Emergent Spacetime and Structural Realism

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Prospects for modeling spacetime as a phenomenon that emerges in the low-energy limit of a quantum liquid.

1. **EFTs in Condensed Matter Systems**
2. **Superfluid Helium**
3. **Emergence and Emergent Spacetime**
4. **Universality, Topology, and Dynamical Structure**

1. EFTs in Condensed Matter Systems

Highly-correlated condensed matter system = ***<u>Non-relativistic</u>*** **many-body quantum system that displays macroscopic quantum effects**

- *superfluids*
- *superconductors*
- *Bose-Einstein condensates*
- *quantum Hall liquids*

Effective Field Theory of condensed matter system = **Theory of low-energy dynamics of system: describes states with energy close to zero**

- *bosonic collective modes of ground state*
- *fermionic excitations above ground state ("quasiparticles")*
- *topological defects ("vortices")*

1. EFTs in Condensed Matter Systems

How to Construct a Condensed Matter EFT

Take Low-energy "limit":

- **Expand Lagrangian in small fluctuations in field variables about ground state and integrate out high-energy fluctuations. (^4He)**

OR

- **Linearize the energy about the values where it vanishes and then construct the corresponding Hamiltonian. (^3He-A)**

For fermionic liquids, the type of EFT that results is ultimately based on *topological* properties of the ground state of system, as opposed to its *symmetries*.

2. "Acoustic" Spacetimes and Superfluid ^4He

- liquid consisting of many ^4He atoms, all phases alligned
- model ground state as *single* quantum particle: $\varphi_0 = \rho_0 e^{i\theta}$

$$\mathcal{L}_{4He} = i\varphi^\dagger\partial_t\varphi - \frac{1}{2m}\partial_i\varphi^\dagger\partial_i\varphi + \mu\varphi^\dagger\varphi - \alpha^2(\varphi^\dagger\varphi)^2$$

kinetic energy

conservation of particle#

SSB potential

Non-relativistic Lagrangian for Superfluid ^4He

Low-energy limit:

- Let $\varphi = \rho e^{i\theta}$, $\rho = \rho_0 + \delta\rho$, $\theta = \theta_0 + \delta\theta$
- Integrate out high-energy fluctuations $\delta\rho$

$$\mathcal{L}_{4He} = \mathcal{L}_0[\rho_0, \theta_0] + \mathcal{L}'_{4He}[\delta\theta]$$

ground state

low-energy fluctuations above ground state (EFT)

2. "Acoustic" Spacetimes and Superfluid ^4He

$$\mathcal{L}'_{4He} = \frac{1}{4\alpha^2}(\partial_t\theta + v_i\partial_i\theta)^2 - \frac{\rho_0}{2m}(\partial_i\theta)^2 \qquad i = 1, 2, 3$$

EFT for Superfluid ^{4}He

$\left(\text{to 2nd order in } \delta\theta: \; v_i = (1/m)\partial_i\theta \right)$

... identical to...

$\Updownarrow$

$$\mathcal{L}'_{4He} = \frac{1}{2}\sqrt{g}\; g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta \qquad \mu, \nu = 0, 1, 2, 4$$

Massless scalar field in curved spacetime!

$$g_{\mu\nu}dx^\mu dx^\nu = (\rho/cm)[-c^2dt^2 + \delta_{ij}(dx^i - v^i dt)(dx^j - v^j dt)]$$

$$c^2 \equiv 2\alpha^2\rho/m$$

Can now model black hole physics:

speed of light = speed of low-energy ocillations (*i.e.*, "sound" modes)

Hence: "acoustic" spacetimes and "acoustic" black holes

2. "Acoustic" Spacetimes and Superfluid ^4He

(i) *What is the kinematic background of acoustic spacetimes?*

Option #1: Minkowski spacetime

$$g_{\mu\nu}dx^\mu dx^\nu = (\rho/cm)[-c^2dt^2 + \delta_{ij}(dx^i - v^i dt)(dx^j - v^j dt)]$$

$$= \eta_{\mu\nu}dx^\mu dx^\nu + g'_{\mu\nu}dx^\mu dx^\nu$$

acoustic metric

background

low-energy fluctuations

✓ **Option #2: Neo-Newtonian spacetime**

Superfluid ^4He in Neo-Newtonian ST $\xrightarrow{\text{1st order } \delta\theta}$ Massless scalar field in Minkowski ST

Superfluid ^4He in Neo-Newtonian ST $\xrightarrow{\text{2st order } \delta\theta}$ Massless scalar field in acoustic ST

2. "Acoustic" Spacetimes and Superfluid ^4He

(ii) *To what extent are "acoustic" spacetimes analogues of GR spacetimes?*

Einstein equations cannot be derived from ^4He EFT. $\Rightarrow$ *Not dynamical analogues!*

Kinematic analogues of GR?

> ... the features of general relativity that one typically captures in an "analogue model" are the *kinematic* features that have to do with how fields (classical or quantum) are defined on curved spacetime, and the *sine qua non* of any analogue model is the existence of some "effective metric" that captures the notion of the curved spacetimes that arise in general relativity. (Barceló, Liberati, Visser 2005, pg. 10.)

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Kinematic analogues of GR?

> The acoustic analogue for black-hole physics accurately reflects half of general relativity -- the kinematics due to the fact that general relativity takes place in a Lorentzian spacetime. The aspect of general relativity that does not carry over to the acoustic model is the dynamics -- the Einstein equations. *Thus the acoustic model provides a very concrete and specific model for separating the kinematic aspects of general relativity from the dynamic aspects.* (Visser 1998, pg. 1790.)

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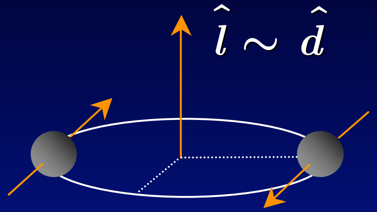
Kinematic analogues of GR?

- If kinematics of GR = Minkowski ST, then *No!*
- Kinematics of GR = *?*

Some features that one normally thinks of as intrinsically aspects of gravity, both at the classical and semiclassical levels (such as horizons and Hawking radiation), can in the context of acoustic manifolds be instead seen to be rather generic features of curved spacetimes and quantum field theory in curved spacetimes, that have nothing to do with gravity *per se*. (Barceló, Liberati, Sonego, Visser 2004, pg. 2.)

2. Standard Model and Superfluid ^3He-A

(Volovik 2003)



- Liquid of many ^3He Cooper Pairs, all phases alligned
- Degrees of freedom: $S_z = 0, \pm 1$; $l_z = 0, \pm 1$
- A-phase: no $S_z = 0$ substates, $\hat{d} \,||\, \hat{l}$,

$$H_{3He\text{-}A} = \chi^\dagger{}_{\alpha\beta}\{(\varepsilon - \mu)\sigma_3 + \vec{V}_{\alpha\beta}(\hat{l}, \hat{d}, \vec{k})\}\chi_{\alpha\beta}$$

$E(\vec{k}) = 0$, for 2 values of $\vec{k}$

Low-energy limit

- Expand $E(\vec{k})$ to 2nd order about zero points:

$$E^2(\vec{k}) \approx g_{ij}(k_i - qA_i)(k_j - qA_j) \qquad g_{ij} \sim l_i l_j\,,\ A_i \sim l_i$$

- Construct $\mathcal{L}'_{3He\text{-}A}$

$$\mathcal{L}'_{3He\text{-}A} = \overline{\Psi} g_{\mu\nu}\gamma^\nu(\partial_\mu - qA_\mu)\Psi$$

$$g_{\mu\nu} \sim (l_i, v_i),\ A_0 \sim l_i v_i$$

Potential field A_μ interacting with matter field Ψ in curved spacetime

2. Standard Model and Superfluid $^3He\text{-}A$

- "Induced QED" (Zeldovich 1967): expand to 2nd order in fluctuations in A_μ

$$\mathcal{L}'_{^3He\text{-}A} = \overline{\Psi} g_{\mu\nu}\gamma^\nu(\partial_\mu - qA_\mu)\Psi + \frac{1}{4\kappa^2}\sqrt{-g}\, g^{\mu\nu} F_{\mu\alpha} F_{\nu\beta}$$

(3+1)-dim QED in curved spacetime

- Extendable in-principle to $SU(n)$ gauge fields $\Rightarrow$ Standard Model
- Similar treatment of effective metric ("induced gravity" Sakharov 1967) fails to reproduce Einstein-Hilbert term

Interpretation

ground state of $^3He\text{-}A$	$\Leftrightarrow$	spacetime ($g_{\mu\nu}$)
low-energy fluctuations (quasiparticles, collective modes)	$\Leftrightarrow$	matter fields, potential fields (Ψ, A_μ)
"induced" vaccum corrections to interactions between Ψ and A_μ	$\Leftrightarrow$	gauge fields ($F_{\mu\nu}$)

3. Emergence...

Claim: Novel phenomena (fields, particles, symmetries, spacetime, *etc.*) *emerge* in low-energy limit of certain condensed matter systems

Emergence in the low-energy limit

(1) *Distinct from emergence via symmetry breaking.*

non-relativistic liquid helum $\xrightarrow{\textit{lower temp}}$ non-relativistic superfluid helium $\xrightarrow{\textit{lower temp}}$ relativistic system

spontaneous symmetry breaking

low-energy limit

$\mathcal{L}$

Effective $\mathcal{L}'$

(2) *Epistemological Emergence*

- Unpredictability
- Irreducibility (and/or unexplainability)
- Causal efficacy

3. … and Emergent Spacetime

Spacetime as emergent…

(a) *… in superfluid ^4He*

- *Motivation*: Spacetime as (some aspect of) solutions to Einstein equations in GR.
- *Prospects*: Limited. ^4He EFT lacks GR dynamics/kinematics.

(b) *… in superfluid ^3He-A*

- *Motivation*: Spacetime as ground state for QFTs of matter, gauge, and metric fields.
- *Prospects*: Still limited. ^3He-A EFT does not fully recover GR.

Research programme:

Determine appropriate condensed matter system that produces relevant matter, gauge and metric fields in low-energy limit.

(Volovik 2003, Wen 2004.)

3. Universality, Topology, ...

Why does ^{3}He-A reproduce the Standard Model?

- *Universal property* = property stable under perturbations

- *Universality class* = class of systems/states characterized by common universal properties

- Universality class of the ground state of a system determines it's low-energy dynamics (*i.e.*, the corresponding EFT)

- Universal properties of ground state are "generic" properties of EFT:
  - *how system behaves at a phase transition*
  - *whether theory is renormalizable*
  - *symmetries of low-energy fluctuations (i.e., symmetries of EFT)*

3. Universality, Topology, ...

Why does ^{3}He-A reproduce the Standard Model?

- **Universality classes of fermionic ground states are characterized by momentum space topology.**

  - *stable regions in k-space where quasiparticle energies $\rightarrow$ 0*

- **^{3}*He-A* and the Standard Model have ground states characterized by the *same* momentum space topology.**

  - *stable point defects ("Fermi points")*

Thus: **^{3}*He-A* and the Standard Model possess the same *universal properties*.**

3. ... and Dynamical Structure

Same k-space topology $\Rightarrow$ Same low-energy dynamics

Irrespective of microscopic details:

- Standard Model
- ^3He-A
- any condensed matter system with "Fermi points"

$\Big\}$ Same dynamical structure

Structural Realist interpretation of spacetime:

- spacetime as a universal property = generic property of an EFT universality class

Qualification: Universality class that best describes spacetime structure still unknown