Why Be Natural?

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1. How to Construct an EFT.
2. Why Be Natural?
   - *Modest empirical success.*
   - *Quantifiable.*
   - *Consistent with "Central Dogma".*

   Less than compelling!
3. Naturalness and Emergence.

   More interesting...
4. Conclusion.

1. How to Construct an EFT.

(Wilson Version): (Polchinski 1994)

Given a theory described by $S[\phi, \partial\phi]$,

1. Choose a cut-off Λ and divide fields into high and low momenta parts with respect to Λ: $\phi = \phi_H + \phi_L$.

2. Integrate out ϕ_H to obtain the Wilsonian effective action $S_\Lambda[\phi_L]$.

$$e^{iS_\Lambda[\phi_L]} \equiv \int \mathcal{D}\phi_H e^{iS[\phi_H, \phi_L]}$$

3. Expand the effective action in a set of local operators $\mathcal{O}_i[\phi_L, \partial\phi_L]$.

$$S_\Lambda = S_0 + \int d^D x \sum_i g_i \mathcal{O}_i$$

encode high-energy DOF

1. How to Construct an EFT.

4. Perform dimensional analysis on S_Λ. For $E \ll \Lambda$:

$$S_\Lambda = S_0 + \int d^D x \sum_i g_i \mathcal{O}_i$$

units E^0 units E^{-D} units $E^{D-\delta_i}$ units E^{δ_i}

Use S_0 to determine δ_i.

[Dimensionful and encode high-energy DOF] $\Rightarrow$ [of order $\Lambda^{D-\delta_i}$]

(i) Define dimensionless $\lambda_i \equiv \Lambda^{\delta_i - D} g_i$ ← Should be of order 1

(ii) The order of the ith term is $\lambda_i (E/\Lambda)^{\delta_i - D}$.

- *Irrelevant term:* $\delta_i > D$. Falls as $E \to 0$. ← **Ideal?** Insensitive to high-energy DOF.
- *Relevant term:* $\delta_i < D$. Grows as $E \to 0$.
- *Marginal term:* $\delta_i = D$. Constant as $E \to 0$.

Worrisome? Indicates sensitivity to high-energy DOF.

1. How to Construct an EFT.

Ex. Scalar field theory in 4-dim ($\Phi \to -\Phi$ symmetry).

$$S_\Lambda[\Phi_L] = \frac{1}{2}\int d^4x(\partial_\mu \Phi_L)^2 + \int d^4x\left[\lambda_{-2}\Lambda^4 + \lambda_0\Lambda^2\Phi_L^2 + \lambda_2\Phi_L^4 + \lambda_4\Lambda^{-2}\Phi_L^6 + \cdots\right]$$

$$+\int d^4x\left[\sum_{n>0}\lambda_n'\Lambda^{-n}(\partial_\mu\Phi_L)^2\Phi_L^n + \sum_{n\geq 0}\lambda_n''\Lambda^{-(n+4)}(\partial_\mu\Phi_L)^4\Phi_L^n + \cdots\right]$$

- Φ_L must have units E^δ satisfying $E^{-4}E^2E^{2\delta} = E^0$, thus $\delta = 1$.

Relevant terms:

Worrisome?

- Additive term: $\lambda_{-2}\Lambda^4$
  - *quartic dependence on cut-off.*

$$m^2_{\text{phys}} = \lambda_0\Lambda^2$$

order 1?

- Mass term: $\lambda_0\Lambda^2\Phi_L^2$
  - *quadratic dependence on cut-off.*

$$m^2_{\text{phys}} = m^2_{\text{bare}} + \kappa\Lambda^2$$

fine-tuning?

2. Why be Natural?

Naturalness (Williams 2015)
No sensitive correlations between low- and high-energy phenomena.

Common to other formulations:

- No parameters with quadratic (or higher) dependence on cutoff/heavy fields.
- No dimensionless parameters that are not order 1, unless protected by a symmetry.
- No bare parameters that require fine-tuning.

Intuition: *Apparent sensitivity is due to presence of new physics.*

2. Why be Natural?

(i) Modest Empirical Success.

- Most parameters in SM are natural.
- *General Claim*: Where naturalness fails, seek new physics.
  - Prediction of charm quark.
  - Postdiction of positron, ρ-meson.

But: Spectacular failures:

- Hierarchy Problem: $\lambda_0 = m^2_{\text{Higgs}}/M^2_{\text{Pl}} \sim 10^{-34}$.
- Cosmological constant Problem: $\lambda_{-2} = \Lambda^4_{\text{C}}/M^4_{\text{Pl}} \sim 10^{-120}$.
- Strong CP Problem: $\theta_{\text{QCD}} < 10^{-10}$.

Where's the new physics?

2. Why be Natural?

(ii) Quantifiable.

(a) Measures of sensitivity of low-energy parameters to high-energy parameters.

(b) Measures of likeliness of fine-tuned values of bare parameters.

***But*:** (Hossenfelder 2018)

- Problems with (a):
  - *Different results*
  - *Different tolerance levels*
- Problems with (b):
  - *Requires a probability distribution.*
  - *Risk of begging the question that fine-tuned parameters are unlikely.*

2. Why be Natural?

(iii) Consistent with "spirit" of EFTs.

The Central Dogma of EFTs (Williams 2015)
Phenomena at widely separated scales should decouple.

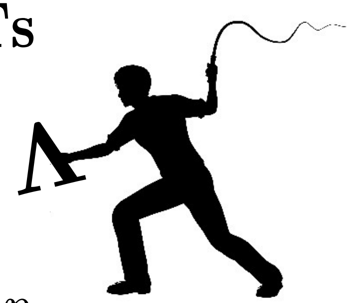
But:

- A failure of naturalness does not signify a failure of decoupling.
- While decoupling may be EFT dogma, naturalness seems dogmatic only for Wilsonian EFTs.

What about "continuum" EFTs?
(Georgi 1993)

2. Why be Natural?

Mass-dependent renormalization and Wilsonian EFTs



- Use the cut-off Λ to regulate divergent integrals.
  - Replace $\int_0^\infty \kappa(p) d^D p$ with $\int_0^\Lambda \kappa(p) d^D p + \int_\Lambda^\infty \kappa(p) d^D p$.
  - Absorb second piece into renormalized parameters.

- Requires a subtraction scheme that is "mass-dependent": renormalized parameters are dependent on the masses of the heavy fields.

Mass-dependent renormalization and Wilsonian EFTs

Advantages:

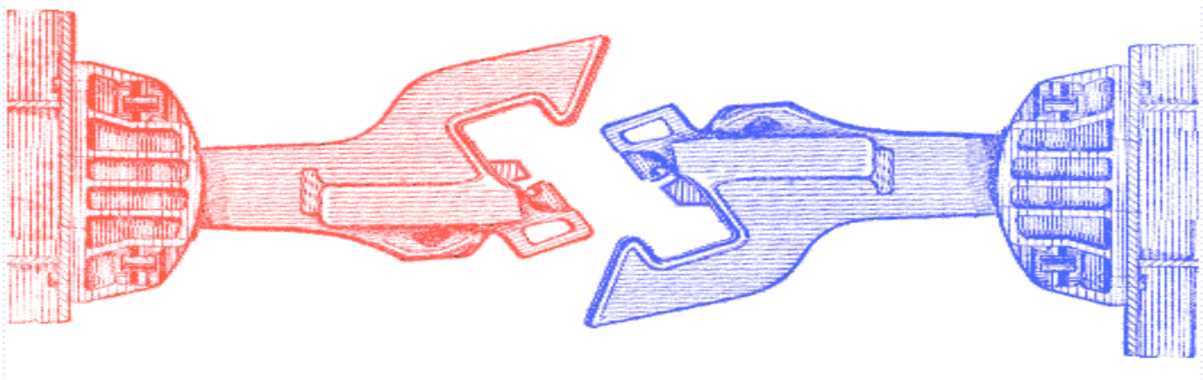
(a) Consistent with image of an EFT as a low-energy approximation to a high-energy theory based on a restriction of the latter to a particular energy scale Λ.

(b) Necessary for proof of the Decoupling Theorem...

Mass-dependent renormalization and Wilsonian EFTs

Decoupling Theorem (Appelquist & Carazzone 1975)

In a perturbartively renormalizable theory with two widely separated mass scales, there is always a mass-dependent renormalization scheme by means of which the effects of the heavy mass can be encoded in the parameters of an effective theory in which only the light mass appears.



2. Why be Natural?

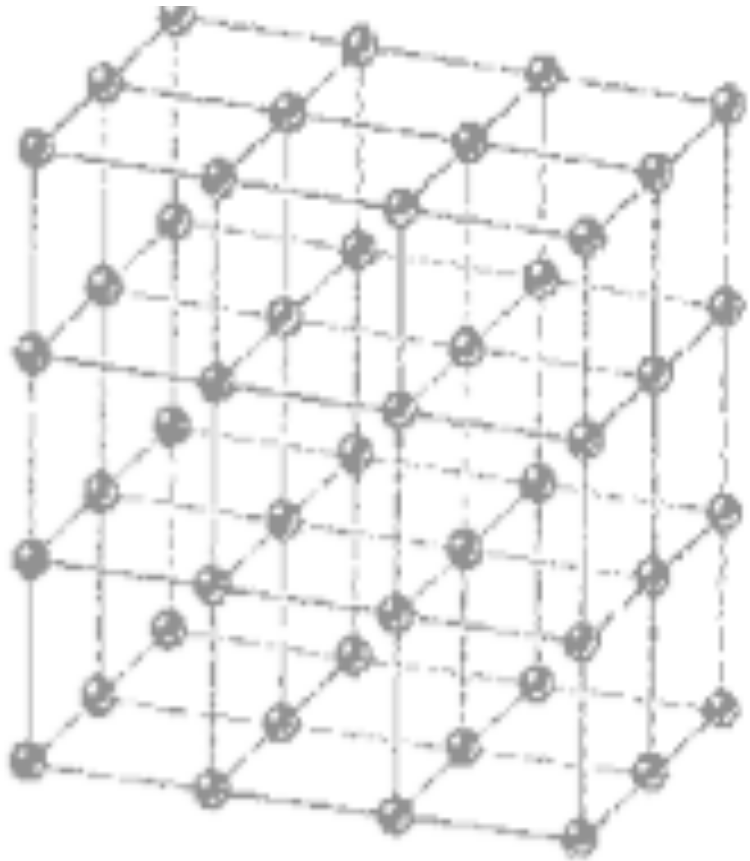
Mass-dependent renormalization and Wilsonian EFTs

Disadvantages:

(a) Momentum cut-off regularization violates Poincaré and gauge invariance.

(b) Dependence of irrelevant terms on orders of E/Λ breaks down for higher-order loop calculations: *Power* dependence of terms on Λ.

- *Higher-order loop calculations cannot ignore irrelevant terms.*



Mass-independent renormalization and continuum EFTs

- Use mass-*independent* subtraction scheme: Energy scale parameter μ appears in loop corrections in *logarithms*.

  - *Irrelevant terms can be ignored at both tree- and high-order loop levels.*

- Use *dimensional regularization*:

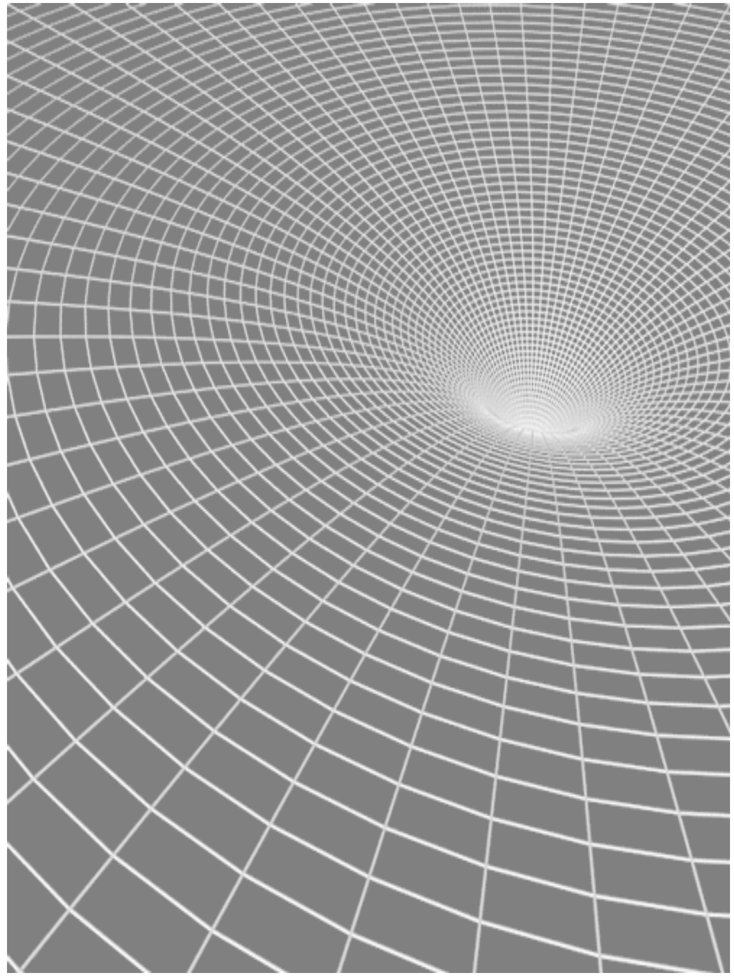
  - Replace $\int_0^\infty \kappa(p) d^D p$ with $\int_0^\infty \kappa(p) d^{D-\varepsilon} p$.
  - Analytically continue $D - \varepsilon$ to D.
  - Absorb poles into (mass-independent) renormalization constants.

Mass-independent renormalization and continuum EFTs

Advantages:

(a) Dimensional regularization respects Poincaré and gauge invariance.

(b) Mass-independent subtraction allows truncation of the effective action to a finite number of terms for both tree-level calculations and higher-order loop corrections.



2. Why be Natural?

Mass-independent renormalization and continuum EFTs

<u>*Disadvantages*</u>:

(a) Violates the "spirit" of an EFT: heavy field terms are present in a dim-regularized action.

(b) Decoupling Theorem does not hold.

What about the Central Dogma?

2. Why be Natural?

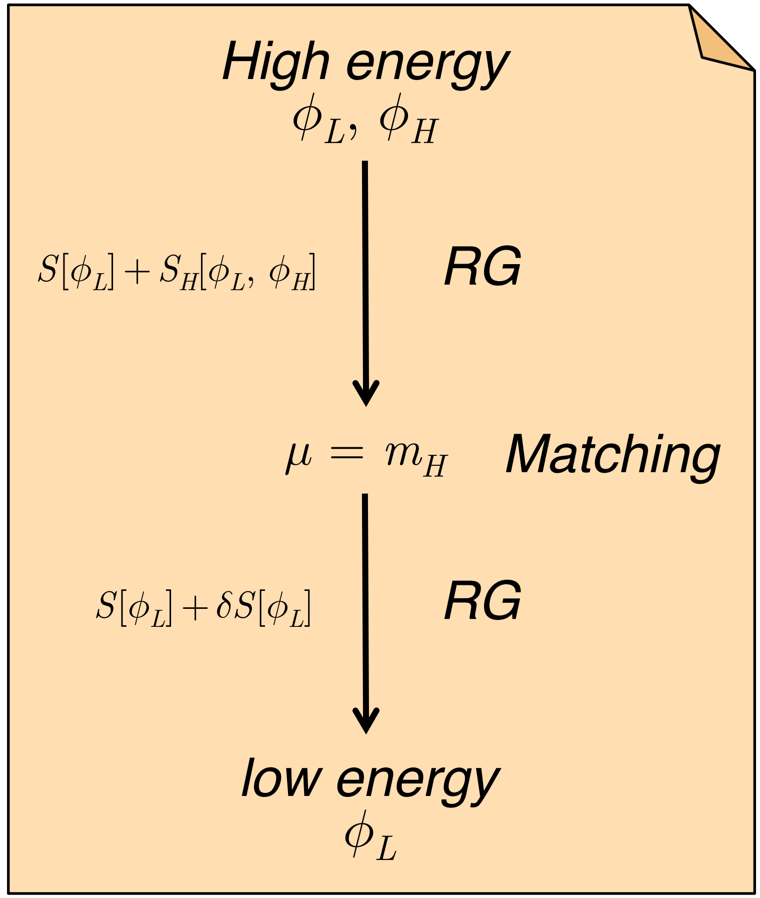
How to construct a continuum EFT

1. Start with $S = S[\phi_L] + S_H[\phi_L, \phi_H]$ at energy scale μ.

2. Evolve action to lower energies *via* RG: $\mu \to \mu - d\mu$.

3. *Matching*: When μ gets below m_H, replace S with $S_{eff} = S[\phi_L] + \delta S[\phi_L]$, where $\delta S[\phi_L]$ encodes a "matching condition" that guarantees S and S_{eff} agree on observables.

"Decoupling by hand" as a guarantee of empirical adequacy



2. Why be Natural?

Wilsonian EFTs: Naturally biased?

Physical cut-off Λ plays double role:

(a) Demarcates low-energy physics from high-energy physics.

(b) Regulates divergent integrals.

- Λ imposes implicit assumptions about the order of effective couplings g_i.

Continuum EFTs: Naturally agnostic?

- Renormalization scale μ plays Role (a).
- Role (b) replaced by dimensional regularization.
- No implicit assumptions about the order of effective couplings.
  - *Fine-tuning: What me worry?*

3. Naturalness and Emergence.

Wilsonian EFTs:

- Motivated by condensed matter physics...

  ...which is enthralled by "emergence":

<u>Informal references to emergence:</u>

- "emergent grativational features in condensed matter systems"; "emergent spacetime symmetries". (Barcelo *et al.* 2005)
- "...an effective electrodynamics emerges from an underlying fermionic condensed matter system." (Dziarmaga 2002)
- "emergent relativistic quantum field theory and gravity"; "emergent nontrivial spacetimes". (Volovik 2003)
- "emergence of relativity". (Zhang & Hu 2001)

3. Naturalness and Emergence.

Emergence = a characteristic of the ontology associated with a physical system (the emergent system), with respect to another physical system (the fundamental system).

Crieteria for Emergence Crowther (2015)

(i) *Dependence.* Emergent system is "ontologically determined" by the fundamental system.

(ii) *Independence.* Emergent system is "robustly novel" with respect to fundamental system.

Task: Resolve tension between *Dependence* and *Independence*.

Suggestion: Natural EFTs accomplish this task.

3. Naturalness and Emergence.

How an EFT satisfies *Dependence*

- Low-energy phenomena decouple from high-energy phenomena.
  - *Low-energy phenomena are low-energy DOF of high-energy phenomena.*
  - *High-energy effects encoded in low-energy dynamics.*
- *Interpretation*: Low-energy phenomena are ontologically determined by high-energy phenomena.

How a natural EFT satisfies *Independence*

- *Naturalness*: No sensitive correlations between low- and high-energy phenomena.
- *Interpretation*: Low-energy phenonena are *robustly dynamically independent* of high-energy phenomena.

Conclusion.

Why be natural?

Not because:

- It's empirically warranted.
- It's quantifiable.
- It underwrites the EFT Central Dogma.

Perhaps because:

- It helps to underwrite a non-trivial notion of emergence associated with EFTs.

Conclusion.

Why be natural?

?

<u>*General Morals*</u>:

(a) Naturalness is an *empirical* hypothesis with *ontological* implications.

(b) As an empirical hypothesis with limited empirical support, one should be cautious in using it as a guiding principle; and one should be cognizant of where it occurs as an assumption in theoretical frameworks (*viz.*, Wilsonian EFTs).

(c) As an ontological hypothesis, there is nothing wrong with the project of examining what the world would be like if it were true, or how current theories might be extended if it were true, as long as one is cognizant of Moral (b).

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