# Interpreting Effective Field Theories

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1. Why Interpret EFTs?
2. How to Construct an EFT.
3. On Renormalization Schemes.
4. Quasi-Autonomous Domains, Discrete Spacetime, and All That.

## A. *What is an EFT?*

- Theory of the dynamics of a physical system at energies $E$ small compared to a cut-off $\Lambda$.
- For systems of interest, low-energy dynamics is effectively independent of high-energy dynamics.

  - Makes calculations easier.
  - No need to worry about renormalizability.
  - Make non-trivial predictions.

## B. *Why interpret EFTs?*

- Current RQFTs of particle physics (QCD, Electroweak, Standard Model) are EFTs of an as yet unknown high-energy theory.
- Many NQFTs of condensed matter systems admit *relativistic* EFTs.
- Provide an arena in which to evaluate notions of reduction, emergence, explanation, *etc.*

## 2. How To Construct an EFT.

Given a field theory described by action $S[\phi]$,

1. Choose a cut-off $\Lambda$ and divide fields into high and low momenta parts with respect to $\Lambda$: $\phi = \phi_H + \phi_L$.

2. Integrate out the $\phi_H$ to obtain the Wilsonian effective action $S_\Lambda[\phi_L]$.

$$\int \mathcal{D}\phi_L \int \mathcal{D}\phi_H e^{iS[\phi_H,\phi_L]} \equiv \int \mathcal{D}\phi_L e^{iS_\Lambda[\phi_L]}$$

3. Expand the effective action in a set of local operators $\mathcal{O}_i$.

$$S_\Lambda = S_0 + \int d^D x \sum_i g^i \mathcal{O}_i$$

## 2. How To Construct an EFT.

4. Perform dimensional analysis on the terms in $S_\Lambda$:

(i) Use free action to determine the dimension $\delta_i$ of the field operators $\mathcal{O}_i$ and, subsequently, coupling constants $g_i$.

(ii) <u>*Result*</u>: For dimensionless coupling constants $\lambda_i = \Lambda^{\delta_i - D} g_i$, the order of the $i$th term is $\lambda_i (E/\Lambda)^{\delta_i - D}$.

(iii) Three types of term:

- *Irrelevant*: $\delta_i > D$. Falls as $E \rightarrow 0$.
- *Relevant*: $\delta_i < D$. Grows as $E \rightarrow 0$.
- *Marginal*: $\delta_i = D$. Constant as $E \rightarrow 0$.

## 2. How To Construct an EFT.

***Example***: Scalar field theories.

- Free action: $S_0[\Phi] = \dfrac{1}{2}\int d^D x \partial_\mu \Phi \partial^\mu \Phi$

- $\Phi$ must have units $E^\delta$ satisfying $E^{-D}E^2E^{2\delta} = E^0$, thus $\delta = D/2 - 1$.

- *In general:* An operator $\mathcal{O}_i$ constructed from $M$ $\Phi$'s and $N$ derivatives will have dimension $\delta_i = M(D/2 - 1) + N$.

- *Thus*: For $D \geq 3$, there are only a finite number of relevant and marginal terms in any $S_\Lambda[\Phi]$.

## 3. On Renormalization Schemes.

**A. *Mass-dependent schemes and "Wilsonian EFTs".***

- Use the cut-off $\Lambda$ to regulate divergent integrals.
  - Replace $\int_0^\infty \kappa(p) d^D p$ with $\int_0^\Lambda \kappa(p) d^D p + \int_\Lambda^\infty \kappa(p) d^D p$.
  - Absorb second piece into renormalization constants.

- Requires a subtraction scheme that is "mass-dependent": renormalization constants are dependent on the masses of the heavy fields.

## 3. On Renormalization Schemes.

**A. *Mass-dependent schemes and "Wilsonian EFTs".***

*Advantages*:

(a) Consistent with image of an EFT as a low-energy approximation to a high-energy theory based on a restriction of the latter to a particular energy scale $\Lambda$.

- $\Lambda$ *plays a double role in designating the appropriate energy scale and in cutting off divergent integrals.*

(b) Necessary for proof of the Decoupling Theorem.

# 3. On Renormalization Schemes.

## A. *Mass-dependent schemes and "Wilsonian EFTs".*

*Decoupling Theorem*: (Appelquist & Carazzone 1975)

"For two coupled systems *with different energy scales* $m_1$, $m_2$ $(m_2 > m_1)$ *and described by a renormalizable theory*, there is always a renormalization condition according to which the effects of the physics at scale $m_2$ can be effectively included in the theory with the smaller scale $m_1$ by changing the parameters of the corresponding theory."

(Hartman 2001)

## 3. On Renormalization Schemes.

### A. *Mass-dependent schemes and "Wilsonian EFTs".*

*Disadvantages*:

(a) Momentum cut-off regularization violates Poincaré and gauge invariance.

(b) Dependence of irrelevant terms on orders of $E/\Lambda$ breaks down for higher-order loop calculations: *Power* dependence of terms on $\Lambda$.

- *Consequence*: An infinite number of terms in $S_\Lambda$.

## 3. On Renormalization Schemes.

**B. *Mass-independent schemes and "continuum EFTs".***

- Use mass-*independent* subtraction scheme: Energy scale parameter $\mu$ appears in loop corrections in *logarithms*.

  - *Consequence*: A finite number of terms in $S_\Lambda$.

- Requires *dimensional regularization*:

  - Replace $\int_0^\infty \kappa(p) d^D p$ with $\int_0^\infty \kappa(p) d^{D-\varepsilon} p$.
  - Analytically continue $D - \varepsilon$ to $D$.
  - Absorb poles into (mass-independent) renormalization constants.

## 3. On Renormalization Schemes.

**B. *Mass-independent schemes and "continuum EFTs".***

*Advantages*:

(a) Dimensional regularization respects Poincaré and gauge invariance.

(b) Mass-independent substraction allows truncation of the effective action to a finite number of terms for both tree-level calculations and higher-order loop corrections.

## B. *Mass-independent schemes and "continuum EFTs".*

*Disadvantages*:

(a) Violates the "spirit" of an EFT: heavy field terms are present in the effective action.

(b) Decoupling Theorem does not hold.

## 3. On Renormalization Schemes.

***How To Construct a continuum EFT*** (Georgi 1993)

1. Start with (dim-regularized) $S = S[\phi_L] + S_H[\phi_L, \phi_H]$ at energy scale $\mu$.

2. Evolve action to lower energies *via* renormalization group: $\mu \to \mu - d\mu$.

3. Insert decoupling "by hand": When $\mu$ gets below mass of $\phi_H$, replace $S$ with effective action $S_{eff} = S[\phi_L] + \delta S[\phi_L]$, where $\delta S[\phi_L]$ encodes a "matching condition".

4. Explicitly calculate $\delta S$ by local operator expansion:

$$S_{eff} = S[\phi_L] + \sum_i \delta S^i[\phi_L]$$

## 3. On Renormalization Schemes.

***Wilsonian EFT***

"Cut-off" $\Lambda$ plays double role:

(a) Demarcates low-energy physics from high-energy physics.

(b) Regulates divergent integrals.

- *Analogous to placing high-energy continuum theory on a discrete lattice.*

***Continuum EFT***

- Renormalization scale $\mu$ plays Role (a).
- Role (b) replaced by dimensional regularization.
  - *No violation of Poincaré invariance: no discrete lattice.*

## 4. On Quasi-Autonomous Domains...

"Thus, with the decoupling theorem and the concept of EFT emerges a hierarchical picture of nature offered by QFT... In this picture, the [physical world] can be considered as layered into *quasi-autonomous domains*, each layer having its own ontology and associated 'fundamental law'." (Cao & Schweber 1993)

"Cao and Schweber's talk of quasi-autonomous domains rests on the validity of the decoupling theorem..." (Hartmann 2001)

"The EFT approach in its extreme version provides a level structure ('tower') of EFTs, each theory connected with the preceding one (going 'up' in the tower) by means of the [renormalization group] equations and the matching conditions at the boundary..." (Castellani 2002)

## 4. On Quasi-Autonomous Domains...

***Wilsonian EFTs:***

- Decoupling Theorem only holds for Wilsonian EFTs for which there exists a corresponding *renormalizable* high-energy theory *with different mass scales*.
  - *Wilsonian EFTs do not, in general, support an ontology of quasi-autonomous domains.*

***Continuum EFTs:***

- Decoupling Theorem does not hold.
- *But*: Decoupling is inserted "by hand" via matching conditions.
  - *Continuum EFTs do, in general, support an ontology of quasi-autonomous domains.*

***How to Realistically Interpret the "Cut-Off":***

(A) The Wilsonian regulator $\Lambda$ should be realistically interpreted.

(B) The parameter that demarcates low-energy physics from high-energy physics ($\Lambda$ or $\mu$) should be realistically interpreted.

- (A) suggests an ontology in which spacetime is discrete.
- *But*: (B) does not entail (A).

## 4. ...Discrete Spacetime...

"If the cut-offs are taken seriously, then they must be interpreted realistically; that is, space is really discrete and of finite extent according to the cut-off variant of QFT."

(Fraser 2009)

- "Cut-off" = demarcator of energy scales.
- Two versions of "the cut-off variant of QFT": Momentum cutoff-regulated QFT and dimensionally-regulated QFT.
- Only the former supports an ontology in which spacetime is discrete.

## 4. ...and All That.

***Idealization and Approximation***

- *Claim (Fraser 2009):* The "cut-off variant of QFT" is an indispensible idealization. It requires an idealized ontology in which spacetime is discrete.

  *But:* To the extent that dim-regulated cut-off QFT does not support such an ontology, it is not an indispensible idealization.

- *Claim (Castellani 2002):* EFTs are "intrinsically approximate and context-dependent".

  *But:* To the extent that an ontology of quasi-autonomous domains suggests otherwise, and is a viable interpretation of continuum EFTs, EFTs in general need not be considered "approximate and context-dependent".

## Conclusion.

- Two conceptually distinct types of EFT: Wilsonian and continuum.
- Admit distinct interpretations.
- Agree on empirically measured quantities (different renormalization schemes ultimately agree on the values of physical quantities).
- Non-trivial example of empirically indistinguishable theories.