A Concept of Emergence for EFTs

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2. The EFT Intertheoretic Relation.
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1. How to Construct an EFT

Given a "high-energy" Lagrangian $\mathcal{L}[\phi(x)]$:

$\boxed{\text{(I) Identify and eliminate high-energy degrees of freedom.}}$

- Choose a cutoff Λ and decompose $\phi(x) = \phi_H(x) + \phi_L(x)$.
- Perform integration over $\phi_H(x)$:

$$Z = \int \mathcal{D}\phi_L \mathcal{D}\phi_H e^{i\int d^D x \mathcal{L}[\phi_L, \phi_H]} = \int \mathcal{D}\phi_L e^{i\int d^D x \mathcal{L}_{eff}[\phi_L]}$$

$\boxed{\text{(II) Construct local operator expansion of } \mathcal{L}_{eff}[\phi_L(x)].}$

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \sum_i g_i \mathcal{O}_i$$

1. How to Construct an EFT

<u>*Characteristics*</u>

(1) $\mathcal{L}[\phi(x)]$ describes ∞ DOF, $\mathcal{L}_{\mathit{eff}}[\phi_L(x)]$ describes finite DOF.

(2) $\mathcal{L}_{\mathit{eff}}[\phi_L]$ is formally distinct from $\mathcal{L}[\phi]$.

(3) $\phi_L(x)$ is "dynamically" distinct from $\phi(x)$.

(4) Relation between $\mathcal{L}_{\mathit{eff}}$ and $\mathcal{L}$ *cannot* be presented as a formal derivation.

- *Step I: Informal choice of cutoff and low-energy DOF.*
- *Step II: Approximation procedure involving informal identification of symmetries.*

- <u>*But*</u>: It *can* be informally characterized through concrete examples...

2. The EFT Intertheoretic Relation

Example 1: Superfluid Helium 3-A

- With respect to T_c, high-energy degrees of freedom are fermionic ^3He atoms arranged in Cooper pairs:

$$\mathcal{L} = \Psi^\dagger\{i\partial_t - (\partial_i^2/2m + \mu)\}\tau_3\Psi + \mathcal{L}_{int}[\Psi, \Delta] \qquad (1)$$

- *Non-relativistic* Lagrangian density. (Schakel 1998)
- Ψ encodes creation/annihilation operators for ^3He atoms.
- Order parameter Δ encodes ^3He-A Cooper pair interaction.

2. The EFT Intertheoretic Relation

Example 1: Superfluid Helium 3-A

- With respect to T_c, low-energy degrees of freedom are bosonic hydrodynamical sound waves $\varphi(x)$:

$$\mathcal{L}_{eff} = -n[\partial_t\varphi + \tfrac{1}{2m}(\partial_i\varphi)^2] + \rho[\partial_t\varphi + \tfrac{1}{2m}(\partial_i\varphi)^2]^2 \qquad (2)$$

- *Non-relativistic* Lagrangian density. (Schakel 1998)
- φ encodes phase of order parameter.
- n and ρ are the fermion number density and density of states.

2. The EFT Intertheoretic Relation

Example 1: Superfluid Helium 3-A

- With respect to ground state, low-energy degrees of freedom are massless fermions coupled to a Maxwell field:

$$\boxed{\mathcal{L}_{eff} = \bar{\Psi}\gamma^{\mu}(\partial_{\mu} - qA_{\mu})\Psi + \mathcal{L}_{Max} \qquad (3)}$$

- *Relativistic* Lagrangian density. (Volovik 2003)
- Ψ encodes creation/annihilation operators for ^3He atoms.
- γ-matrices are determined by a Lorentz-signature "metric" $g^{\mu\nu}$ that encodes ^3He-A Cooper pair degrees of freedom.
- qA_{μ} encodes position of "Fermi points" in 4-momentum space.

2. The EFT Intertheoretic Relation

<u>*Comparison*</u>

$$\mathcal{L} = \Psi^\dagger\{i\partial_t - (\partial_i^2/2m + \mu)\}\tau_3\Psi + \mathcal{L}_{int}[\Psi, \Delta] \qquad (1)$$

$$\mathcal{L}_{eff} = -n[\partial_t\varphi + \tfrac{1}{2m}(\partial_i\varphi)^2] + \rho[\partial_t\varphi + \tfrac{1}{2m}(\partial_i\varphi)^2]^2 \qquad (2)$$

$$\mathcal{L}_{eff} = \bar{\Psi}\gamma^\mu(\partial_\mu - qA_\mu)\Psi + \mathcal{L}_{Max} \qquad (3)$$

a. High-energy theory (1) is formally and dynamically distinct from low-energy EFTs (2) and (3).
   - High-energy theory (1) is a non-relativistic QFT describing *fermionic* degrees of freedom.
   - EFT of T_c (2) is a non-relativsitic QFT describing *bosonic* degrees of freedom.
   - EFT of ground state (3) is a *relativistic* QFT.

2. The EFT Intertheoretic Relation

Comparison

$$\mathcal{L} = \Psi^\dagger\{i\partial_t - (\partial_i^2/2m + \mu)\}\tau_3\Psi + \mathcal{L}_{int}[\Psi, \Delta] \qquad (1)$$

$$\mathcal{L}_{eff} = -n[\partial_t\varphi + \tfrac{1}{2m}(\partial_i\varphi)^2] + \rho[\partial_t\varphi + \tfrac{1}{2m}(\partial_i\varphi)^2]^2 \qquad (2)$$

$$\mathcal{L}_{eff} = \bar{\Psi}\gamma^\mu(\partial_\mu - qA_\mu)\Psi + \mathcal{L}_{Max} \qquad (3)$$

b. (1), (2) and (3) describe distinct physical systems:

- (1) describes non-relativistic fermionic ^3He atoms.
- (2) describes non-relativistic bosonic sound waves.
- (3) describes relativistic fermions coupled to a Maxwell field.

2. The EFT Intertheoretic Relation

Suggests:

1. *Failure of law-like deducibility*: The laws of $\mathcal{L}_{eff}$ are not deducible consequences of the laws of $\mathcal{L}$.

2. *Ontological distinctness*: Degrees of freedom of $\mathcal{L}_{eff}$ are (typically) associated with physical systems that are distinct from the physical systems associated with the DOF of $\mathcal{L}$.

3. *Ontological dependence*: DOF of $\mathcal{L}_{eff}$ are exactly the low-energy DOF of $\mathcal{L}$. (Physical systems described by an EFT do not "float free" of those described by its high-energy theory.)

2. The EFT Intertheoretic Relation

Example 2: 2-dim Quantum Hall Liquid

- High-energy degrees of freedom are electrons coupled to an external magnetic field A_i and a Chern-Simons field a_μ:

$$\mathcal{L} = -\psi^\dagger\{\partial_t - ie(A_0 - a_0)\}\psi - \tfrac{1}{2m}\psi^\dagger\{\partial_i - ie(A_i + a_i)\}\psi + \mu\psi^\dagger\psi + \vartheta\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \tag{4}$$

- *Non-relativistic* Lagrangian density. (Schakel 1998)
- ϑ chosen so that electrons ψ have an even number of magnetic fluxes ("composite" fermions).
- Quantum Hall Effect: $\sigma = \nu(e^2/h)$,

$$\nu = \frac{(\#\,electrons)}{(\#\,states\ per\ energy\ level)} = \text{integer or fraction}$$

2. The EFT Intertheoretic Relation

Example 2: 2-dim Quantum Hall Liquid

- "Low-energy" degrees of freedom of bulk liquid are two Chern-Simons fields:

$$\mathcal{L}_{eff} = \vartheta\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + \vartheta'\varepsilon^{\mu\nu\lambda}(A_\mu + a_\mu)\partial_\nu(A_\lambda + a_\lambda) \qquad (5)$$

- *Topological quantum field theory.* (Schakel 1998)
- a_μ, $(A_\mu + a_\mu)$ are two Chern-Simons fields.
- ϑ' chosen to produce integer QHE.
- An EFT of the Fractional QHE, but not a *low-energy* EFT.

2. The EFT Intertheoretic Relation

Example 2: 2-dim Quantum Hall Liquid

- Low-energy degrees of freedom of edge are bosonic hydrodynamical sound waves $\phi(x)$:

$$\boxed{\mathcal{L}_{\textit{eff-edge}} = (1/8\pi)\{(\partial_t\phi)^2 - (\partial_x\phi)^2\} \qquad (6)}$$

- *Relativistic* (1+1)-dim Lagrangian density. (Wenn 1990)

2. The EFT Intertheoretic Relation

<u>*Comparison*</u>

$$\mathcal{L} = -\psi^{\dagger}\{\partial_{t} - ie(A_{0} - a_{0})\}\psi - \frac{1}{2m}\psi^{\dagger}\{\partial_{i} - ie(A_{i} + a_{i})\}\psi + \mu\psi^{\dagger}\psi + \vartheta\varepsilon^{\mu\nu\lambda}a_{\mu}\partial_{\nu}a_{\lambda} \tag{4}$$

$$\mathcal{L}_{eff} = \vartheta\varepsilon^{\mu\nu\lambda}a_{\mu}\partial_{\nu}a_{\lambda} + \vartheta'\varepsilon^{\mu\nu\lambda}(A_{\mu} + a_{\mu})\partial_{\nu}(A_{\lambda} + a_{\lambda}) \tag{5}$$

$$\mathcal{L}_{eff\text{-}edge} = (1/8\pi)\{(\partial_{t}\phi)^2 - (\partial_{x}\phi)^2\} \tag{6}$$

a. High-energy theory (4) is formally and dynamically distinct from EFTs (5) and (6):
   - High-energy theory (4) is a non-relativistic QFT.
   - EFT of bulk (5) is a topological QFT.
   - Low-energy EFT of edge (6) is a relativistic QFT.

2. The EFT Intertheoretic Relation

Comparison

$$\mathcal{L} = -\psi^\dagger\{\partial_t - ie(A_0 - a_0)\}\psi - \tfrac{1}{2m}\psi^\dagger\{\partial_i - ie(A_i + a_i)\}\psi + \mu\psi^\dagger\psi + \vartheta\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \tag{4}$$

$$\mathcal{L}_{eff} = \vartheta\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + \vartheta'\varepsilon^{\mu\nu\lambda}(A_\mu + a_\mu)\partial_\nu(A_\lambda + a_\lambda) \tag{5}$$

$$\mathcal{L}_{eff\text{-}edge} = (1/8\pi)\{(\partial_t\phi)^2 - (\partial_x\phi)^2\} \tag{6}$$

b. (4), (5) and (6) describe distinct physical systems:

- (4) describes non-relativistic composite electrons.
- (5) describes two topological Chern-Simons fields.
- (6) describes relativistic massless bosons.

3. Emergence in EFTs

Two General Notions of Emergence:

(a) *Emergence as descriptive of the ontology (entities, properties) associated with a physical system with respect to another.*

- To say phenomena associated with an EFT are emergent is to say the entities or properties described by the EFT emerge from those described by a high-energy theory.

(b) *Emergence as a relation between theories.*

- To say phenomena associated with an EFT are emergent is to say the EFT stands in a certain relation to a high-energy theory.

3. Emergence in EFTs

My Approach:

- Use the (informal) intertheoretic relation between an EFT and its high-energy theory to inform an ontological notion of emergence appropriate for EFTs.

- *Thus*: Emergence (under this view) is not a formal characteristic of theories; but rather an interpretation-dependent characteristic.

3. Emergence in EFTs

Disiderata

(i) Emergence should involve *microphysicalism*: The emergent system should ultimately be composed of microphysical systems that comprise the fundamental system and that obey the fundamental system's laws.

(ii) Emergence should involve *novelty*: The properties of the emergent system should not be deducible from the properties of the fundamental system.

- (i) and (ii) are underwritten in the EFT context by the elimination of degrees of freedom (DOF)...

3. Emergence in EFTs

How the properties of a system described by $\mathcal{L}_{eff}$ emerge from a fundamental system described by $\mathcal{L}$:

(i) *Microphysicalism*: High-energy DOF are integrated out of $\mathcal{L}$, which entails that the DOF of $\mathcal{L}_{eff}$ are exactly the low-energy DOF of $\mathcal{L}$.

(ii) *Novelty*: $\mathcal{L}_{eff}$ is expanded in a local operator expansion. The result is dynamically distinct from $\mathcal{L}$ in the sense of a failure of lawlike deducibility from $\mathcal{L}$ of the properties described by $\mathcal{L}_{eff}$.

4. Other Notions of Emergence

(A) ***New Emergentism.***

- *Claim (Mainwood 2006)*: *Microphysicalism* and *novelty* characterize the "New Emergentism" of Anderson (1972) and Laughlin and Pines (2000).
- *But*: The mechanisms that underwrite New Emergentism are spontaneous symmetry breaking and universality.
- *And*: These mechanisms are typically *not* present in EFTs:
  - Present in EFTs for superfluid ^{3}He-A.
  - Not present in EFTs for quantum Hall liquids.

4. Other Notions of Emergence

(B) *Wilson's (2010) Weak Ontological Emergence.*

- *Claim*: Elimination of DOF plays two roles:

(a) Secures the lawlike deducibility of an emergent entity's behavior from its composing parts (*physicalism*).

(b) Entails that an emergent entity is characterized by different law-governed properties and behavior than those of its composing parts (*non-reductionism*).

- Applicable to EFTs?
- *No*: DOF elimination in an EFT is characterized by:

(a) A *failure* of lawlike deducibility (*novelty*).

(b) The retention, in the EFT, of the low-energy degrees of freedom of the high-energy theory (*microphysicalism*).

4. Other Notions of Emergence

(C) *The Failure of a Limiting Relation.*

- Necessary conditions for the existence of an emergent property described by a theory T' with respect to a more fundamental theory T (Batterman 2000):

(i) There must be a limiting relation between T and T'.

(ii) The limiting relation must fail in the context in which the emergent property is identified; in particular, there must be a *physical singularity* associated with the emergent property.

4. Other Notions of Emergence

(C) *The Failure of a Limiting Relation.*

<u>Example (i)</u>: Properties associated with phase transitions involving spontaneously broken symmetries.
T = statistical mechanical description.
T' = thermodynamical description.
Limiting relation = $N,\ V \to \infty,\ N/V = \text{const.}$

- Limiting relation fails at a critical point/fixed point.
- Physical singularity = divergence in correlation length.
- Emergent properties = properties associated with the phase transition.

4. Other Notions of Emergence

(C) *The Failure of a Limiting Relation.*

<u>Example (ii)</u>: Properties associated with a cutoff-regulated theory.
T = renormalizable continuum theory.
T' = cutoff-regulated theory.
Limiting relation = $\Lambda(s) \to \infty$, $[\textit{bare parameters}] \to \infty$, $[\textit{renormalized parameters}] = [\textit{bare parameters}]/\Lambda(s) = \text{const.}$

- Limiting relation fails at a fixed point (scale independence).
- Physical singularity = divergence in Green's functions.
- Emergent properties = properties associated with system at a fixed point.

(C) *The Failure of a Limiting Relation.*

Example (ii): Properties associated with a cutoff-regulated theory.
T = renormalizable continuum theory.
T' = cutoff-regulated theory.
Limiting relation = $\Lambda(s) \to \infty$, $[bare\ parameters] \to \infty$, $[renormalized\ parameters] = [bare\ parameters]/\Lambda(s) = \text{const}$.

- T = high-energy theory; T' = EFT?
- *No*: Not all EFTs are obtained from renormalizable high-energy theories.
- *Moveover*: T and T' are formally identical in Example (ii), whereas an EFT and its high-energy theory are not.

5. Conclusion

- Emergence in an EFT can be characterized by the elimination of DOF from a high-energy theory.
- This results in an EFT that can be interpreted as describing *novel* entities or properties in the sense of being dynamically independent of, and thus not deducible from, the entities or properties associated with a high-energy theory.
- These novel entities or properties can be said to ultimately be composed of the entities or properties that are constitutive of a high-energy theory (*microphysicalism*), insofar as the DOF exhibited by the former are exactly the low-energy DOF exhibited by the latter.