A Duality Between Topology and Entanglement? ER=EPR, Part 1

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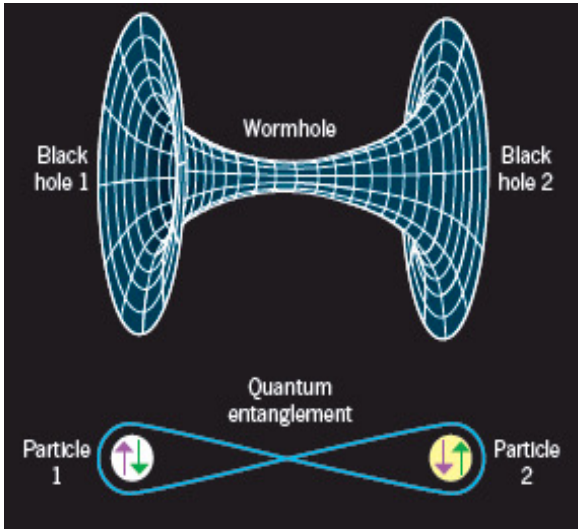
0. ER=EPR: Introduction

The ER = EPR Hypothesis
The subsystems of a composite system in a quantum entangled state ("EPR") are connected by an Einstein-Rosen wormhole ("ER").

Maldaceno & Susskind (2013)



- A way to reconcile quantum theory with general relativity?
- A duality between quantum entanglement and spacetime topology?
- A solution to the Firewall Problem?
- Intriguing similarities between entangled states and wormholes:
  - *Both can't be used to send superluminal signals.*
  - *Both are not affected by "local operations".*

Part 1: Review and Initial Motivation

1. ER Wormholes
   - *Warm-Up: Minkowski Spacetime*
   - *Extended Schwarzschild Spacetime*
   - *Extended AdS-Schwarzschild Spacetime*
2. Quantum Entanglement and Spacetime Structure
   - *Quantum Entanglement in Minkowski Spacetime*
   - *Quantum Entanglement in Extended Schwarzschild Spacetime*
3. Initial Motivation: Maldacena's (2003) Example
   - *Extended AdS-Schwarzschild Spacetime and the CFT Thermofield Double State*

Part 2: Additional Motivations

1. Entanglement Entropy and Cross-Sectional Area of Wormhole Throat
2. "No-Go" Theorems
3. A Solution to the Firewall Problem?
4. A Causal Explanation of Entanglement Correlations?

1. ER Wormholes

Carroll (2004)
Harlow (2016)

1.1. Warm-Up: Minkowksi Spacetime

Def. 1. **Minkowski spacetime** is the unique zero curvature vacuum solution to the Einstein equations with spacetime interval in (t, x, y, z) coordinates given by

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

Minkowski spacetime interval in spherical coordinates (t, r, θ, ϕ):

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

Minkowski spacetime as an extension of Rindler spacetime

- Consider "right wedge" of Minkowski spacetime $x > |t|$.

- Introduce Rindler coordinates (η, ξ):

$$x = \xi \cosh \eta \qquad \xi = \sqrt{x^2 - t^2}$$

$\xi > 0$ for $x > |t|$

$$t = \xi \sinh \eta \qquad \eta = \tanh^{-1}(t/x)$$

- An η-"time" translation is equivalent to a (t, x)-Lorentz boost:

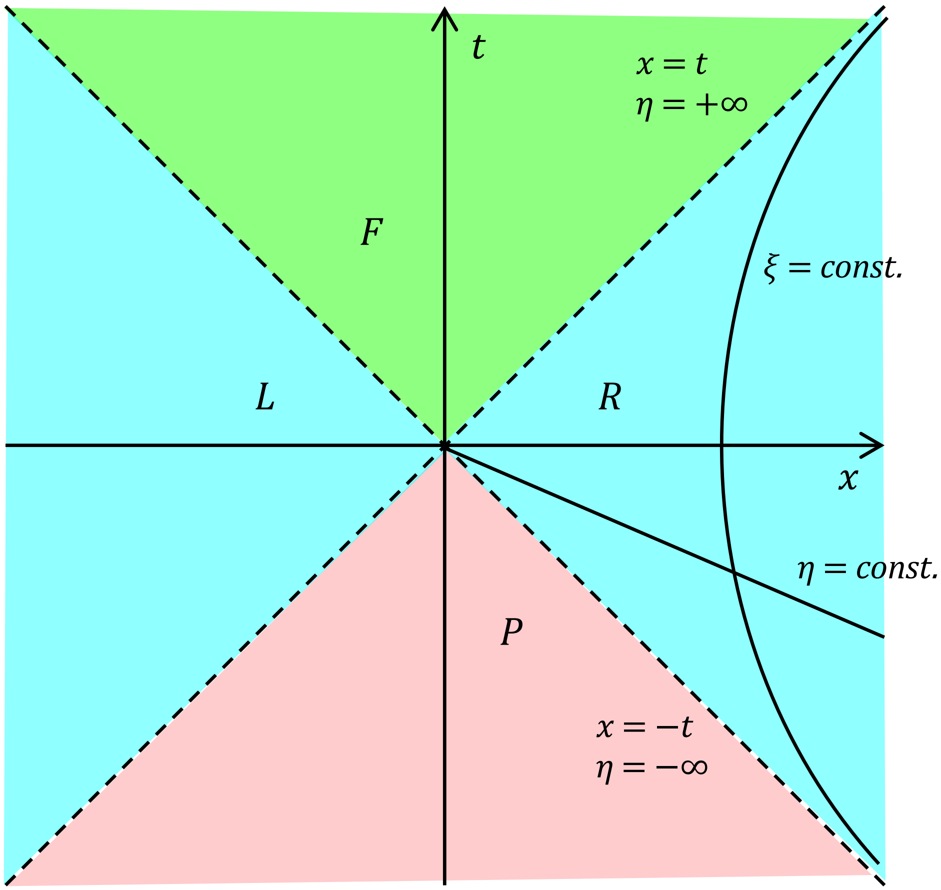
$$\underbrace{(\partial/\partial\eta)^a}_{\text{generator of } \eta\text{-time translations}} = \underbrace{t(\partial/\partial x)^a + x(\partial/\partial t)^a}_{\text{generator of Lorentz boosts}}$$

Killing vector = generator of symmetry of metric

- Minkowski interval in (η, ξ) coordinates: $ds^2 = -\xi^2 d\eta^2 + d\xi^2$

> **Def 2. Rindler spacetime** is the $x > |t|$ "right wedge" of Minkowski spacetime with spacetime interval in (η, ξ) coordinates:
>
> $$ds^2 = -\xi^2 d\eta^2 + d\xi^2, \qquad -\infty < \eta < \infty, \;\; \xi > 0$$

Minkowski spacetime as an extension of Rindler spacetime



Region R = Rindler spacetime

Curves of constant ξ are:

- hyperbolae $x^2 - t^2 = const$ with asymptotes at $x = \pm t$ (dashed lines)
- orthogonal to $\eta = const$ surfaces (Cauchy foliation)
- integral curves of
$$(\partial/\partial\eta)^a = t(\partial/\partial x)^a + x(\partial/\partial t)^a$$

Timelike Killing vector distinct from $(\partial/\partial t)^a$

Claim. The null surface $x = \pm t$ is a *bifurcate Killing horizon* generated by $(\partial/\partial\eta)^a$.

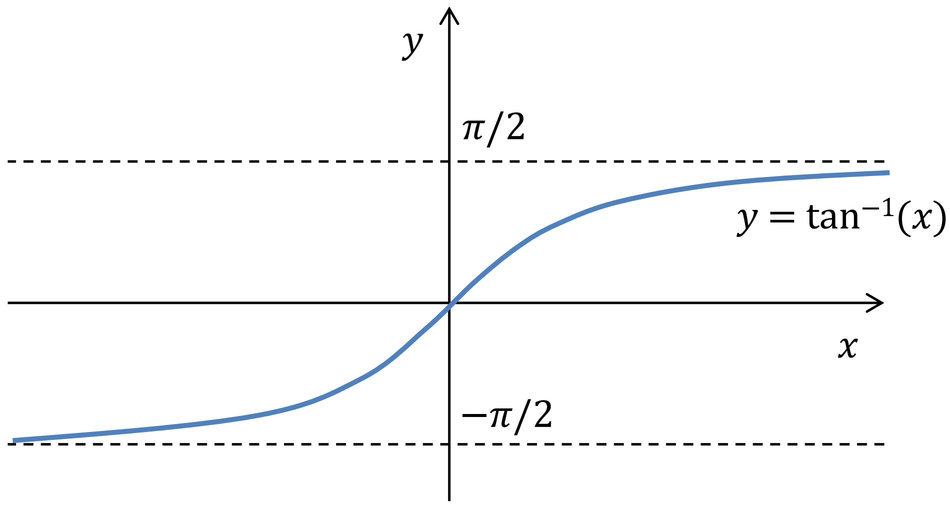
Fine print: Bifurcate Killing horizon = the union of two null hypersurfaces that intersect in a spacelike 2-dim surface (the bifurcation surface) and that are both Killing horizons (null hypersurface everywhere normal to a Killing vector) with respect to the same Killing vector.

Bottom-line: A spacetime with a bifurcate Killing horizon:

- decomposes into 4 regions;
- admits two distinct timelike Killing vectors.

Conformal Diagram for Minkowski Spacetime

- Construct new coordinates that "pull in" temporal, spatial, and null infinities.
- *Intuition*: $\tan^{-1}(x)$ maps $(-\infty, +\infty)$ to $(-\pi/2, \pi/2)$



Motivation for Penrose coordinates...

Conformal Diagram for Minkowski Spacetime

Penrose Coordinates (T', R') for Minkowski Spacetime

$T'+R' = 2\tan^{-1}(t+r)$, $\quad T'-R' = 2\tan^{-1}(t-r)$

$T' = \tan^{-1}(t+r) + \tan^{-1}(t-r)$, $\quad R' = \tan^{-1}(t+r) - \tan^{-1}(t-r)$

$-\pi < T' < \pi \quad 0 \leq R' < \pi$

Minkowski spacetime interval in Penrose coordinates (T', R', θ, ϕ):

$$ds^2 = \frac{1}{(\cos T' + \cos R')^2}\left[-dT'^2 + dR'^2 + \sin^2 R'(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

Pre-factor blows up at:

$T' = \pm\pi, \quad R' = 0$

$T' = 0, \quad R' = \pi$

$T' = \pi - R', \quad 0 < R' < \pi$

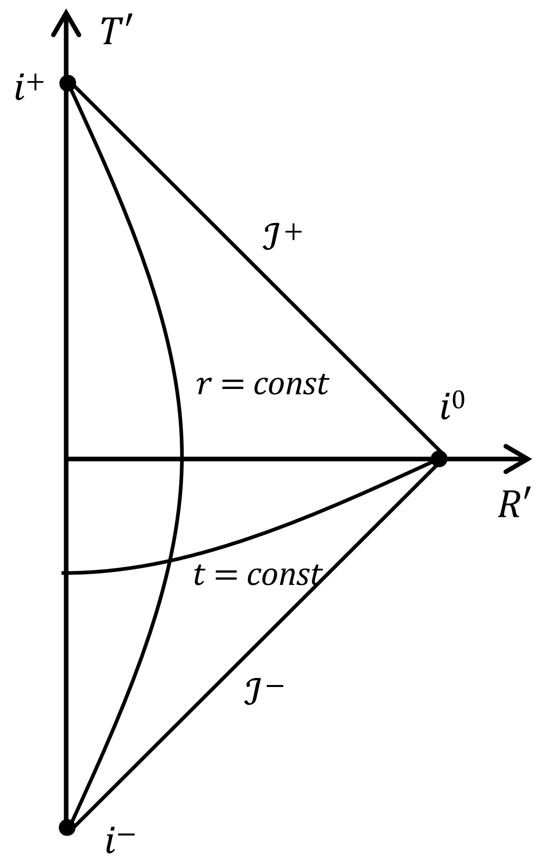
$T' = -\pi + R', \quad 0 < R' < \pi$

Points at infinity corresponding to $t = \pm\infty, r = \pm\infty$

Encode them in the boundary of a conformally equivalent metric with interval

$$\widetilde{ds}^2 = -dT'^2 + dR'^2 + \sin^2 R'(d\theta^2 + \sin^2\theta d\phi^2)$$

Conformal Diagram for Minkowski Spacetime



$i^0 = spacelike\ infinity\ (T' = 0, R' = \pi)$

$i^+ = future\ timelike\ infinity\ (T' = \pi, R' = 0)$

$i^- = past\ timelike\ infinity\ (T' = -\pi, R' = 0)$

$\mathcal{J}^+ = future\ null\ infinity\ (T' = \pi - R', 0 < R' < \pi)$

$\mathcal{J}^- = past\ null\ infinity\ (T' = -\pi + R', 0 < R' < \pi)$

1.2. Extended Schwarzschild Spacetime

Def. 3. Schwarzschild spacetime is the unique spherically symmetric vacuum solution to the Einstein equations with spacetime interval in (t, r, θ, ϕ) coordinates:

$$ds^2 = -\left(1 - \frac{r_S}{r}\right) dt^2 + \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

<u>*Some properties*</u>:

1. *Exterior*. For $r \gg r_S$, $r_S = 2GM$. ← *geodesic equation is 2nd Law for force due to grav field of spherically symmetric mass M.*

2. *Physical singularity*. At $r = 0$, ds^2 diverges. ← $R^{\sigma\mu\nu\rho}R_{\sigma\mu\nu\rho} = 12{r_S}^2/r^6$ *diverges.*

3. *Coordinate singularity*. At $r = r_S$, ds^2 diverges. ← *No corresponding divergence in a curvature scalar.*

4. *Event horizon*. At $r = r_S$, signature changes.

   $(-dt^2, +dr^2)$, for $r > r_S$ $\quad$ $(+dt^2, -dr^2)$, for $r < r_S$

r-coordinate behaves like a temporal coordinate $\Rightarrow$ *Future-directed timelike/null curves eventually hit singularity at $r = 0$.* $\Rightarrow$ *Event horizon ≈ surface beyond which future-directed timelike/null curves cannot escape to infinity.*

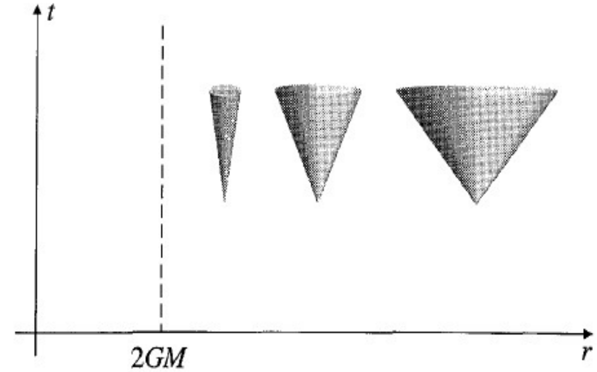
Getting rid of the $r = r_S$ coordinate singularity...

- For radial (θ, ϕ constant) null ($ds^2 = 0$) curves:

$$dt/dr = \pm(1 - r_S/r)^{-1}$$

Nicely behaved for $r \gg r_S$: $dt/dr = \pm 1$.

Badly behaved for $r \to r_S$: $dt/dr \to \pm\infty$.



Lightcones "close up"!

- Pick coordinates so that slope of a radial null curve is always ± 1.
  Require: (*temporal coordinate*) $= \pm$(*radial coordinate*) + *const.*

- <u>*Initial attempt*</u>: *tortise coordinate* r_*

$$t = \pm r_* + const., \qquad \text{where } r_* = r + r_S \ln(r/r_S - 1)$$

t-coord as function of r for radial null curve

<u>*Problem*</u>: $t, r_* = \pm\infty$ at $r = r_S$

<u>*Null Kruskal coordinates (U, V)*</u>

$$U = -e^{(r_* - t)/2r_S}, \quad V = e^{(r_* + t)/2r_S}$$

$$r = r_S \quad \Rightarrow \quad U, V = 0$$

$$r = 0 \quad \Rightarrow \quad UV = (1 - r/r_S)e^{r/r_S} = 1$$

Null coords: Curves of constant U, V are null.

<u>*Problem solved*</u>, *but at expense of null coordinates...*

<u>*Kruskal Coordinates* (T, R)</u>

$T = ½(U+V), \quad R = ½(V-U)$

$U, V = const. \quad \Rightarrow \quad T = \pm R + const.$

$r = r_S; U, V = 0 \quad \Rightarrow \quad T = \pm R$

$r = 0; UV = 1 \quad \Rightarrow \quad T = \pm\sqrt{1 + R^2}$

$r \gg r_S \quad \Rightarrow \quad R \gg T; R \ll T$

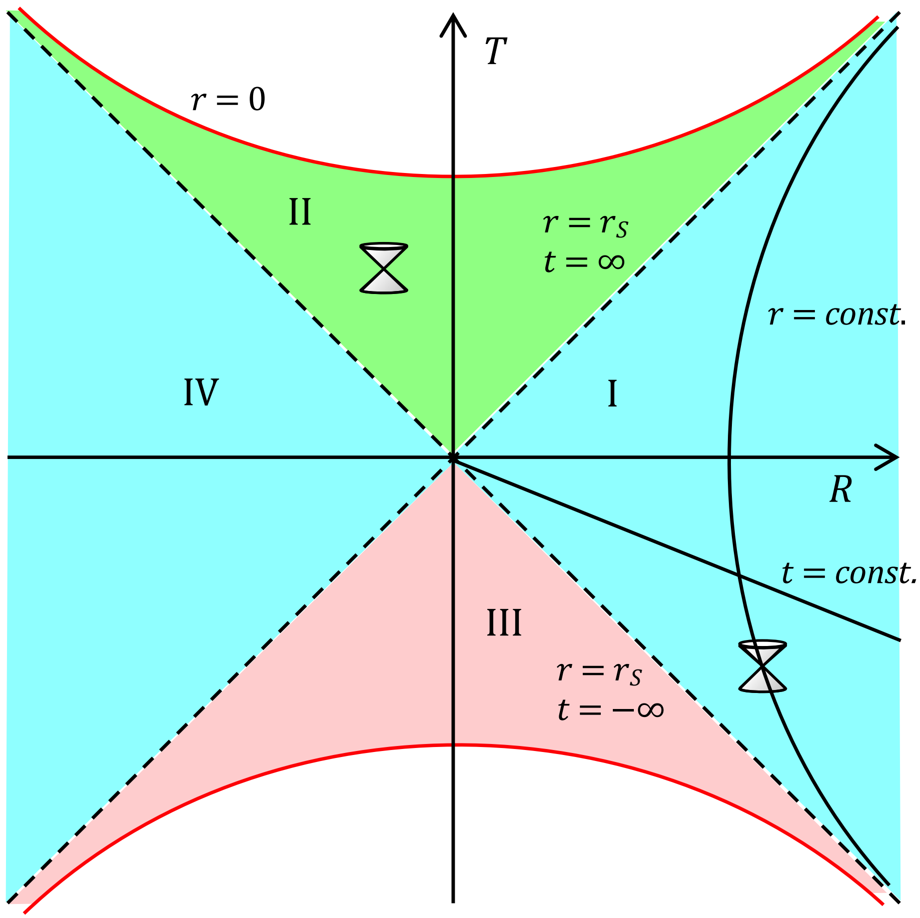
Kruskal coordinates cover Schwarzschild spacetime twice!

Schwarzschild spacetime as a wedge in a larger spacetime (call it "Extended Schwarzschild spacetime") covered by Kruskal coordinates.

Extended Schwarzschild spacetime interval in (T, R, θ, ϕ) coordinates:

$$ds^2 = \frac{4{r_S}^3}{r} e^{-r/r_S}(-dT^2 + dR^2) + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Extended Schwarzschild spacetime as an extension of Schwarzschild spacetime



Region I = Schwarzschild spacetime

Curves of constant r are:

- hyperbolae $R^2 - T^2 = const$ with asymptotes at $X = \pm T$ (dashed lines)
- orthogonal to $t = const$ surfaces (Cauchy foliation)
- integral curves of

$$(\partial/\partial t)^a = T(\partial/\partial R)^a + R(\partial/\partial T)^a$$

Timelike Killing vector distinct from $(\partial/\partial T)^a$

Claim. The event horizons $R = \pm T$ form a *bifurcate Killing horizon* generated by $(\partial/\partial t)^a$.

Next Up: Interpret $T = const.$ spatial slices as a wormhole that connect Regions I and IV...

The Extended Schwarzschild Einstein-Rosen (ER) Wormhole

- Consider geometry of $T = 0$ spatial slice in (t, r, θ, ϕ) coords. Let $\theta = \pi/2$ for simplicity:

$$ds^2 = \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 + r^2 d\phi^2$$

2-dim surface

- Embed it in 3-dim Euclidean space to visualize it:

3-dim Euclidean space

$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2 = \left[1 + \left(\frac{dz}{dr}\right)^2\right] dr^2 + r^2 d\phi^2$$

- Equate coefficients in front of dr^2 and solve for $z(r)$:

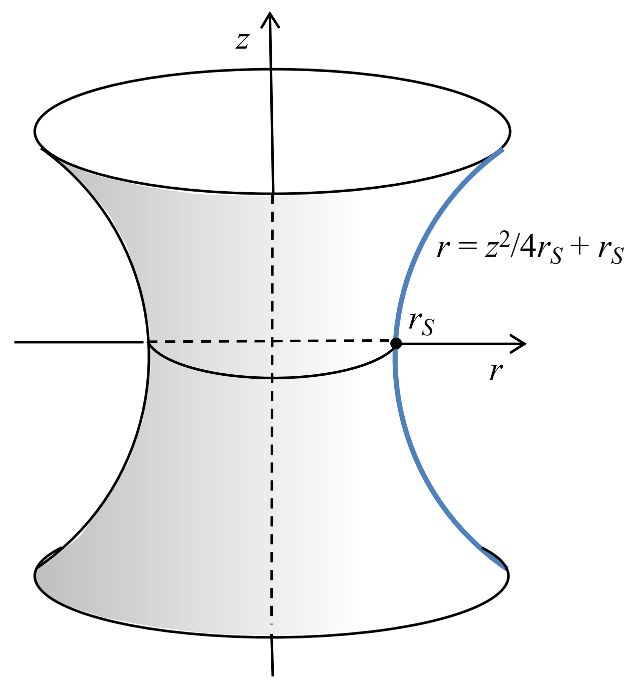
$$\left(\frac{dz}{dr}\right)^2 = \frac{r_S}{r - r_S} \qquad \Rightarrow \qquad z(r) = \pm 2\sqrt{r_S}\sqrt{r - r_S}$$

- Can now solve for r:

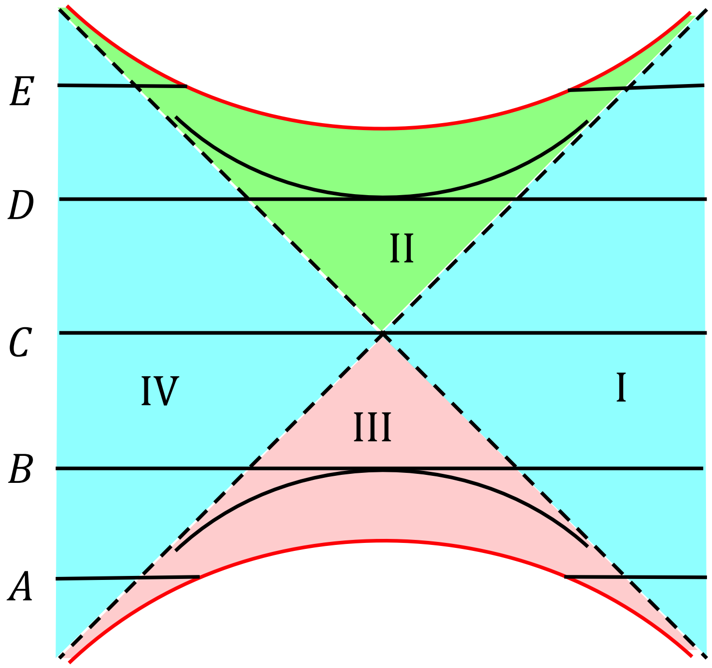
$$r = \frac{z^2}{4r_S} + r_S$$

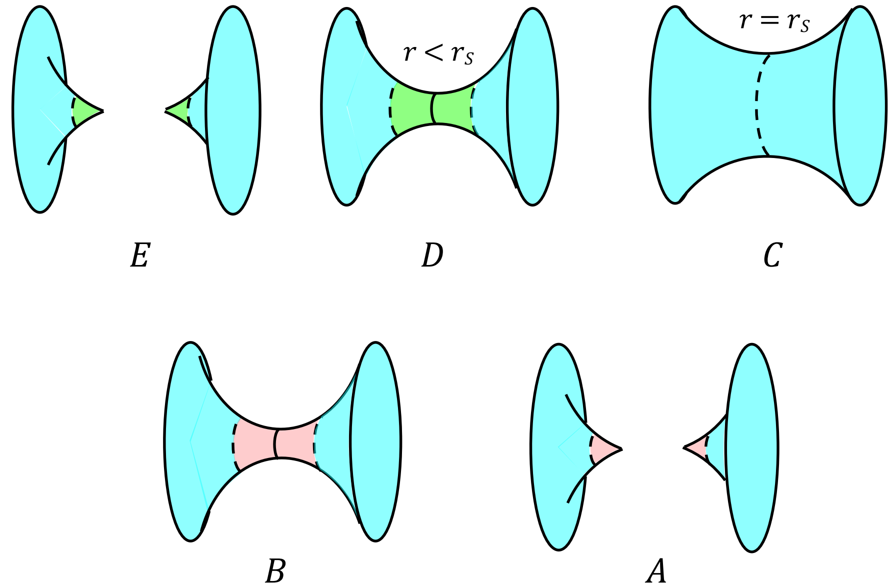
A parabola in the r-z plane!

To get 2-dim $T = 0$ spatial slice, rotate about z-axis to restore ϕ dimension...



The Extended Schwarzschild Einstein-Rosen (ER) Wormhole





- Constant-T spatial slices A, B, C, D, E are paraboloids of revolution.
- C slice ($T = 0$) has greatest minimal cross-sectional area $4\pi r_S$.
- B, D slices have minimal cross-sectional area $4\pi r$, $r < r_S$.
- A, E slices are disconnected.

Extended Schwarzschild ER Wormhole

Wormhole-at-an-instant: Spatial slice of extended Schwarzschild spacetime

Wormhole throat: Region of spatial slice between left and right event horizons.

Conformal Diagram for Extended Schwarzschild Spacetime

Penrose Coordinates (T'', R'') for Extended Schwarzschild Spacetime

$$T''+R'' = \tan^{-1}\left(\frac{T+R}{\sqrt{r_S}}\right), \quad T''-R'' = \tan^{-1}\left(\frac{T-R}{\sqrt{r_S}}\right)$$

$$T''= ½\left\{\tan^{-1}\left(\frac{T+R}{\sqrt{r_S}}\right) + \tan^{-1}\left(\frac{T-R}{\sqrt{r_S}}\right)\right\}, \quad R'' = ½\left\{\tan^{-1}\left(\frac{T+R}{\sqrt{r_S}}\right) - \tan^{-1}\left(\frac{T-R}{\sqrt{r_S}}\right)\right\}$$

(1) Singularity $r = 0$; $T = \pm\sqrt{1+R^2}$ or $(T+R) = 1/(T-R)$

$$T'' = \begin{cases} \pi/4, & T-R > 0 \\ -\pi/4, & T-R < 0 \end{cases}$$

$$\tan^{-1}(1/x) = \begin{cases} \pi/2 - \tan^{-1}(x), & x > 0 \\ -\pi/2 - \tan^{-1}(x), & x < 0 \end{cases}$$

$$R'' = \begin{cases} \pi/4 - \tan^{-1}\left(\frac{T-R}{\sqrt{r_S}}\right), & T-R > 0 \\ -\pi/4 - \tan^{-1}\left(\frac{T-R}{\sqrt{r_S}}\right), & T-R < 0 \end{cases}$$

In both cases $R'' \in [-\pi/4, \pi/4]$

$$0 \le \tan^{-1}(x) < +\pi/2, \quad x > 0$$

$$-\pi/2 < \tan^{-1}(x) \le 0, \quad x < 0$$

(T, R)-hyperbolae become (T'', R'')-straight lines

Conformal Diagram for Extended Schwarzschild Spacetime

Penrose Coordinates (T'', R'') for Extended Schwarzschild Spacetime

$$T''+R'' = \tan^{-1}\left(\frac{T+R}{\sqrt{r_S}}\right), \quad T''-R'' = \tan^{-1}\left(\frac{T-R}{\sqrt{r_S}}\right)$$

$$T''= ½\left\{\tan^{-1}\left(\frac{T+R}{\sqrt{r_S}}\right) + \tan^{-1}\left(\frac{T-R}{\sqrt{r_S}}\right)\right\}, \quad R'' = ½\left\{\tan^{-1}\left(\frac{T+R}{\sqrt{r_S}}\right) - \tan^{-1}\left(\frac{T-R}{\sqrt{r_S}}\right)\right\}$$

(2) Event horizon $r = r_S$; $T = \pm R$:

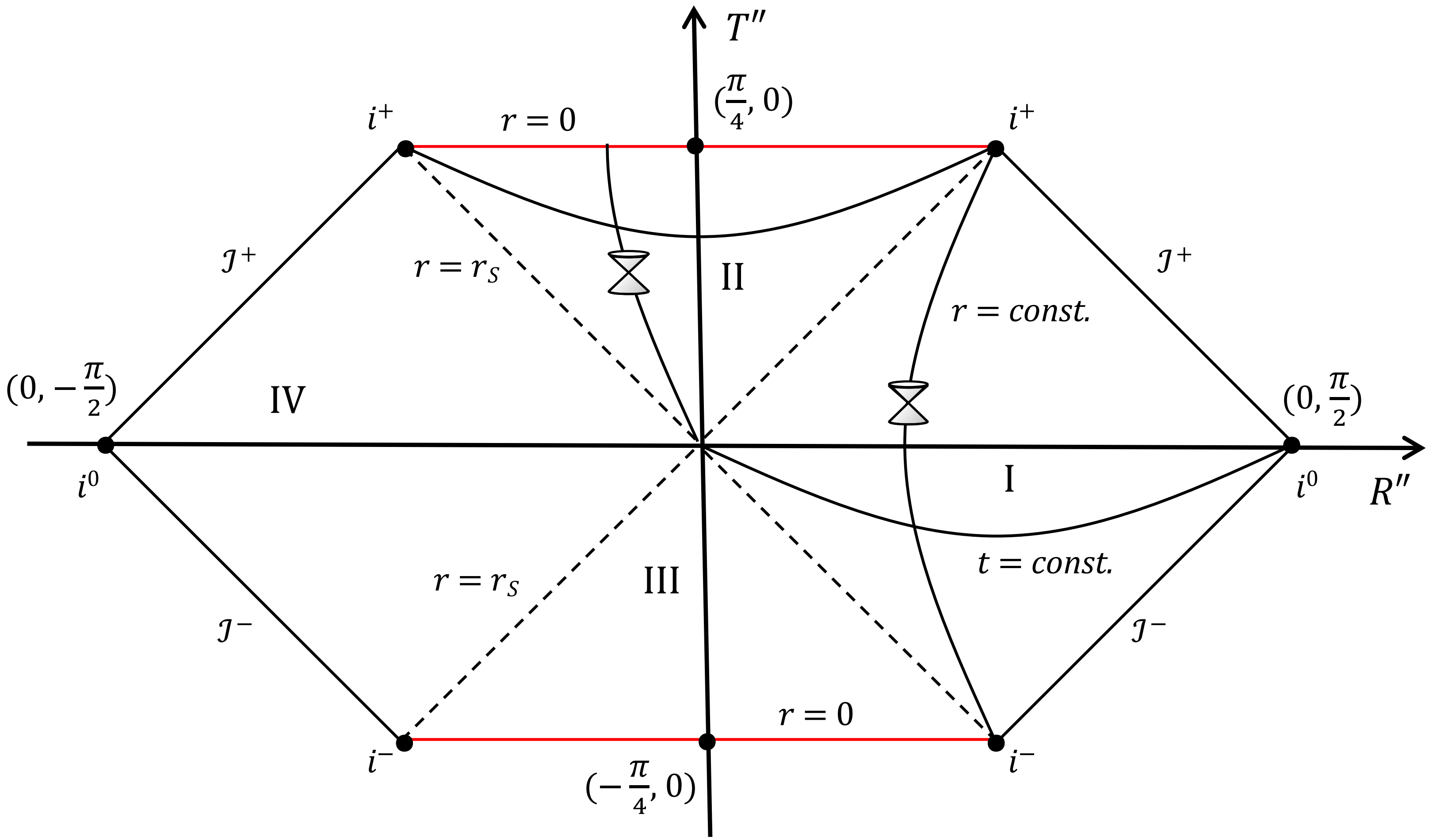
$$T''= \begin{cases} ½\tan^{-1}\left(\frac{2X}{\sqrt{r_S}}\right) = R'', & T = R \\ ½\tan^{-1}\left(\frac{-2X}{\sqrt{r_S}}\right) = -R'', & T = -R \end{cases}$$

(T, R) straight lines remain
(T'', R'') straight lines

(3) If $T = \pm R + const.$, then $T'' = \pm R'' + const.$

Penrose coords preserve invariant lightcone structure of Kruskal coords.

Conformal Diagram for Extended Schwarzschild Spacetime



1.3. Extended AdS-Schwarzschild spacetime

Def. 4. Anti-de Sitter (AdS) spacetime is the unique vacuum solution to the Einstein equations with constant negative curvature and spacetime interval in (t, r, θ, ϕ) coordinates:

$$ds^2 = -\left(1 + \frac{r^2}{a^2}\right) dt^2 + \left(1 + \frac{r^2}{a^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- a is radius of curvature
- As $a \to \infty$, curvature goes to zero, AdS interval becomes Minkowski interval.

Def. 5. AdS-Schwarzschild spacetime is the unique spherically symmetric vacuum solution to the Einstein equations with constant negative curvature and spacetime interval in (t, r, θ, ϕ) coordinates:

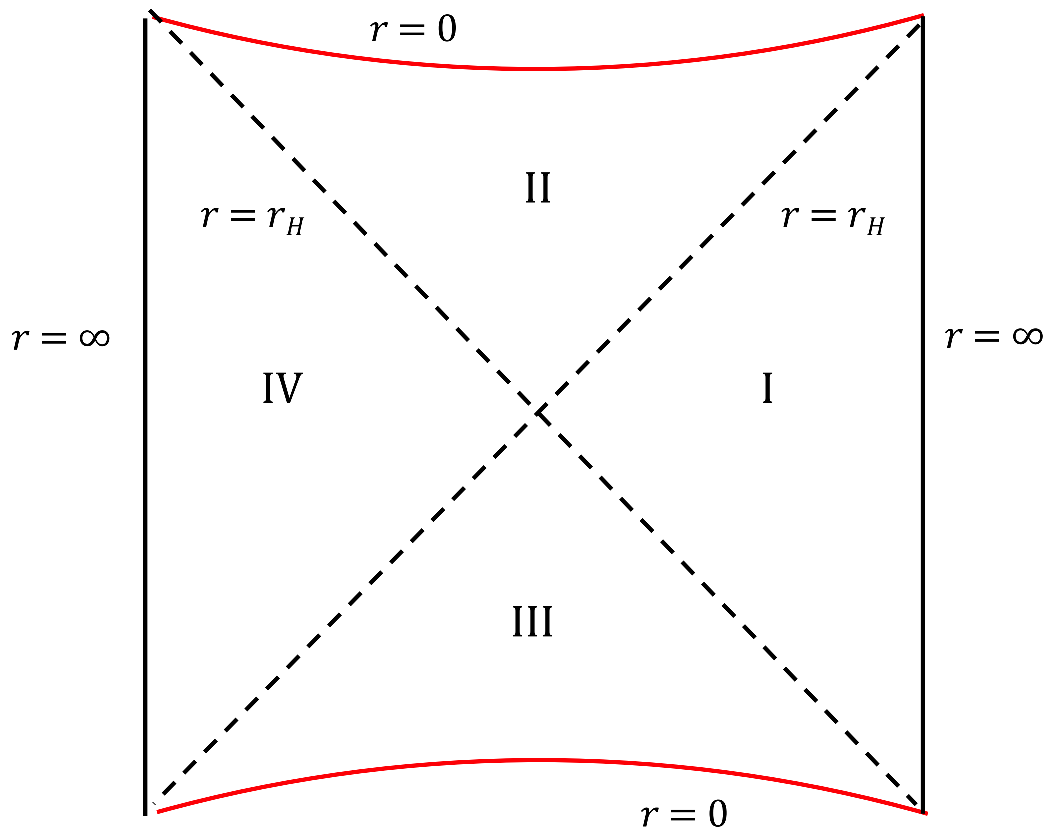
$$ds^2 = -\left(1 - \frac{r_S}{r} + \frac{r^2}{a^2}\right) dt^2 + \left(1 - \frac{r_S}{r} + \frac{r^2}{a^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- For large r, AdS-Schwarzschild interval becomes AdS interval.
- For small r, AdS-Schwarzschild interval becomes Schwarzschild interval.
- Coordinate singularity at the value r_H of r that solves $1 - \frac{r_S}{r} + \frac{r^2}{a^2} = 0$

Now: Transform to Kruskal-esque coordinates to get rid of coordinate singularity in a lightcone invariant way.

Claim. Extended AdS-Schwarzschild spacetime admits an interpretation in terms of an ER wormhole.

Conformal Diagram for Extended AdS-Schwarzschild Spacetime



	Spacetime	*Extended Spacetime*	*Wormhole?*
(a)	Rindler	Minkowski	no
(b)	Schwarzschild	Extended Schwarzschild	yes
(c)	AdS-Schwarzschild	Extended AdS-Schwarzschild	yes

What do these spacetimes have to do with quantum entanglement?

Story to come

- For (a), (b), (c), the vacuum state of a quantum field in the extended spacetime can be expressed as an entangled state across the left and right wedges.
- Maldacena (2003) reinterpreted entangled state in (c) as an entangled state in a boundary CFT.

Part 1: Review and Initial Motivation

1. ER Wormholes
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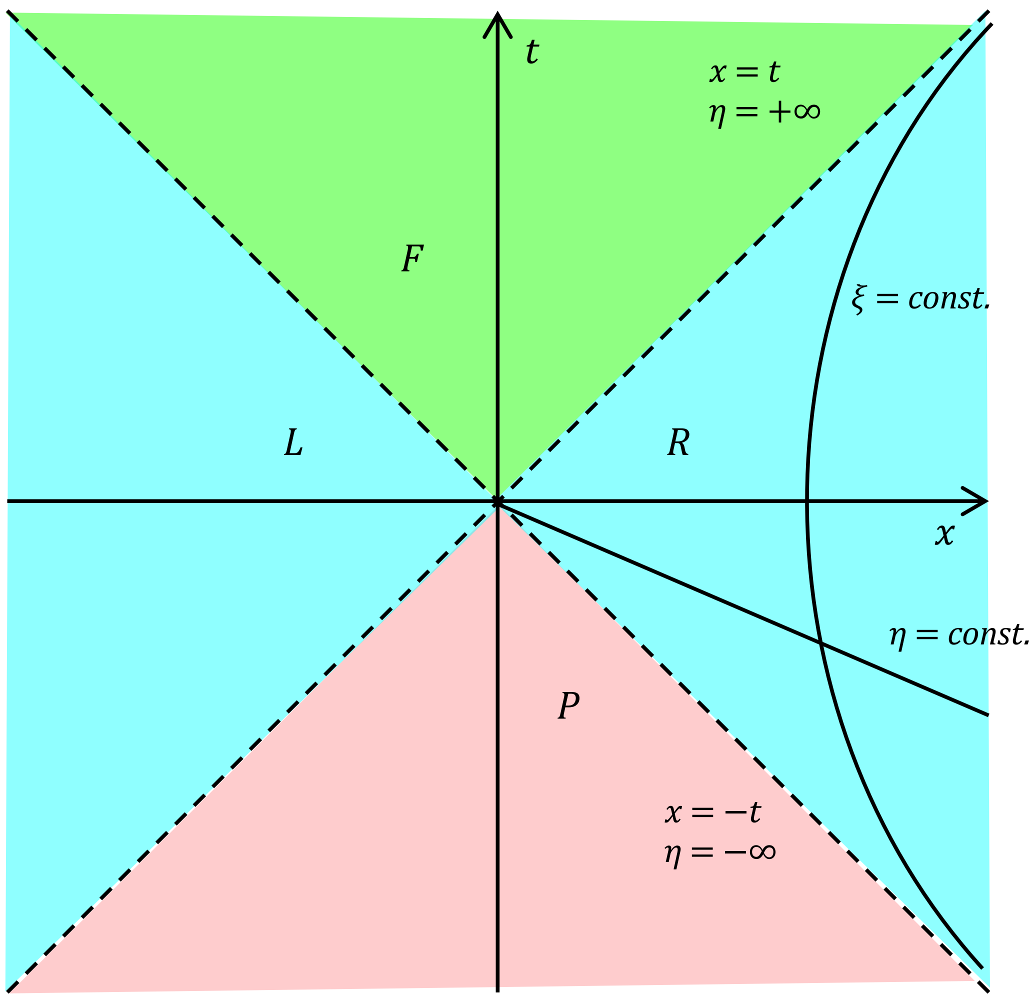
2. Quantum Entanglement and Spacetime Structure

2.1. Quantum Entanglement in Minkowksi Spacetime

Claim. The vacuum state of a quantum field in Minkowski spacetime can be expressed as an entangled state over the left and right Rindler wedges L, R.



3 Routes to a demonstration...

Route 1: Standard Quantization Procedure

1. Decompose field into negative and positive frequency modes.
2. Equip space of pos freq solutions with inner product and complete to form single-particle Hilbert space $\mathcal{H}$.
3. Define multi-particle Fock space $\mathcal{F} = \bigoplus_{n=0}^{\infty} \mathcal{H}^{(n)}$, $\mathcal{H}^{(n)} = \bigotimes_n \mathcal{H}$.

- Step 1 can be done in R via $(\partial/\partial\eta)^a$ or $(\partial/\partial t)^a$.
  - Fock space built on η: Fulling representation with vacuum state $|0_R\rangle$.
  - Fock space built on t: Minkowski representation with vacuum state $|0_M\rangle$.

Claim. Let $\mathcal{H}_L, \mathcal{H}_R$ be one-particle Hilbert spaces with respect to $\pm(\partial/\partial\eta)^a$ on L and R, and let $|0_M\rangle|_R$ be the restriction of $|0_M\rangle$ to R. Then

$$|0_M\rangle\Big|_R \propto \sum_n e^{-\beta E_n/2} |n_L n_R\rangle$$

where $\beta = 2\pi/\kappa$, κ = surface gravity of Killing horizon, and $|n_L\rangle \in \mathcal{H}_L$, $|n_R\rangle \in \mathcal{H}_R$ are eigenstates of energy in L and R.

Problem: Minkowski and Fulling representations are unitarily inequivalent!

Arageorgis (1995); Clifton & Halvorson (2001); Arageorgis, Earman, Ruetsche (2003)

Harlow (2016)
Jacobson (2013)
Hartman (2015)

Route 2: Euclidean Path Integral Procedure

1. Express $\langle \phi | 0_M \rangle$ as a Euclidean path integral $\mathrm{Int}_E[t_E, \phi]$.
2. Change variables: $\mathrm{Int}_E[t_E, \phi] \to \mathrm{Int}_E[\pm\eta_E, \phi_L, \phi_R]$.
3. Express $\mathrm{Int}_E[\pm\eta_E, \phi_L, \phi_R]$ as a transition amplitude $\langle \phi_L \phi_R | [\cdots] | n_L n_R \rangle$.

$t_E = it$

$\eta_E = i\eta$

Step 1. $\langle \phi | 0_M \rangle \propto \lim_{t_E \to \infty} \langle \phi | e^{-t_E H} | \chi \rangle \propto \int_{\phi(t_E=-\infty)=\chi}^{\phi(t_E=0)=\phi} \mathcal{D}\phi e^{-S}$

Path integral over field configurations between χ and ϕ in lower half of Euclidean spacetime $-\infty < t_E < 0$.

Argument (1): t_E evolution "damps" out excited states.

Argument (2): At finite temperatures $t_E = 1/temp$; hence large t_E states are low temperature states.

Harlow (2016)
Jacobson (2013)
Hartman (2015)

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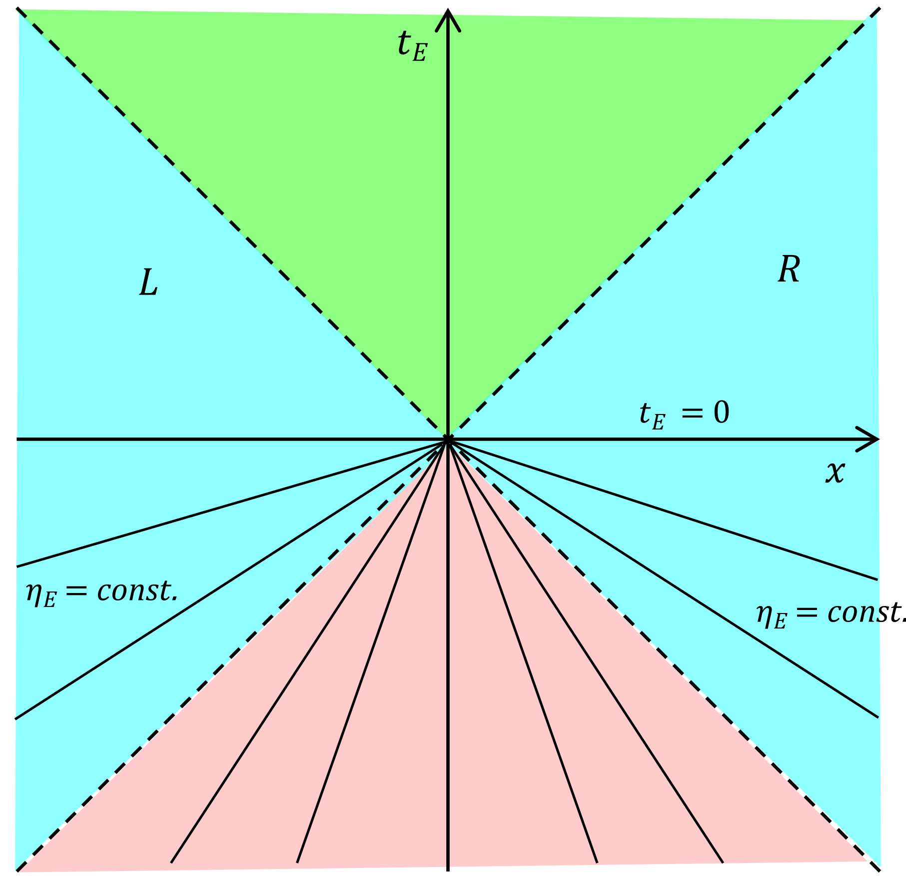
$t_E = it$

$\eta_E = i\eta$

Step 2. $$\int_{\phi(t_E=-\infty)=\chi}^{\phi(t_E=0)=\phi} \mathcal{D}\phi e^{-S} = \int_{\phi(\eta_E=\pi)=\Theta\phi_L}^{\phi(\eta_E=2\pi)=\phi_R} \mathcal{D}\phi e^{-S}$$

Antiunitary $\Theta = CP^1T$
maps $\mathcal{H}_L$ *to* $\mathcal{H}_R$.



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2. Change variables: $\mathrm{Int}_E[t_E,\phi] \to \mathrm{Int}_E[\pm\eta_E,\phi_L,\phi_R]$.
3. Express $\mathrm{Int}_E[\pm\eta_E,\phi_L,\phi_R]$ as a transition amplitude $\langle\phi_L\phi_R|[\cdots]|n_L n_R\rangle$.

Harlow (2016)
Jacobson (2013)
Hartman (2015)

$t_E = it$

$\eta_E = i\eta$

Step 3. $$\int_{\phi(\eta_E=\pi)=\Theta\phi_L}^{\phi(\eta_E=2\pi)=\phi_R} \mathcal{D}\phi e^{-S} \propto \left\langle\phi_R\middle|e^{-\pi H_\eta}\Theta\middle|\phi_L\right\rangle$$

$$= \sum_n e^{-\pi E_n}\langle\phi_R|n_R\rangle\langle n_R|\Theta|\phi_L\rangle$$

Complete set of eigenstates of H_η

$$= \sum_n e^{-\pi E_n}\langle\phi_R|n_R\rangle\left\langle\phi_L\middle|\Theta^\dagger\middle|n_R\right\rangle$$

Antilinearity of Θ

$$= \sum_n e^{-\pi E_n}\langle\phi_L\phi_R|n_L^* n_R\rangle$$

- <u>So</u>: $\langle\phi|0_M\rangle \propto \sum_n e^{-\pi E_n}\langle\phi_L\phi_R|n_L^* n_R\rangle$
- <u>*Hence (?)*</u>: $|0_M\rangle \propto \sum_n e^{-\pi E_n}|n_L^* n_R\rangle$

<u>Problem</u>: Minkowski and Fulling representations are unitarily inequivalent!

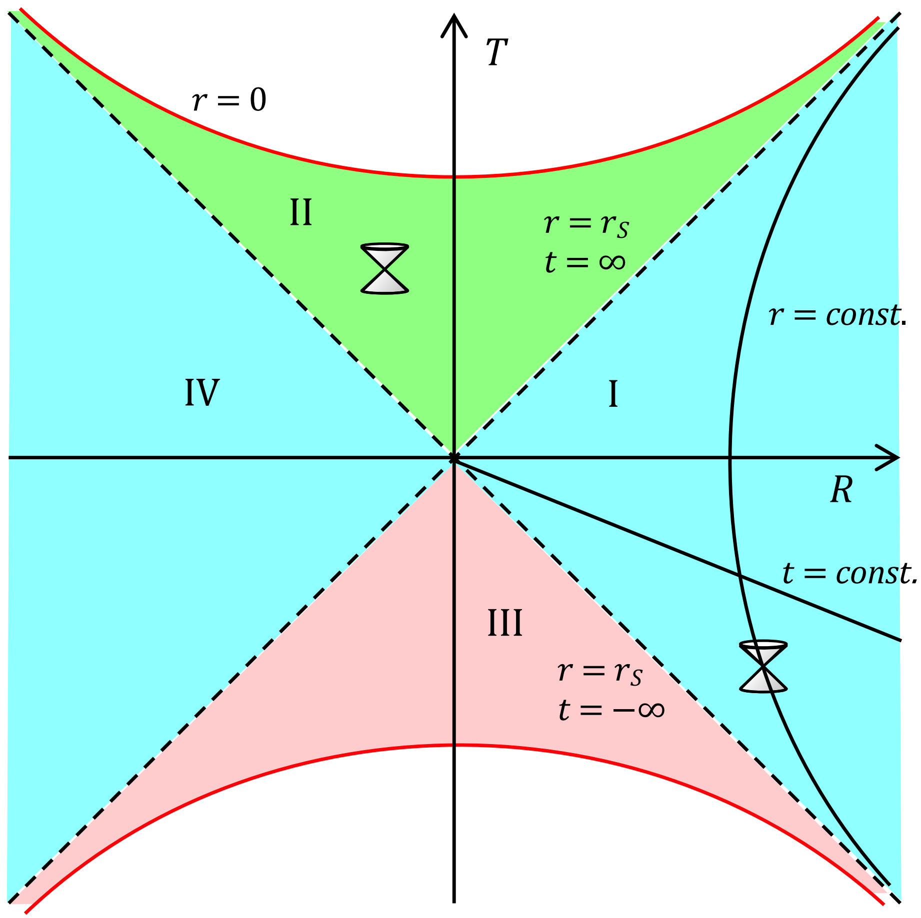
Claim. The vacuum state of a quantum field in Minkowski spacetime can be expressed as an entangled state over the left and right Rindler wedges L, R.

Route 3 Options:

- Insert cut-off; or,
- Appeal to Reeh-Schiedler Theorem for general claim concerning vacuum entanglement across spacelike separated regions.

2.2. Quantum Entanglement in Extended Schwarzschild Spacetime

Claim. The vacuum state of a quantum field in extended Schwarzschild spacetime can be expressed as an entangled state over regions IV and I.



Quick and Dirty Argument

1. Near $r = r_S$, Schwarzschild spacetime is identical to Rindler spacetime.
2. Thus, near $r = r_S$,

$$|0_{HH}\rangle \propto \sum_n e^{-\beta E_n/2} |n_{\mathrm{IV}}^* n_{\mathrm{I}}\rangle$$

"Hartle-Hawking" vacuum state for Fock space built on T.

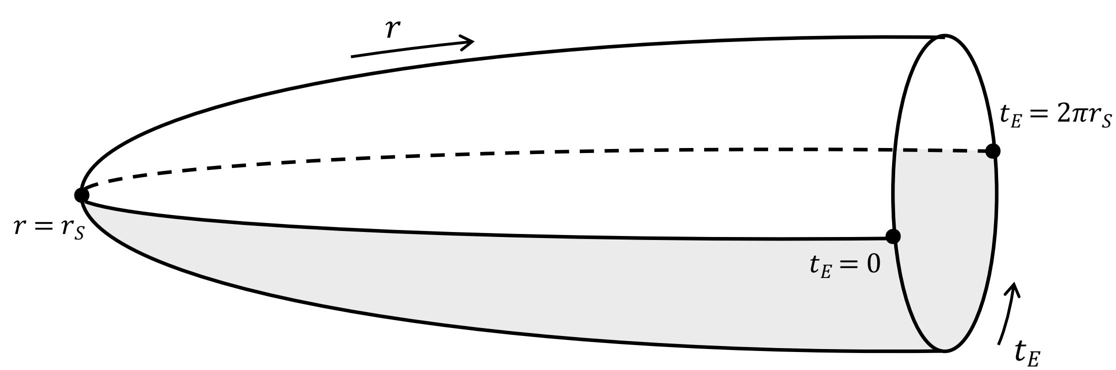
$\beta = 4\pi r_S$

Euclidean Path Integral Procedure

1. For Euclidean Schwarzschild spacetime t_E is periodic: $t_E = t_E + 4\pi r_S$.
2. Express $\langle\phi|0_{HH}\rangle$ as a Euclidean path integral $\mathrm{Int}_E[t_E, \phi]$.
3. Express $\mathrm{Int}_E[t_E, \phi]$ as a transition amplitude $\langle\phi_L\phi_R|[\cdots]|n_L n_R\rangle$.

Step 1. Euclidean Schwarzschild interval in (t_E, r) coordinates:

$$ds^2 = \left(1 - \frac{r_S}{r}\right) dt_E{}^2 + \left(1 - \frac{r_S}{r}\right)^{-1} dr^2$$



- Let $\rho = 2r_S\sqrt{1 - \frac{r_S}{r}}$

- For $r \sim r_S$:

$$ds^2 = \rho^2 \left(\frac{dt_E}{2r_S}\right)^2 + \left(\frac{r}{r_S}\right)^2 d\rho^2$$

$$\approx \rho^2 \left(\frac{dt_E}{2r_S}\right)^2 + d\rho^2$$

Singularity at $\rho = 0$ $(r = r_S)$ can be treated as origin of 2-dim Euclidean plane, provided t_E plays role of angular polar coordinate:

$$\frac{t_E}{2r_S} \equiv \theta = \theta + 2\pi$$

or $\quad t_E = t_E + 4\pi r_S$

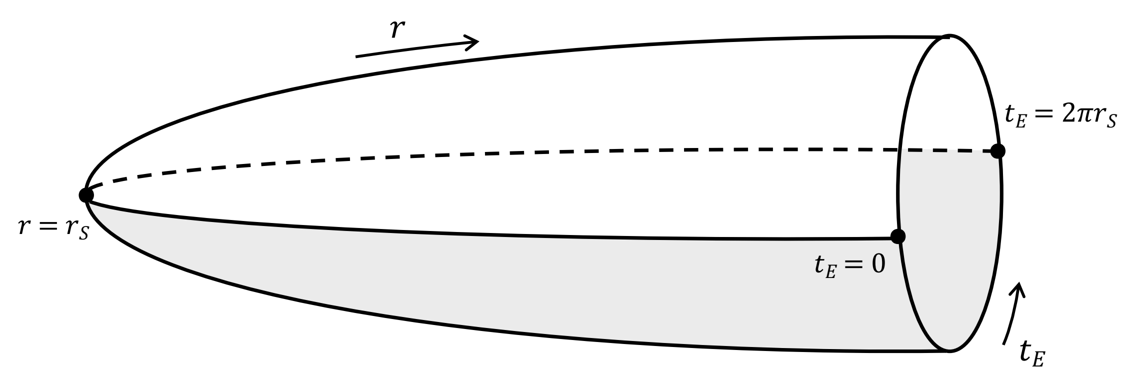
Looks like flat Euclidean space in polar coordinates $ds^2 = d\rho^2 + \rho^2 d\theta^2$

Euclidean Path Integral Procedure

1. For Euclidean Schwarzschild spacetime t_E is periodic: $t_E = t_E + 4\pi r_S$.
2. Express $\langle \phi | 0_{HH} \rangle$ as a Euclidean path integral $\mathrm{Int}_E[t_E, \phi]$.
3. Express $\mathrm{Int}_E[t_E, \phi]$ as a transition amplitude $\langle \phi_L \phi_R | [\cdots] | n_L n_R \rangle$.

Step 2. $\langle \phi | 0_{HH} \rangle \propto \lim_{t_E \to \infty} \langle \phi | e^{-t_E H} | \chi \rangle \propto \int_{\phi(t_E=0)=\chi}^{\phi(t_E=2\pi r_S)=\phi} \mathcal{D}\phi e^{-S}$

Path integral over field configs between χ and ϕ in lower half of Euclidean Schwarzschild spacetime $0 < t_E < 2\pi r_S$.



Euclidean Path Integral Procedure

1. For Euclidean Schwarzschild spacetime t_E is periodic: $t_E = t_E + 4\pi r_S$.
2. Express $\langle\phi|0_{HH}\rangle$ as a Euclidean path integral $\mathrm{Int}_E[t_E, \phi]$.
3. Express $\mathrm{Int}_E[t_E, \phi]$ as a transition amplitude $\langle\phi_L\phi_R|[\cdots]|n_L n_R\rangle$.

Step 3.
$$\int_{\phi(t_E=0)=\Theta\phi_L}^{\phi(t_E=2\pi r_S)=\phi_R} \mathcal{D}\phi e^{-S} \propto \langle\phi_R|e^{-2\pi r_S H}\Theta|\phi_L\rangle$$
$$= \sum_n e^{-2\pi r_S E_n}\langle\phi_L\phi_R|n_L^* n_R\rangle$$

- *So*: $\langle\phi|0_{HH}\rangle \propto \sum_n e^{-2\pi r_S E_n}\langle\phi_L\phi_R|n_L^* n_R\rangle$

- *Hence (?)*: $|0_{HH}\rangle \propto \sum_n e^{-\beta E_n/2}|n_L^* n_R\rangle$

Problem: $|0_{HH}\rangle$ and $|n_L^* n_R\rangle$ belong to unitarily inequivalent representations!

$\beta = 4\pi r_S$

Part 1: Review and Initial Motivation

1. ER Wormholes
   - *Warm-Up: Minkowski Spacetime*
   - *Extended Schwarzschild Spacetime*
   - *Extended AdS-Schwarzschild Spacetime*
2. Quantum Entanglement and Spacetime Structure
   - *Quantum Entanglement in Minkowski Spacetime*
   - *Quantum Entanglement in Extended Schwarzschild Spacetime*
3. Initial Motivation: Maldacena's (2003) Example
   - *Extended AdS-Schwarzschild Spacetime and the CFT Thermofield Double State*

3. Maldacena's (2003) Example

- *First glance*: Quantum entanglement in extended AdS-Schwarzschild spacetime.
- *New element*: An appeal to the AdS-CFT correspondence to justify this expression.
- *Alternatively*: An appeal to this expression to justify an entry in the AdS-CFT dictionary.

Old Claim. The vacuum state of a quantum field in extended AdS-Schwarzschild spacetime can be expressed as an entangled state over regions IV and I.

Maldacena's Proposal. Under the AdS/CFT dictionary, extended AdS-Schwarzschild spacetime is dual to a CFT thermofield double state.

Prelude: Thermofield double state

- For a given QFT with Hamiltonian $H|n\rangle = E_n|n\rangle$, $|n\rangle \in \mathcal{H}$, consider thermal mixed state:

$$\rho = e^{-\beta H} = \sum_n e^{-\beta E_n}|n\rangle\langle n| \ , \qquad \beta = 1/\text{temperature}$$

- Now "purify" ρ: find a pure state of a larger system such that ρ is the reduced density operator for a given bi-partition.

Thermofield double state = purification of a thermal state ρ obtained by:
- Doubling degrees of freedom to form $\mathcal{H}_1 \otimes \mathcal{H}_2$, where $\mathcal{H}_1, \mathcal{H}_2$ are copies of $\mathcal{H}$.
- Constructing pure state $|\text{TFD}\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2$ such that $\rho = \text{Tr}_2(\,|\text{TFD}\rangle\langle\text{TFD}|)$

- *Result*:

$$|\text{TFD}\rangle \propto \sum_n e^{-\beta E_n/2}|n_1 n_2\rangle$$

Standard interpretation: physical system (subsystem 1) in thermal equilibrium with a heat bath (subsystem 2).

Looks like entangled vacuum state of a QFT across left and right wedges of a spacetime admitting a bifurcate Killing horizon!

What does this have to do with AdS/CFT?

1. Take $\mathcal{H}_1, \mathcal{H}_2$ to be two copies of a CFT.
2. Express $\langle\phi|\mathrm{TFD}\rangle$ as a Euclidean path integral $\mathrm{Int}_E[t_E, \phi]$ over a spatiotemporal region $\mathcal{S} = \Sigma \times \mathrm{Interval}_{t_E}$.
3. Use AdS/CFT dictionary to identify corresponding bulk spacetime.

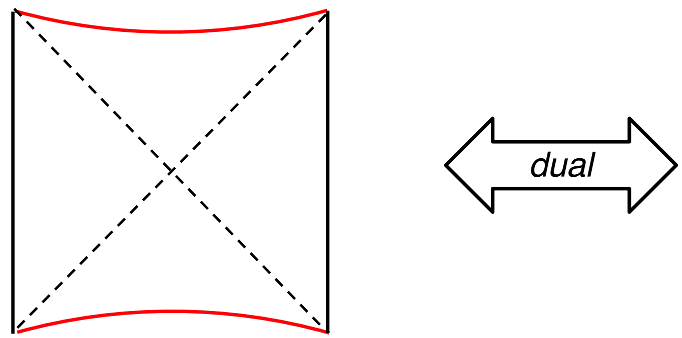
Step 2. $\langle\phi|\mathrm{TFD}\rangle \propto \lim_{t_E\to\infty} \langle\phi|e^{-t_E H_{\mathrm{TFD}}}|\chi\rangle \propto \int_{\phi(t_E=0)=\chi}^{\phi(t_E=\beta/2)=\phi} \mathcal{D}\phi e^{-S}$

Path integral over field configs between χ and ϕ on $\mathcal{S} = \Sigma \times [0, \beta/2]$

Step 3. AdS/CFT differentiate dictionary:

$$Z_{\mathrm{bulk}}[\partial M = \mathcal{S}] = Z_{\mathrm{CFT}}[\mathcal{S}]$$

Boundary of lower half of Euclidean extended AdS-Schwarzschild spacetime $= \mathbb{S}^{d-1} \times [0, \beta/2]$, $\beta = 4\pi r_S$!



$$|\mathrm{TFD}\rangle \propto \sum_n e^{-\beta E_n/2} |n_1 n_2\rangle$$

Two Interpretations:

Maldacena & Susskind (2013)

1. A single black hole in thermal equilibrium.

$$H_{\mathrm{TFD}} = H_1 - H_2\,, \qquad H_{\mathrm{TFD}}|\mathrm{TFD}\rangle = 0|\mathrm{TFD}\rangle$$

2a. Two entangled black holes in disconnected spaces I, IV with a common time.

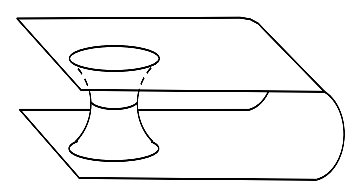
$$H_{\mathrm{TFD}} = H_1 + H_2$$

$$|\mathrm{TFD}(t)\rangle \propto \sum_n e^{-\beta E_n/2} e^{-iE_n t} |n_1^* n_2\rangle$$

2b. Two distant entangled black holes in the same space.

"Two-sided" black hole

A "rather suprising consequence": "two completely noninteracting systems can nonetheless have an alternate description where there is a single connected geometry where observers from the right and the left can jump in and meet each other in the middle." (Harlow 2016, pg. 40)



Spacetime	*Extended Spacetime*	*Wormhole?*	*Entangled State*
Rindler	Minkowski	no	$\lvert 0_M\rangle \propto \sum e^{-\beta E_n/2}\lvert n^*{}_L n_R\rangle$
Schwarzschild	Extended Schwarzschild	yes	$\lvert 0_{HH}\rangle \propto \sum e^{-\beta E_n/2}\lvert n^*{}_L n_R\rangle$
AdS-Schwarzschild	Extended AdS-Schwarzschild	yes	$\lvert \mathrm{TFD}\rangle \propto \sum e^{-\beta E_n/2}\lvert n^*{}_1 n_2\rangle$

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