Emergence and Mechanism in the Fractional Quantum Hall Effect

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1. Two Versions of Emergence

2. The Quantum Hall Effect

3. The Underdetermination of Mechanism in the FQHE

4. Law-Centric Emergence in the FQHE

5. Conclusion

1. Two Versions of Emergence

Emergence

> Crowther (2015)
>
> (i) *Dependence.* Emergent system is "ontologically determined" by the fundamental system.
>
> (ii) *Independence.* Emergent system is *novel* with respect to fundamental system.

Task: Resolve tension between *Dependence* and *Independence*.

1. Two Versions of Emergence

Mechanism-Centric Emergence

"...emergent properties are not a panacea, to be appealed to whenever we are puzzled by the properties of large systems. In each case, we must produce **a detailed physical mechanism** for emergence, which rigorously explains the qualitative difference that we see with the microphysical." (Mainwood 2006)

"...understanding emergent phenomena in terms of **symmetry breaking** —a structural dynamical feature of physical systems...—clarifies both how and why emergent phenomena are independent of any specific configuration of their microphysical base." (Morrison 2012)

"...the electrons in FQHE states cannot just be correlated over small distances; **they must have a mechanism** of achieving long-range correlations. This mechanism is Chen *et al.*'s conception of long-range entanglement." (Lancaster & Pexton 2015)

1. Two Versions of Emergence

Mechanism-Centric Emergence

> *Claim*: To avoid triviality, novelty must be underwritten by a **mechanism**.

- *Microphysical mechanism*: a particular collection of entities and activities that are organized in such a way that they realize a regularity or principle (Weber *et al.* 2013).

- *High-level mechanism*: a general physical process that can be instantiated by any number of microphysical processes.

  - "a structural/dynamical feature of physical systems" (Morrison 2012).

  - "a higher organizing principle" (Laughlin & Pines 2000).

1. Two Versions of Emergence

Law-Centric Emergence

Claim: Novelty is underwritten by **distinct dynamical laws**.

- Exemplified by effective field theories (EFTs):

$$\frac{\partial \mathcal{L}}{\partial \phi} - \frac{\partial}{\partial x}\frac{\partial \mathcal{L}}{\partial(\partial \phi)} = 0 \qquad (1) \qquad \begin{array}{c} \textit{high-energy} \\ \textit{(fundamental) system} \end{array}$$

$$\frac{\partial \mathcal{L}_{eff}}{\partial \theta} - \frac{\partial}{\partial x}\frac{\partial \mathcal{L}_{eff}}{\partial(\partial \theta)} = 0 \qquad (2) \qquad \begin{array}{c} \textit{low-energy} \\ \textit{(emergent) system} \end{array}$$

- $\mathcal{L}[\phi] \neq \mathcal{L}_{eff}[\theta]$
- Low-energy behavior (2) is _dynamically independent_ of, and _dynamically robust_ with respect to, high-energy behavior (1).

1. Two Versions of Emergence

> *Story to come:*
> Mechanism-centric views of emergence fail to adequately account for emergence in the FQHE.

2. The Quantum Hall Effect
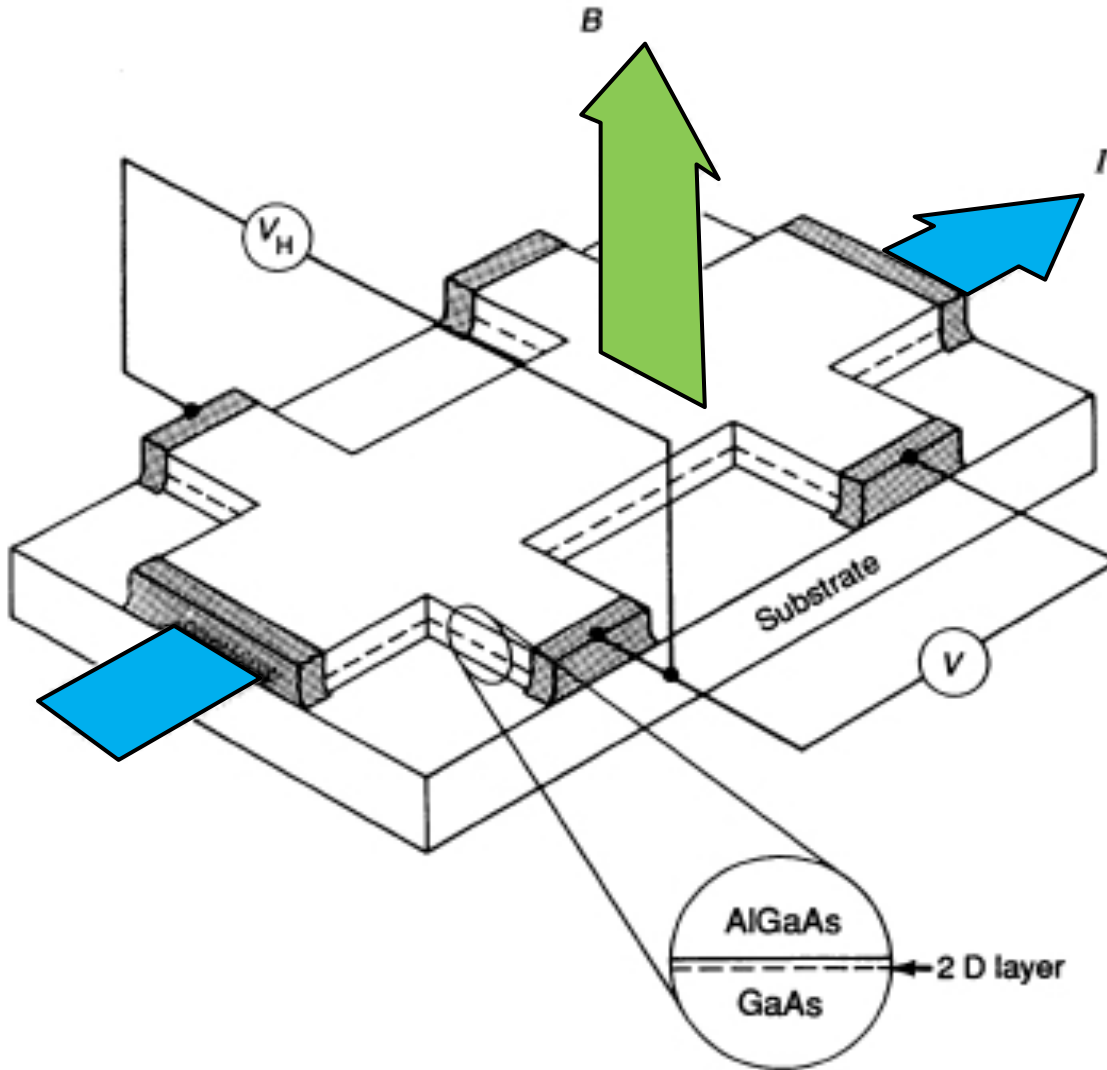
B

I

V_H

Substrate

V

AlGaAs

2 D layer

GaAs

Hall resistance

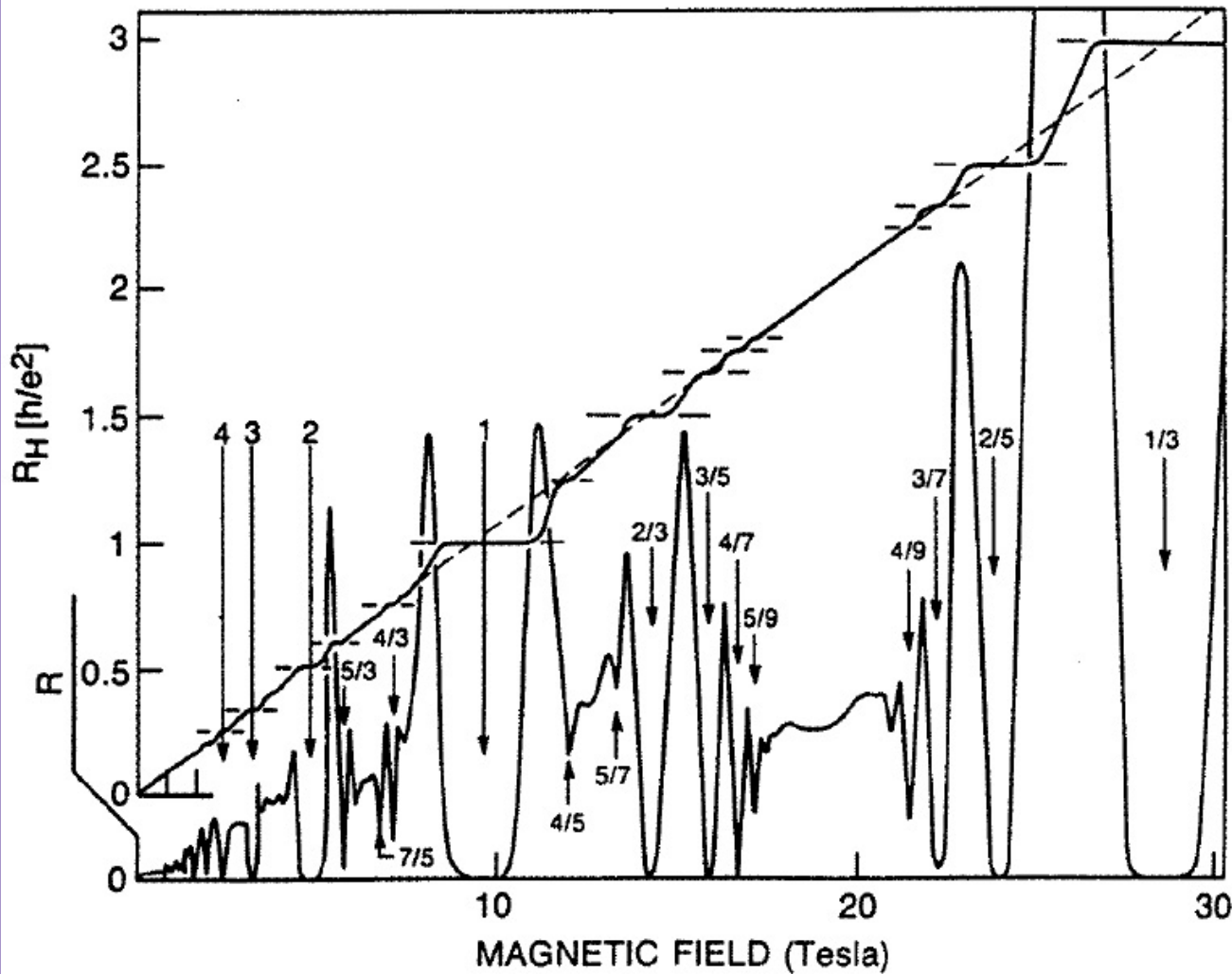$$R_H = \frac{V_H}{I} = \frac{B}{eN}$$

$N = \#\ electrons$
$per\ area$

Longitudinal resistance

$R = V/I$

2. The Quantum Hall Effect

Hall resistance

"Plateaus" at

$$R_H = \frac{h}{e^2}\frac{1}{n}$$

n = integer or fraction

Longitudinal resistance

$R = 0$ at plateaus in R_H.

- Low temperature (~0.02 K)
- Large magnetic field (~30 T)

2. The Quantum Hall Effect

Integer Quantum Hall Effect (IQHE)

2-dim one-body Hamiltonian for electron in mag field:
- Discrete "Landau levels", with degeneracy $D = eB/h$.
- "Filling factor" $\nu \equiv N/D = Nh/eB = \#$ filled levels.
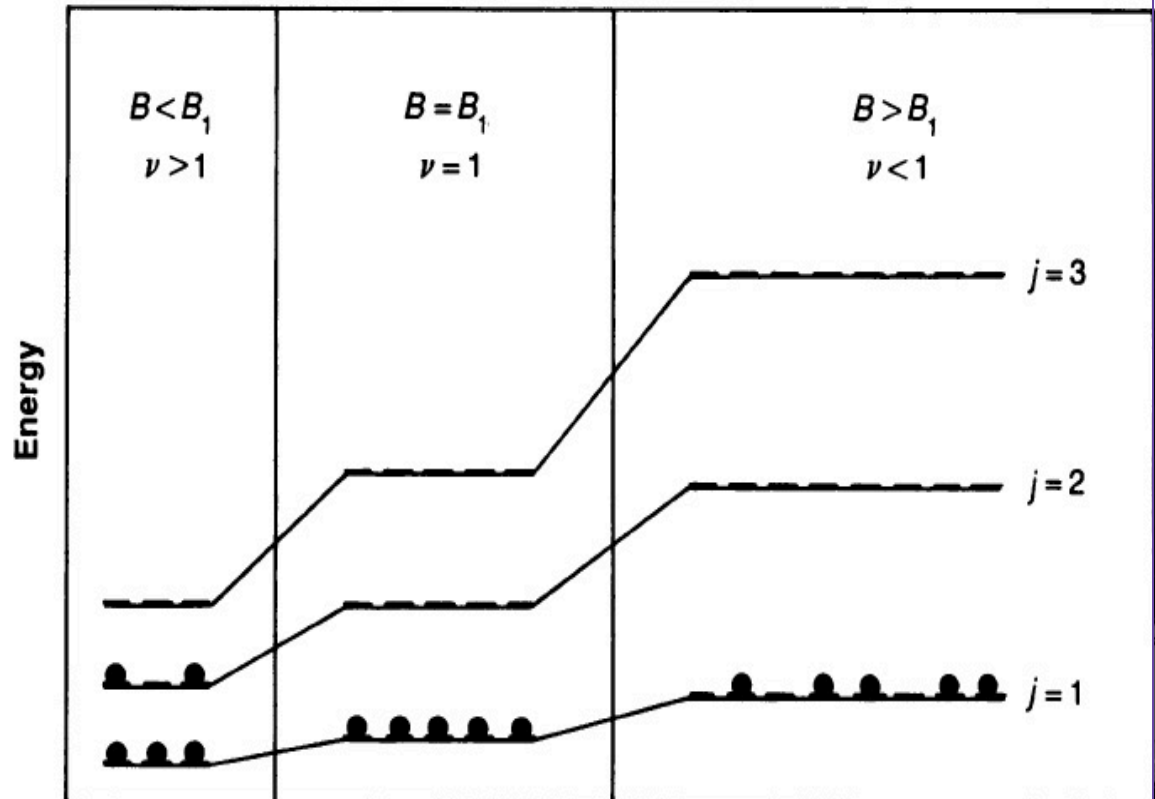
When $\nu = n =$ integer:

1. System becomes *incompressible*.
   $\Rightarrow R = 0$

2. $B = B_n = Nh/en$
   $\Rightarrow R_H = B/eN$
   $$= \frac{h}{e^2}\frac{1}{n}$$

$B < B_1$ $\nu > 1$ $B = B_1$ $\nu = 1$ $B > B_1$ $\nu < 1$

Energy

$j = 3$

$j = 2$

$j = 1$

2. The Quantum Hall Effect

Integer Quantum Hall Effect (IQHE)

1. Why is the system incompressible at integer values of ν? (Or: Why does R vanish at plateaus in R_H?)

Ans. *Gapped structure of energy spectrum; Pauli Exclusion Principle (PEP) for electrons.*

2. Why does incompressibility persist for small changes in B? (Or: Why does the plot of R_H exhibit plateaus?)
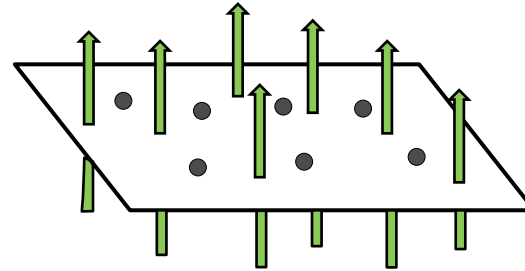
Ans. *Impurities trap conduction electrons ("localization").*

- IQHE is a *one-body* effect: single electron coupled to mag field.
- FQHE is a *one-body* or *many-body* effect: depending on the type of particle invoked.

  - *Four different mechanistic accounts...*

3. The Underdetermination of Mechanism in the FQHE

(i) _Laughlin Ground State Account_ (Laughlin 1983)

- Particles = electrons.
- _Many-body_ electron–electron interactions prevent electrons in partially filled highest Landau level from moving to unoccupied states.

$$\Psi_m(\mathbf{r}_1,...,\mathbf{r}_N) = \prod_{1 \le j < k \le N} (z_j - z_k)^{1/\nu} e^{\sum |z_i|^2/4}$$

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1. Why is the system incompressible at fractional values of ν? (Or: Why does R vanish at plateaus in R_H?)

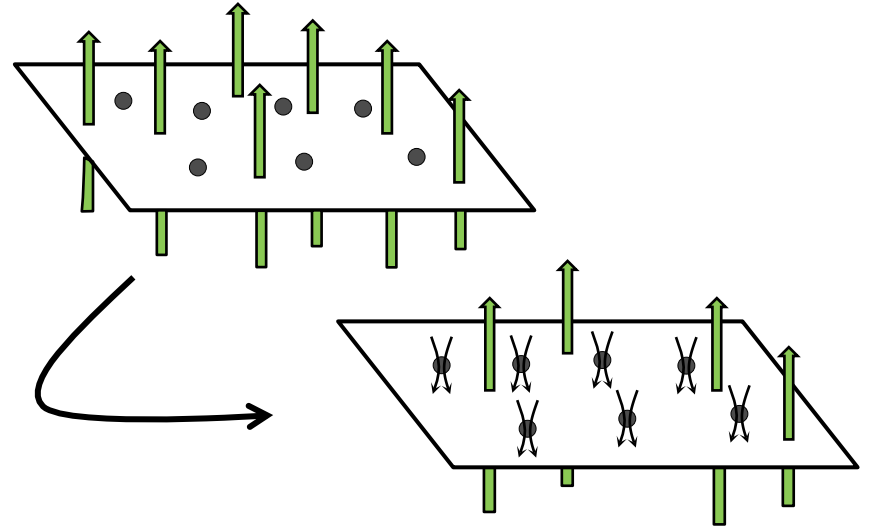_Ans._ _Electron–electron interactions; PEP for electrons._

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2. Why does incompressibility persist for small changes in B? (Or: Why does the plot of R_H exhibit plateaus?)

Ans. _Quasiparticle–impurity interactions ("localization")._

3. The Underdetermination of Mechanism in the FQHE

(ii) _Composite Fermion Account_   (Jain 1989)

- Particles $=$ electrons with _even_ attached Chern–Simons fluxes.
- Fluxes reduce B field to IQHE values.
- FQHE as _1-body_ IQHE of composite fermions.

1. Why is the system incompressible at fractional values of ν? (Or: Why does R vanish at plateaus in R_H?)

Ans.   _Electron–CS field interaction; gapped structure of energy spectrum; PEP for composite fermions._
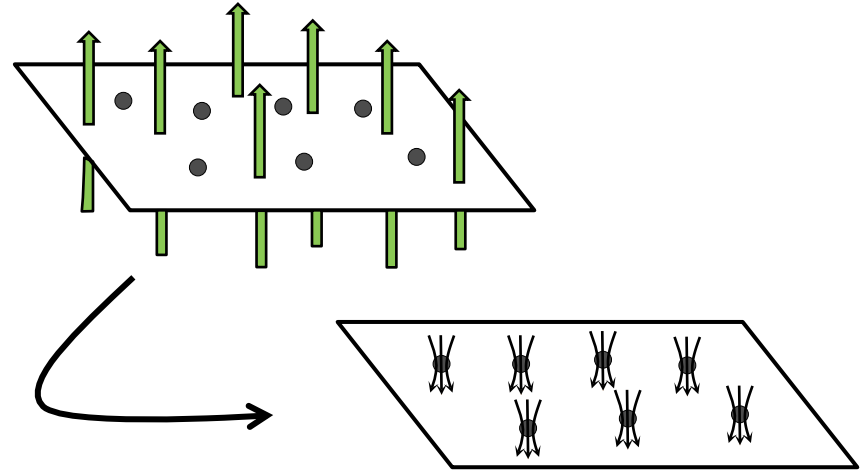
2. Why does incompressibility persist for small changes in B? (Or: Why does the plot of R_H exhibit plateaus?)

Ans.   _Composite fermion–impurity interactions ("localization")._

3. The Underdetermination of Mechanism in the FQHE

(iii) _Composite Boson Account_ (Girvin & MacDonald 1987)

- Particles $=$ electrons with _odd_ attached Chern–Simons fluxes.
- Fluxes _cancel B_ field.
- FQHE as _1-body_ effect of condensing composite bosons.

1. Why is the system incompressible at fractional values of ν? (Or: Why does R vanish at plateaus in R_H?)

Ans. _Electron–CS field interaction; BEC formation via spontaneous symmetry breaking; Meissner effect._
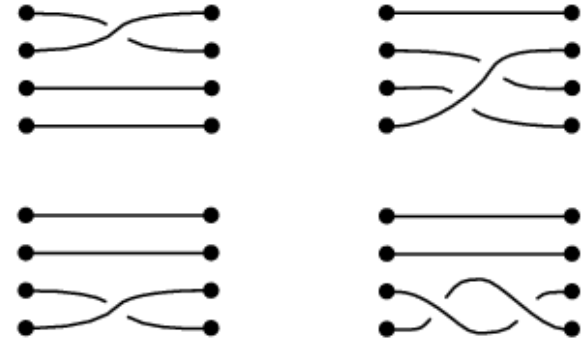
2. Why does incompressibility persist for small changes in B? (Or: Why does the plot of R_H exhibit plateaus?)

Ans. _Vortex–impurity interactions ("localization")._

3. The Underdetermination of Mechanism in the FQHE

(iv) *Topological Order Account*   (Wen 1990)

- Particles = *long-range entangled* (LRE) electrons.
- LRE state: cannot be "disentangled" by a local unitary evolution.
- *Claim*: LRE reflects "topological entanglement".

1. Why is the system incompressible at fractional values of ν? (Or: Why does R vanish at plateaus in R_H?)

Ans. Long-range correlations.

2. Why does incompressibility persist for small changes in B? (Or: Why does the plot of R_H exhibit plateaus?)

Ans. Quasiparticle–impurity interactions ("localization").

3. The Underdetermination of Mechanism in the FQHE

	Mechanism	
	$R = 0$	Plateaus in R_H
Laughlin ground state	Many-body Coulomb effect of strongly interacting electrons.	**Localization**: quasiparticle–impurity interactions.
Composite fermion	One-body IQHE effect of non-interacting composite fermions.	**Localization**: composite fermion–impurity interactions.
Composite boson	One-body effect of weakly-interacting composite bosons undergoing **SSB**.	**Localization**: vortex–impurity interactions.
Topological order	Many-body **long-range entangled** effect of electrons.	**Localization**: quasiparticle–impurity interactions.

- Four distinct microphysical mechanist accounts.

- Three distinct high-level mechanist accounts.

- *Concern*: Shouldn't there be just a *single* ontological mechanistic account of the emergence (and persistence) of incompressibility in the FQHE?

4. Law-Centric Emergence in the FQHE

Claim: The novelty that characterizes incompressibility in an FQH system is underwritten by the **distinct dynamical laws** that govern it, compared to those that govern the fundamental 2-dim conductor.

$$\mathcal{L}[\psi, A_\mu] = \psi^\dagger i (\partial_t - iA_t)\psi + \frac{1}{2m}\psi^\dagger(\partial_i - iA_i)^2\psi + V(\psi^\dagger, \psi) \tag{3}$$

$$\mathcal{L}_{eff}[a_\mu, A_\mu, j^\mu] = -\frac{p}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu\,\partial_\nu a_\lambda + \frac{e}{2\pi}\epsilon^{\mu\nu\lambda}A_\mu\,\partial_\nu a_\lambda + j^\mu a_\mu \tag{4}$$

- Low-energy behavior (4) of FQH system is _dynamically independent_ of, and _dynamically robust_ with respect to, high-energy behavior (3) of 2-dim conductor.

5. Conclusion

- There is an underdetermination of mechanistic accounts of the FQHE, at both the microphysical level and the level of higher organizing principles.

- This underdetermination is pernicious for mechanism-centric views of emergence.

- A law-centric view of emergence avoids underdetermination by avoiding reference to mechanisms.

- Under a law-centric view, the novelty exhibited by an FQH system, with respect to the fundamental electron system from which it emerges, is explained by appealing to the distinct dynamical laws that govern both systems.

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