Spacetime as a Quantum Error-Correcting Code?

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1. Mystery-Mongering.
2. The Bulk-Locality Paradox and the QECC Interpretation.
3. The HaPPY Code Realization.
4. Spacetime as a QECC?

Not literally, but perhaps in an emergent sense...

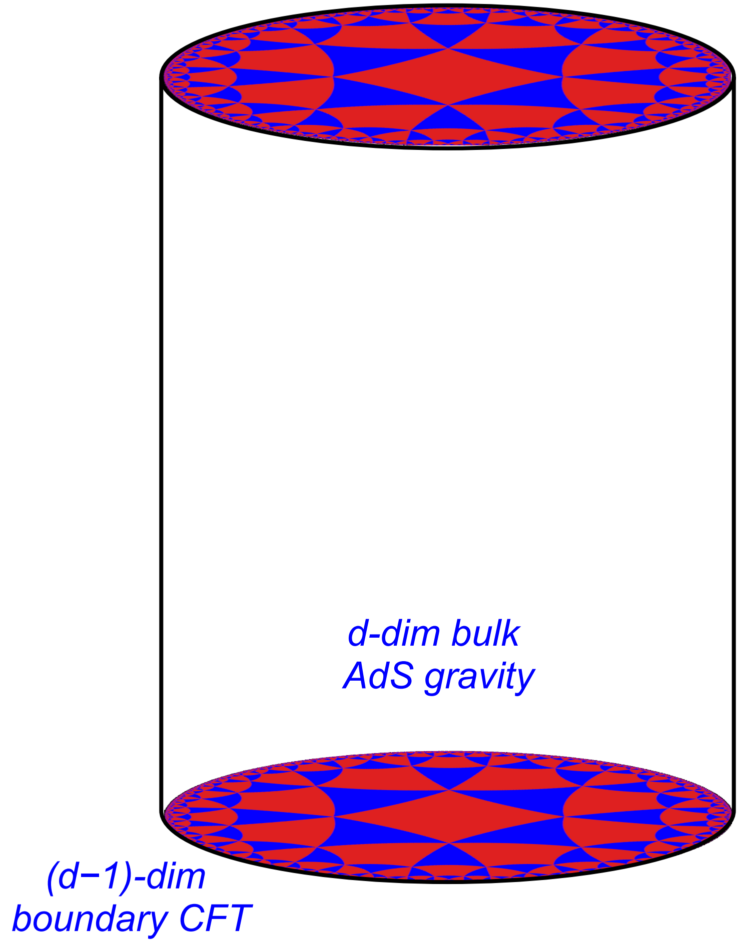
1. Mystery-Mongering

AdS-CFT Correspondence

- *Redundant aspect*: boundary degrees of freedom in (d–1)-dim are mapped to bulk degrees of freedom in d-dim.

- Suggests a quantum error-correcting code (QECC): A way to protect information *redundantly* against environmental degradation.

- Fodder for mystery-mongering...

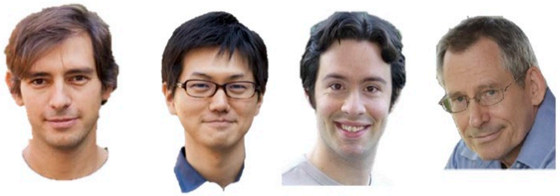


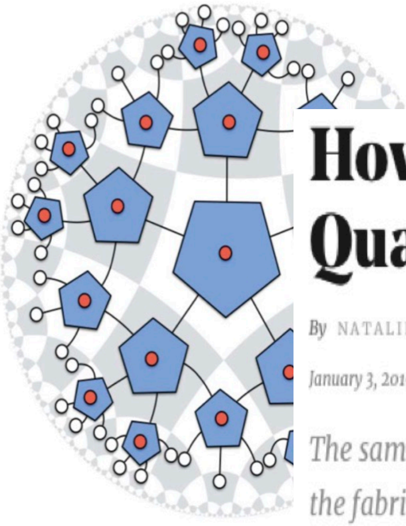
1. Mystery-Mongering

Is spacetime a quantum error-correcting code?

Fernando Pastawski, Beni Yoshida,
Daniel Harlow, John Preskill
= *HaPPY*

J. of High Energy Physics 06 (2015) 149





How Space and Time Could Be a Quantum Error-Correcting Code

By NATALIE WOLCHOVER

January 3, 2019

The same codes needed to thwart errors in quantum computers may also give the fabric of space-time its intrinsic robustness.

IQIM
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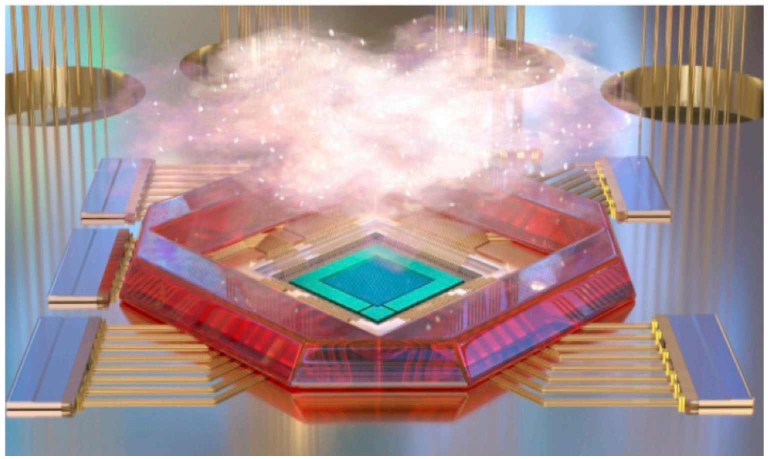
NSF

GORDON AND BETTY MOORE FOUNDATION

John Preskill
STOC in Montreal
20 June 2017



1. Mystery-Mongering

Towards De-Mystification

- *"Bulk locality paradox"*: Under reasonable assumptions about locality, the standard way of representing a local bulk field on the boundary entails it is trivial.

- *Analogous condition in a QECC*: Local operators that detect and correct errors act like the identity on the codespace.

- *QECC interpretation of AdS/CFT*: A certain collection of bulk states consists of a subspace of boundary states that forms the codespace for a QECC.

But what does this have to do with spacetime?

2. The Bulk Locality Paradox...

The AdS/CFT Dictionary

<u>Differentiate version</u> (Gubser *et al.* 1998; Witten 1998)

$$Z_{bulk}[\phi_0] = Z_{CFT}[\phi_0]$$

$$\left\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \right\rangle_{CFT} = \left[\frac{\delta}{\delta\phi_0(x_1)} \cdots \frac{\delta}{\delta\phi_0(x_n)} Z_{bulk}[\phi_0] \right]_{\phi_0 = 0}$$

<u>Extrapolate version</u> (Banks *et al.* 1998)

$$\lim_{r\to\infty} r^{\Delta} \phi(r, x) = \mathcal{O}(x)$$

$$\left\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \right\rangle_{CFT} = \lim_{r\to\infty} r^{n\Delta} \left\langle \phi(r, x_1) \cdots \phi(r, x_n) \right\rangle_{bulk}$$

Equivalent!

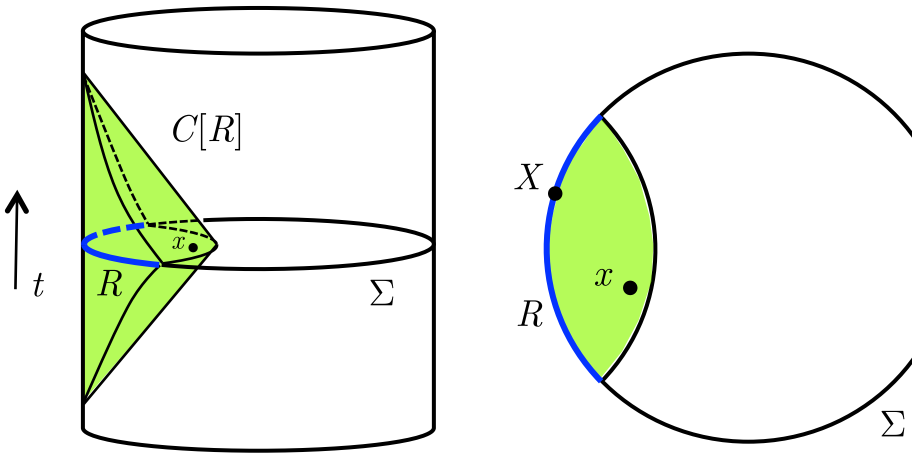
(Harlow & Stanford 2011)

- What about local bulk fields far from the boundary?
- What about observables lurking behind bulk horizons?

"We'd like to back off of the extrapolate dictionary" *(Harlow 2018)*

2. The Bulk Locality Paradox...

Reconstruction: Boundary representation of a local bulk field



Causal wedge of R

$$C[R] = J^+_{bulk}[D_{bnd}[R]] \cap J^-_{bulk}[D_{bnd}[R]]$$

All bulk points that are causally accessible from the boundary region causally determined by R.

AdS-Rindler representation of bulk field

$$\phi(x)\Big|_{x\in C[R]} = \int_{D_{bnd}[R]} K(x;X)\,\mathcal{O}(X)\,dX \equiv \mathcal{O}_{(\phi;R)}$$

Bulk field at a point as a boundary operator on a region

Causal wedge reconstruction conjecture

For any boundary spatial region R, any bulk field in $C[R]$ can be represented by a boundary operator on R.

A powerful entry in the AdS/CFT dictionary?

- trivial?

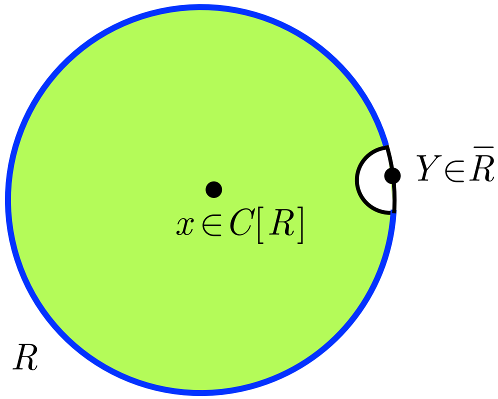
2. The Bulk Locality Paradox...

Almheiri *et al.* (2015); Pastawski *et al.* (2015); Harlow (2018)

Claim. The AdS-Rindler representation $\mathcal{O}_{(\phi;R)}$ is trivial.

Proof sketch.

(i) For any $\phi(x)$ and any local $O(Y)$ on the same timeslice, there is an $\mathcal{O}_{(\phi;R)}$ such that $O(Y)$ lies in $\bar{R}$ and hence $[\mathcal{O}_{(\phi;R)}, O(Y)] = 0$ (*local commutativity*).

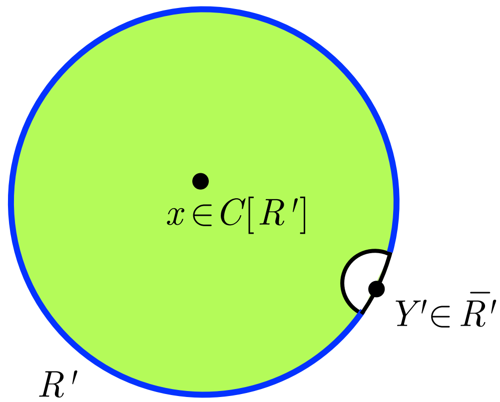


Implication: Since a bulk field admits multiple AdS-Rindler reps, each one commutes with some arbitrary boundary operator on the same timeslice.

$$\mathcal{O}_{(\phi;R)} = \mathcal{O}_{(\phi;R')}$$

(ii) *Uniqueness*: For a given $\phi(x)$, $\mathcal{O}_{(\phi;R)} = \mathcal{O}_{(\phi;R')} = \ldots \equiv \mathcal{O}_{(\phi)}$.

How might this Fail?



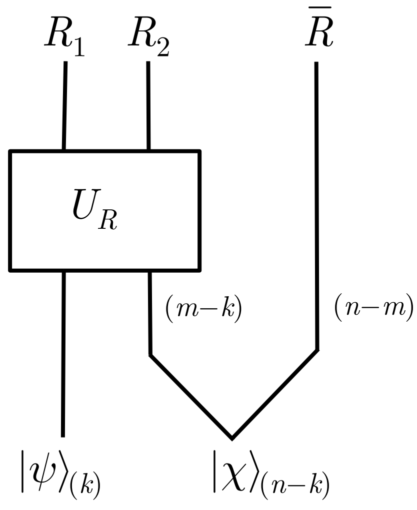
(iii) By the timeslice axiom, $\mathcal{O}_{(\phi)}$ must be a multiple of the identity.

2. ...and the QECC Interpretation

Erasure-Protection QECC

(i) n-qudit "physical" Hilbert space $\mathcal{H}^{(n)} = \mathcal{H}_R^{(m)} \otimes \mathcal{H}_{\bar{R}}^{(n-m)}$, where $\mathcal{H}_R^{(m)}$ and $\mathcal{H}_{\bar{R}}^{(n-m)}$ consist of m- and $(n-m)$-qudit states with support in R and $\bar{R}$.

(ii) Map that encodes k "logical" qudits in "encoded logical" qudits of a "codespace" $\mathcal{H}_C \subset \mathcal{H}^{(n)}$ in such a way that the former can be recovered if access is limited to some set R of $m < n$ of the latter.



<u>*Example*</u>: 3 qutrit code ($k = 1$, $n = 3$, $m = 2$)

- Encodes one logical qutrit in three physical qutrits.
- Protects against erasure of one of the three physical qutrits by representing 3-qutrit operators that act entirely on $\mathcal{H}_C$ as 2-qutrit operators.

2. ...and the QECC Interpretation

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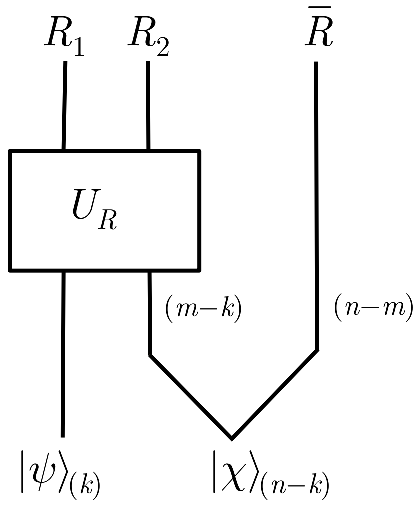


Necessary and sufficient conditions for (ii)

(a) *QECC Condition*: Any $(n-m)$-qudit operator on $\bar{R}$ acts like a multiple of the identity on $\mathcal{H}_C$.

(b) *Erasure-Protection Condition*: Any n-qudit operator on $\mathcal{H}_C$ ("encoded logical operator") can be expressed as an m-qudit operator with support on R.

Bulk locality paradox?

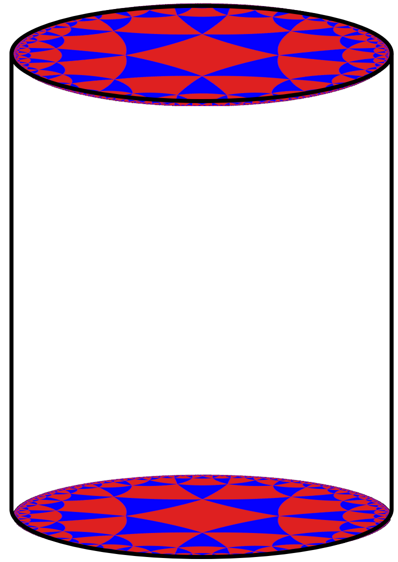
Causal wedge reconstruction conjecture!

2. ...and the QECC Interpretation

QECC	AdS-CFT
Physical qudit Hilbert space $\mathcal{H}^{(n)} = \mathcal{H}_R^{(m)} \otimes \mathcal{H}_{\bar{R}}^{(n-m)}$.	CFT Hilbert space $\mathcal{H} = \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}$, for boundary subregion R.
Code supspace $\mathcal{H}_C \subset \mathcal{H}^{(n)}$ (*encoded logical qudits*).	Subspace $\mathcal{H}_C \subset \mathcal{H}$ of CFT states that represent bulk states in $C[R]$.
<u>*Erasure-Protection Condition*</u>: Any n-qudit operator on $\mathcal{H}_C$ can be expressed as an m-qudit operator on R.	<u>*Causal Wedge Reconstruction*</u>: Any bulk field on $C[R]$ can be expressed as a CFT operator on R.
<u>*QECC Condition*</u>: Any $(n-m)$-qudit operator on $\bar{R}$ acts as a multiple of the identity on $\mathcal{H}_C$.	Any local CFT operator on $\bar{R}$ acts as a multiple of the identity on bulk states in $C[R]$.

- <u>*Redundancy 1*</u>: Info associated with a boundary operator on R is protected against erasure of $\bar{R}$ by encoding it redundantly in a bulk field in $C[R]$.
- <u>*Redundancy 2*</u>: The same bulk field can encode different boundary operators. Interpretation: $\mathcal{H}_C$ is space of low-energy CFT states.

2. ...and the QECC Interpretation



Bulk theory is a way of protecting boundary degrees of freedom against erasure.

<u>What does this have to do with spacetime?</u>

- A QECC isn't the sort of thing associated with a spacetime.
- A QECC can be *realized* by physical systems, which can possess spatiotemporal properties.

<u>Suggestion</u>: To the extent that the bulk emerges from the boundary, and the boundary has the structure of a QECC, perhaps (bulk) spacetime emerges from a QECC.

- <u>*But*</u>: Bulk/boundary duality is *symmetrical* (*i.e.*, exact), and emergence is *asymmetrical*.

Teh (2013), Dieks *et al.* (2015), de Haro *et al.* (2016), de Haro (2017).

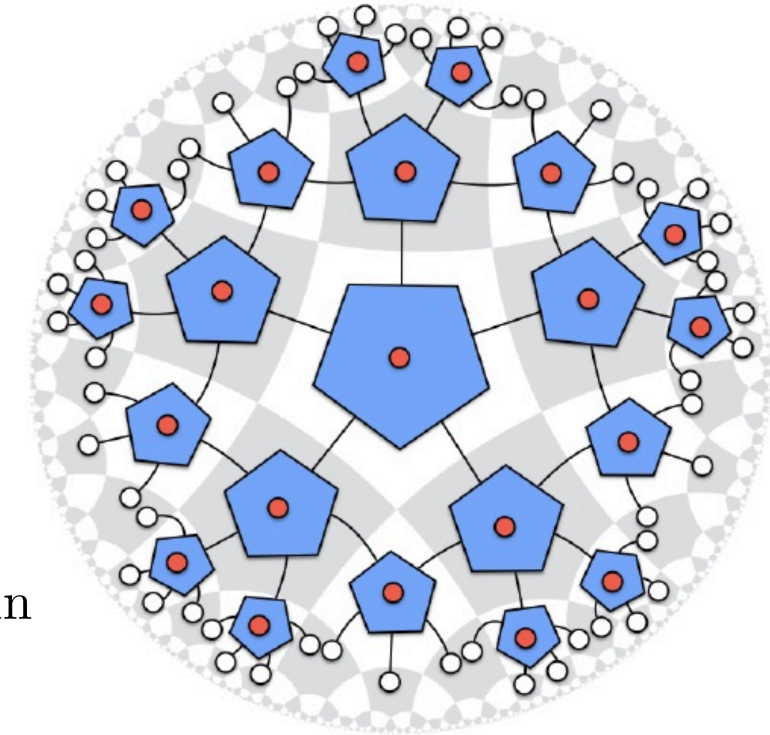
What might underwrite the asymmetry needed for emergence?

3. The HaPPY Code Realization

Pastawski, Yoshida, Harlow, Preskill (2015)

Discrete Lattice System of Qudits

- (5,4) tiling of hyperbolic plane, one 6-index tensor per lattice face.
- 1 free bulk index per face (logical qudit), free indices on boundary (physical qudits).
- _Claim_: Tensor network forms an erasure-protection QECC encoding 1 logical qudit in 5 physical qudits against erasure of any 2.



Continuum limit: $(\#vertices) \to \infty$, $area \to 0$, $(\#vertices)/area = \text{const.}$

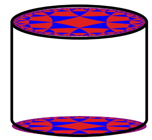
- Expect to recover AdS/CFT correspondence!

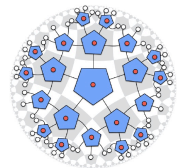
Both CFT and AdS gravity emerge in continuum limit of Happy Code discrete lattice system.

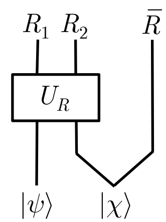
4. Spacetime as a QECC?

$$\begin{array}{ccc} CFT & \xleftrightarrow{\text{duality}} & AdS \\ \uparrow{\scriptstyle \text{continuum limit}} & & \uparrow{\scriptstyle \text{continuum limit}} \\ \text{boundary qudits} & & \text{bulk qudits} \\ \uparrow & & \uparrow \\ \text{physical qudits} & & \text{encoded logical qudits} \end{array}$$

$$\begin{array}{c} AdS/CFT \\ \uparrow{\scriptstyle \text{emergence of spacetime}} \\ \text{HaPPY code} \\ \uparrow{\scriptstyle \text{realization}} \\ \text{QECC structure} \end{array}$$





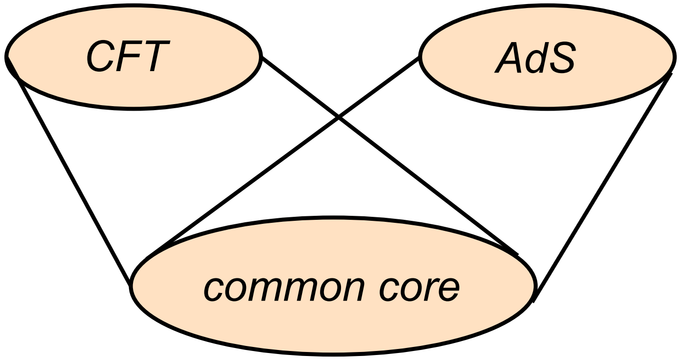


Both CFT and AdS gravity emerge in continuum limit of Happy Code discrete lattice system.

4. Spacetime as a QECC?

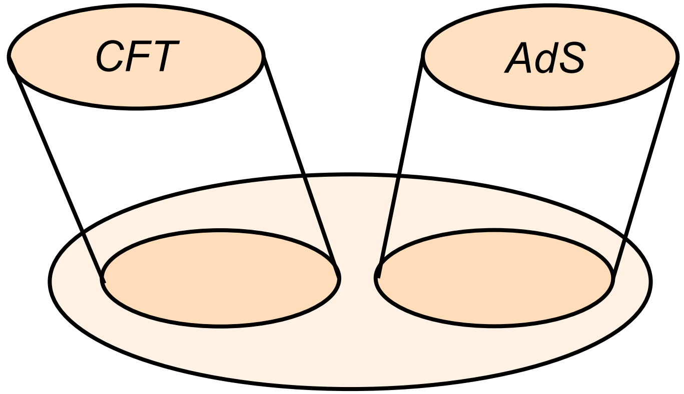
Le Bihan & Read (2018)

Common Core or Overarching Structure?

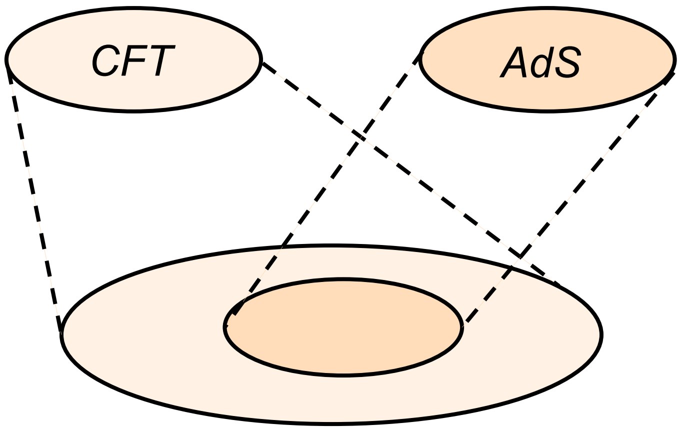


Common core

Vistarini (2017), de Haro (2019),
de Haro & Butterfield (2018; 2019)



Overarching structure



HaPPY Code Interpretation

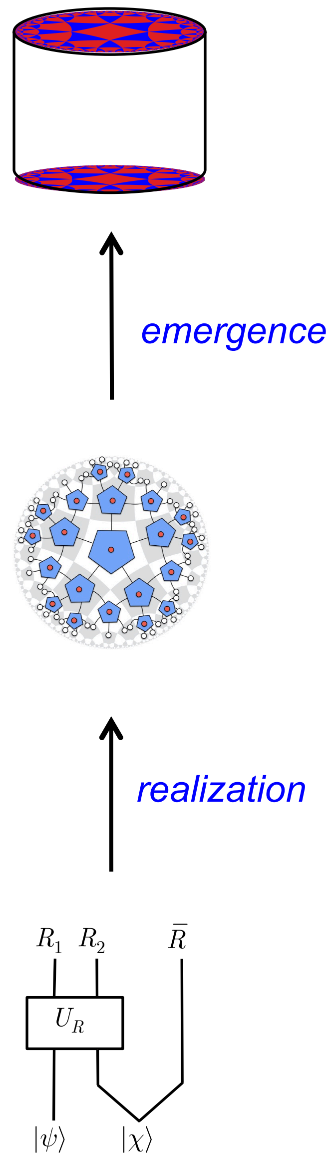
Conclusion

- Is Spacetime a QECC?

  - No!

- Does spacetime emerge in a continuum limit from a discrete physical system on a lattice whose kinematically possible states realize a QECC?

  - Possibly!



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