Principles of Quantum Gravity in the Condensed Matter Approach

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1. Some Principles of QG.
2. The Condensed Matter Approach: Two Versions.
3. Asymptotic Safety.
4. Relative Locality.
5. Holography.
6. Conclusion.

1. Some Principles of Quantum Gravity.

(i) **Asymptotic Safety**: *A theory of QG must scale towards a UV fixed point with a finite number of UV-irrelevant couplings.*

(ii) **Relative Locality**: *A theory of QG must entail that coincidence of events in spacetime ("locality") is relative to an observer's energy/momentum.*

(iii) **Holography**: *A theory of QG must entail that the number of fundamental degrees of freedom in any region $\mathcal{O}$ of spacetime cannot exceed $A/4$, A = surface area of $\mathcal{O}$.*

2. The Condensed Matter Approach to QG...

- *Goal*: To construct an *effective field theory* (EFT) of a condensate that mimicks GR and Standard Model.

Renormalization Group (RG) transformation:

(i) Given theory $S[\mathbf{g}] = \sum_a g_a \mathcal{O}_a$, impose cutoff $\Lambda(s) = s\Lambda_0$.

(ii) Integrate out "high-energy" modes.

(iii) Absorb changes into re-definitions of couplings.

- *RG flow*: Generated by sucessive RG transformations.

- *Fixed point*: Point $\mathbf{g}^*$ that is invariant under RG transformations (encodes a theory with scale-invariant parameters).

2. The Condensed Matter Approach to QG...

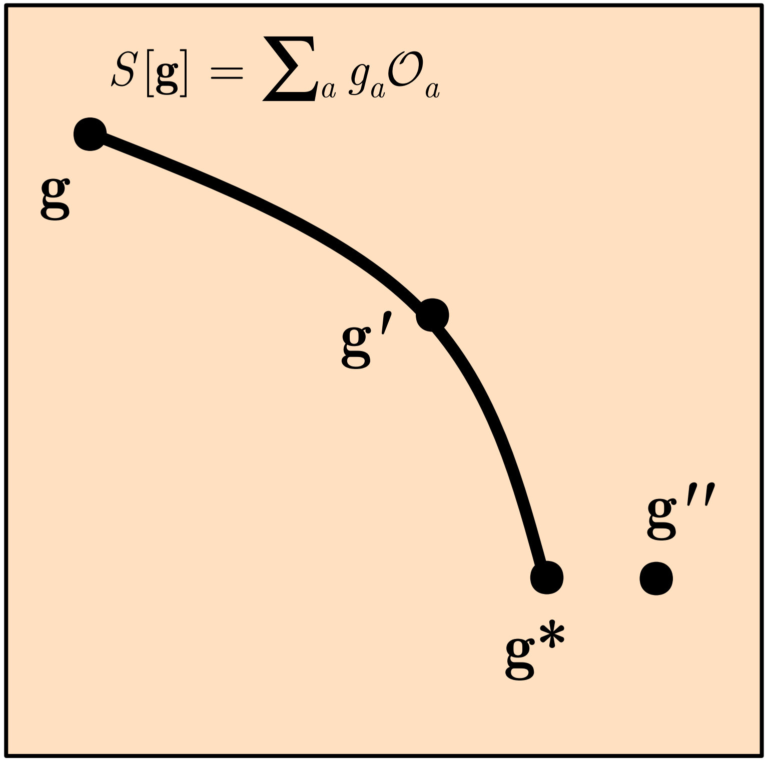
- *Goal*: To construct an *effective field theory* (EFT) of a condensate that mimicks GR and Standard Model.

- *Three Notions of an EFT*:

(a) A fixed point $\mathbf{g}^*$ (*renormalized theory*): $S[\mathbf{g}^*]$.

(b) A point $\mathbf{g}'$ on an RG flow that intersects a fixed point (*renormalizable theory*): $S[\mathbf{g}'] = \sum_a g'_a \mathcal{O}_a$.

(c) A point $\mathbf{g}''$ in the neighborhood of a fixed point (*non-renormalizable theory*): $S[\mathbf{g}''] = S[\mathbf{g}^*] + \sum_a g''_a \mathcal{O}'_a$.



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- *Universality class.*
- *Spontaneously broken symmetry.*
- *Order characterized by symmetry.*

2. The Condensed Matter Approach to QG: Two Versions.

- *Goal*: To construct an *effective field theory* (EFT) of a condensate that mimicks GR and Standard Model.

Version A: Focus on condensates characterized by spontaneously broken symmetries and universality.

- *Example*: EFT of superfluid ^{3}He-A belongs to same universality class as massless sector of Standard Model above electroweak symmetry breaking. (Volovik 2003)
- *Essential features*:
  - ∘ EFT is characterized by an RG universality class.
  - ∘ Order characterized by symmetry.

2. The Condensed Matter Approach to QG: Two Versions.

- *Goal*: To construct an *effective field theory* (EFT) of a condensate that mimicks GR and Standard Model.

Version B: Focus on condensates characterized by "topological order".

- *Example*: EFT of edge of 4-dim fractional quantum Hall (FQH) liquid describes (3+1)-dim zero-rest-mass fields of all helicities. (Zhang & Hu 2001)
- *Essential features*:
  - EFT not characterized by an RG universality class.
  - Order characterized by topology. (Wen 2004)

3. Asymptotic Safety.

Asymptotic Safety: *A theory of QG must scale towards a UV fixed point with a finite number of UV-irrelevant couplings.* (Weinberg 1979)

- *UV fixed point*: fixed point associated with $s \to \infty$.
- *IR fixed point*: fixed point associated with $s \to 0$.

Irrelevant coupling w.r.t. $\mathbf{g}^*$: coupling that decreases towards $\mathbf{g}^*$.
- UV (resp. IR) irrelevant: decreases as $s \to \infty$ (resp. 0).

Relevant coupling w.r.t. $\mathbf{g}^*$: coupling that increases towards $\mathbf{g}^*$.
- UV (resp. IR) relevant: increases as $s \to \infty$ (resp. 0).

3. Asymptotic Safety.

Renormalized theory	$S[\mathbf{g}_{IR}{}^*] = \sum_a g^*{}_a \mathcal{O}_a$	no IR-irrelevant finite # IR-relevant
Renormalizable theory	$S[\mathbf{g}'] = \sum_a g'{}_a \mathcal{O}_a$	finite # IR-irrelevant finite # IR-relevant
Non-renormal-izable theory	$S[\mathbf{g}''] = S[\mathbf{g}_{IR}{}^*] + \sum_a g''{}_a \mathcal{O}'{}_a$	infinite # IR-irrelevant finite # IR-relevant
Asymptotically safe theory	$S[\mathbf{g}''] = S[\mathbf{g}_{UV}{}^*] + \sum_a g''{}_a \mathcal{O}'{}_a$	finite # UV-irrelevant infinite # UV-relevant

- *Non-renormalizable example*: GR as an RQFT.
  - Infinite # IR-irrelevant couplings *supposedly* blow-up as $s \to \infty$.
- *Aymptotically safe examples*: QCD; GR as an RQFT?
  - If GR as an RQFT possesses a UV fixed point (with a finite number of UV-irrelevant couplings), it would be a well-behaved theory of QG!

3. Asymptotic Safety.

- *Claim*: The EFTs in both versions of the condensed matter approach should aspire to be ASTs (to the extent that they attempt to reproduce the QCD sector of the Standard Model, and, possibly, the GR sector of QG).

- *Which would seem to mean*: The EFTs in both versions aspire to be associated with two fixed points:
  - An IR fixed point associated with the "high-energy" theory of the condensate.
  - A UV fixed point associated with QCD/GR sector of QG.

- *But*: Is it consistent to consider an EFT as an AST?
  - AST = fundamental theory to all orders.
  - EFT = effective theory restricted to a given energy scale, beyond which new physics arises.

4. Relative Locality.

Relative Locality: *A theory of QG must entail that coincidence of events in spacetime ("locality") is relative to an observer's energy/momentum.* (Amelino-Camelia *et al.* 2011)

- *Idea*: Due to curvature of momentum space.

Γ (phase space x^μ, p_μ)	$\mathcal{M}$ (config. space x^μ)	$\mathcal{P}$ (mo. space p_μ)
$\Gamma^{SR} = \mathcal{M} \times \mathcal{P}$	flat	flat
$\Gamma^{GR} = T^*(\mathcal{M})$	curved	flat
$\Gamma^{RL} = T^*(\mathcal{P})$	flat	curved

- In RL, there's a separate $\mathcal{M}_p = T_p{}^*(\mathcal{P})$ for each $p \in \mathcal{P}$.
- If $\mathcal{P}$ is curved, the $\mathcal{M}_p$'s will differ from point to point.

4. Relative Locality.

Relative Locality: *A theory of QG must entail that coincidence of events in spacetime ("locality") is relative to an observer's energy/momentum.* (Amelino-Camelia *et al.* 2011)

- *Motivation*: $\mathcal{P}$-space curvature entails non-commutativity of spacetime coordinates.
  - *And*: Various approaches to QG employ non-commutative geometry.
- *Moreover*: $\mathcal{P}$-space curvature encodes corrections to relativistic particle dynamics of order $E/(Planck\ mass)$.
  - *And*: Such corrections may have measureable effects at low-energies/long distances: time of arrival of γ-ray bursts measured by Fermi telescope. (Amelino-Camelia & Smolin 2009)

4. Relative Locality.

"...just as some condensed matter or fluid systems provide analogues for relativity and gravity, it may be that condensed matter systems with curved momentum spaces may give us analogues to the physics of relative locality" (Amelino-Camelia *et al.* 2011, 084010-12)

- ***Both Versions***: Encode aspects of EFTs in aspects of $\mathcal{P}$-space topology.
- ***And***: These latter can be related to $\mathcal{P}$-space curvature.

Ex: Gauss-Bonnet-Chern theorem

$$2(1-g) = \frac{1}{2\pi}\int_S K\, dA$$

$g =$ *integer* $=$ *Chern number*
$K =$ *adiabatic curvature*
$S =$ *torus*

characterizes topology of parameter space

characterizes geometry of parameter space

4. Relative Locality.

Version A: (Volovik 2003)

(1) Encode *low-energy dynamics* in $\mathcal{P}$-space.

- Green's function $\mathcal{G}(p_0, \mathbf{p}) = [ip_0 - \mathcal{H}(\mathbf{p})]^{-1}$.
- Low-energy excitations are poles in $\mathcal{G}$ = "Fermi points" $\in \mathcal{P}$.

(2) Demonstrate stability of low-energy dynamics.

$$\mathscr{N} = \frac{1}{24\pi^2}\epsilon_{\mu\nu\lambda\gamma}\mathrm{Tr}\int_\Sigma dS^\gamma \mathcal{G}\,\partial_{p_\mu}\mathcal{G}^{-1}\mathcal{G}\,\partial_{p_\nu}\mathcal{G}^{-1}\mathcal{G}\,\partial_{p_\lambda}\mathcal{G}^{-1}$$

- Defines nontrivial winding number of map from Σ to space of matrices $\mathcal{G}$ (element of nontrivial homotopy group).
- Invariant under continuous deformations of $\mathcal{G}$.
- *Which means*: Invariant under low-energy perturbations.
- *Which means*: Defines fixed point/universality class.

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(3) Relate $\mathscr{N}$ to $\mathcal{P}$-space curvature.

- *Intuitively*: LHS encodes topology, RHS encodes geometry.
- *Can show*: In IQHE, quantized Hall conductance σ given by topological invariant obtained from $\mathscr{N}$ *via* dim reduction.
- *And*: σ is given by adiabatic curvature. (Thouless *et al.* 1982)

4. Relative Locality.

Version B: (Wen 2004)

(1) Encode *order* in ground state degeneracy (GSD).

- Two distinctly ordered FQH states (distinct filling factors) can have same symmetries but different GSD.
- *Thus*: Internal order of FQH states cannot be characterized by symmetry, but can be (partially) characterized by GSD.

4. Relative Locality.

<u>*Version B*</u>: (Wen 2004)

(1) Encode *order* in ground state degeneracy (GSD).

(2) Demonstrate stability of GSD. (Wen & Niu 1990)

- For FQH states, GSD depends on spatial topology.
- GSD is robust against arbitrary permutations.

(3) Relate GSD to $\mathcal{P}$-space curvature. (Wen 1990)

- FQH states classified by matrices K, and described by TQFT $\mathcal{L} = (1/4\pi)K_{IJ}\,\epsilon_{\mu\nu\lambda}a_{I\mu}\,\partial_\nu a_{J\lambda}$.
- GSD $= |K|$.
- K can be encoded in Berry phase characterizing adiabatic deformations of FQH Hamiltonian.

5. Holography.

Holography: *A theory of QG must entail that the number of fundamental degrees of freedom N in any region $\mathcal{O}$ of spacetime cannot exceed $A/4$, A = surface area of $\mathcal{O}$.*

- *Informally*: The "information" encoded in a physical system is contained, not in its volume, but in its boundary.
- *Version A*: No explicit reference to such things.
- *Version B*: Edge states of FQH liquid (partially) encode order of bulk states.

"This phenomenon of 2-dim topological orders being encoded in 1-dim edge states shares some similarities with the holomorphic principle in superstring theory and quantum gravity." (Wen 2004, pg. 346.)

5. Holography.

Holography: *A theory of QG must entail that the number of fundamental degrees of freedom N in any region $\mathcal{O}$ of spacetime cannot exceed $A/4$, A = surface area of $\mathcal{O}$.*

- *Two Steps*: (Bousso 2002)

(I) Identify $N \equiv \ln\mathcal{N}$ = Boltzmann entropy S_B, where $\mathcal{N}$ = (# states).

(II) Appeal to various entropy bounds.

Ex. Spherical Entropy Bound: (Susskind 1995)
$S(\mathcal{O}) \leq A/4$, $\mathcal{O}$ is spatial region with radius R,
A is area of black hole with radius R.

- *Motivation for (II)*: Generalized 2nd Law (GSL): (Bekenstein 1973)

$\Delta S_{bh} + \Delta S \geq 0$, where $S_{bh} = A/4$.

5. Holography.

- ***So***: Holography really requires *three* steps:

(1) Positing a relation between N and $\mathcal{N}$: $N = \ln \mathcal{N}$.
(2) Using this to identify N with Boltzmann entropy S_B.
(3) Assuming the entropy of matter S in the GSL is S_B.

- Concern with Step (1).
  - Motivated by theories with Boolean degrees of freedom for which (#Boolean DOF) $= \log_2 \mathcal{N}$.

> *The number of Boolean DOF in any region of spacetime cannot exceed* $A/(4 \ln 2)$. ('t Hooft 1993)

  - This suggests the naive generalization $N = \log_{(\#values)} \mathcal{N}$.

5. Holography.

- *So*: Holography really requires *three* steps:

> (1) Positing a relation between N and $\mathcal{N}$: $N = \ln \mathcal{N}$.
> (2) Using this to identify N with Boltzmann entropy S_B.
> (3) Assuming the entropy of matter S in the GSL is S_B.

- Concern with Step (2).
  - The naive generalization $N = \log_{(\#values)}\mathcal{N}$ is disanalogous with the definition of Boltzmann entropy $S_B = \ln\mathcal{N}$.
  - *Recall*: One motivation for the latter is that S_B is supposed to be an *additive version* of $\mathcal{N}$.

$$S_{12} = S_1 + S_2$$
$$\mathcal{N}_{12} = \mathcal{N}_1 \times \mathcal{N}_2$$

  - *But*: N is *conceptually distinct* from $\mathcal{N}$, and not just an additive version of $\mathcal{N}$:

$$N = (\#\text{essential properties}),\ \mathcal{N} = (\#\text{states}).$$

5. Holography.

- *So*: Holography really requires *three* steps:

> (1) Positing a relation between N and $\mathcal{N}$: $N = \ln \mathcal{N}$.
> (2) Using this to identify N with Boltzmann entropy S_B.
> (3) Assuming the entropy of matter S in the GSL is S_B.

- Concern with Step (3).
  - Requires a "Boltzmann version" of black hole entropy S_{bh}.
  - *Which requires*: Identifying the microstates of a black hole and relating them to the area.
  - Some results in string theory and loop quantum gravity (Strominger & Vafa 1996; Ashetekar, *et al.* 1998).

6. Conclusion.

Principle	*Version A*	*Version B*
asymptotic safety	yes (?)	yes (?)
relative locality		
holography		

- *Asymptotic safety*: Both versions should aspire to be ASTs.
  - *But*: Can an AST be an EFT?

6. Conclusion.

Principle	*Version A*	*Version B*
asymptotic safety	yes (?)	yes (?)
relative locality	yes	yes
holography		

- *Relative locality*: Both versions encode relevant quantities in $\mathcal{P}$-space topological invariants.
- *And*: These invariants generate nontrivial $\mathcal{P}$-space curvature.
  - *But*: Version A uses topological invariants to encode low-energy *dynamics* (universality classes of interacting theories).
  - Version B uses topological invariants to encode low-energy *kinematics* (categorization of ordered states).

6. Conclusion.

Principle	*Version A*	*Version B*
asymptotic safety	yes (?)	yes (?)
relative locality	yes	yes
holography	no	yes (?)

- *Holography*: Version A makes no appeal to holography; Version B, charitably, does.
  - *But*: Holography is suspect...

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