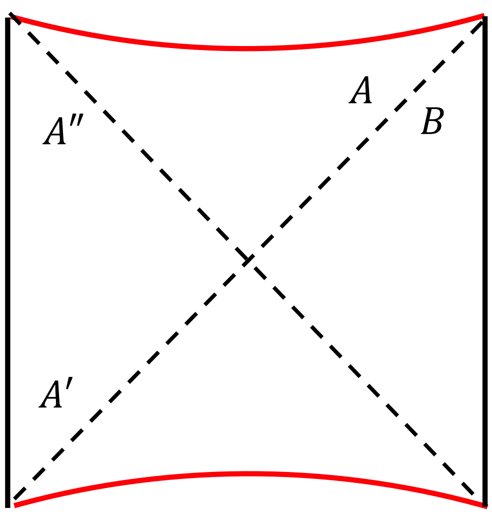


ER=EPR Resolution II: "A=R_B"

Maldacena & Susskind (2013)

- Claim there is a wormhole connecting R_B and A, and this underwrites and equivalence between R_B and A.
- *Thus*: B is entangled with a single mode A/R_B.



- A' is maximally entangled with B (A' and B are subsystems in a CFT thermofield double state).
- A'' is mode on same timeslice as A that is determined by A' via boundary time evolution.
- A'' is a boundary operator that represents A in the bulk.
- A'' can be understood as R_B.

But: In what sense does a wormhole connecting A'' and A underwrite their equivalence?

1. Entanglement Entropy vs. Minimal Area Cross-Section
   - *Conservation Under "Local Operations"*
   - *Monogamy*
2. "No-Go" Theorems
   - *No-Signaling vs. Non-traversability*
   - *No Cloning vs. No Topology Change*
   - *Non-detectability*
3. A Solution to the Firewall Problem?
4. A Causal Explanation of Entanglement Correlations?

4. A Causal Explanation of Entanglement Correlations?

Does the presence of a wormhole connecting two regions that support observables that exhibit an entanglement correlation causally explain the latter?

Set-up:

Let AB be a bipartite system in a pure entangled state, with subsystems A, B localized in regions X, Y. Observables O_X, O_Y associated with A and B exhibit an **entanglement correlation** just when:

(a) O_X and O_Y are correlated.

(b) The distance between X and Y exceeds an appropriate bound on causal signal propagation. ← *No direct cause explanation of (a).*

(c) There is no random variable λ such that O_X and O_Y are conditionally statistically independent with respect to λ. ← *No common cause explanation of (a).*

Now suppose ER=EPR:

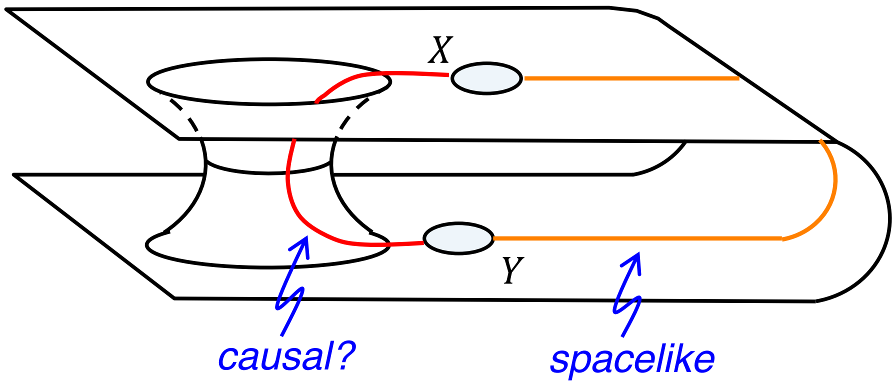
(d) X and Y are connected by a wormhole.

Can (d) provide the missing direct cause explanation of (a)?

Can (d) provide the missing common cause explanation of (a)?

Can a wormhole provide the missing direct cause explanation of an entanglement correlation?

No!



- Suppose X, Y cannot be connected by a causal path in the exterior spacetime.
- Can X, Y be connected by a causal path through the wormhole?

Not for typical general relativistic spacetimes...

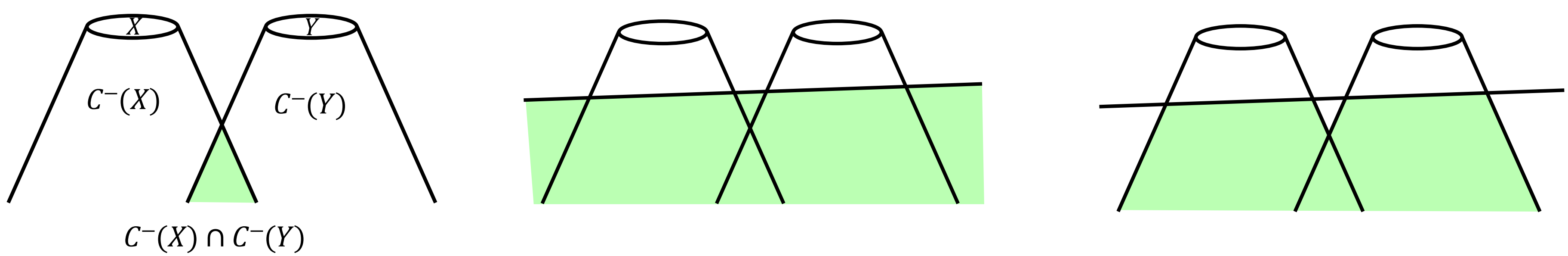
Can a wormhole provide the missing common cause explanation of an entanglement correlation?

<u>*Two senses of common cause*</u>

- Observables O_X, O_Y share a *statistical common cause* just when there is a random variable that screens one off from the other.
- Regions X, Y can share a *spatiotemporal common cause* just when they share a *common past*.

Many possible definitions!

Butterfield (1989)



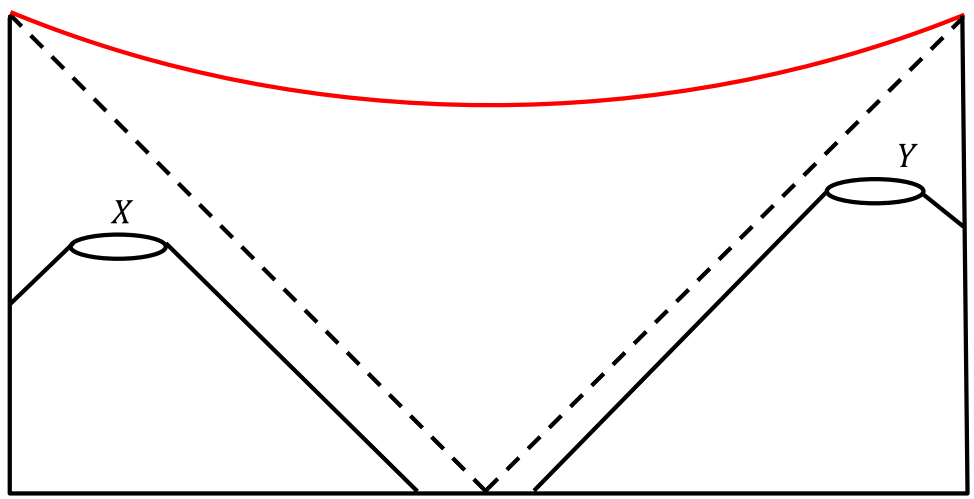
<u>*Mysterious nature of entanglement*</u>

Entanglement-correlated observables lack a statistical common cause even when their regions of support may share a spatiotemporal common cause.

Can ER=EPR provide the "missing link" between statistical and spatiotemporal senses of common cause?

Is the lack of a statistical common cause exhibited by entangled observables O_X, O_Y mirrored by the lack of a spatiotemporal common cause for wormhole-connected regions of support X, Y?

In certain constrained scenarios, wormhole-connected regions lack a spatiotemporal common cause (for a given definition of common cause)...



Are the bulk regions X, Y that do not share a common causal past dual to boundary regions that support entangled boundary observables?

Truncated extended AdS-Schwarzschild spacetime

Does the presence of a wormhole connecting two regions that support observables that exhibit an entanglement correlation causally explain the latter?

Probably not...

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