Condensed Matter Physics and the Nature of Spacetime

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Prospects for modeling spacetime as a phenomenon that emerges in the low-energy limit of a quantum liquid.

1. **EFTs in Condensed Matter Systems**
2. **Superfluid Helium and Quantum Hall Liquids**
3. **Emergence and Emergent Spacetime**
4. **Universality, Dynamical Structure and Structural Realism**

1. EFTs in Condensed Matter Systems

Highly-correlated condensed matter system = **_Non-relativistic_ many-body quantum system that displays macroscopic quantum effects**

- *superfluids*
- *superconductors*
- *Bose-Einstein condensates*
- *quantum Hall liquids*

Effective Field Theory of condensed matter system = **Theory of low-energy dynamics of system: describes states with energy close to zero**

- *bosonic collective modes of ground state*
- *fermionic excitations above ground state ("quasiparticles")*
- *topological defects ("vortices")*

1. EFTs in Condensed Matter Systems

How to Construct a Condensed Matter EFT

Take Low-energy "limit":

- **Expand Lagrangian in small fluctuations in field variables about ground state and integrate out high-energy fluctuations. (^4He)**

OR

- **Linearize the energy about the values where it vanishes and then construct the corresponding Hamiltonian. (^3He-A)**

For fermionic liquids, the type of EFT that results is ultimately based on *topological* properties of the ground state of system, as opposed to its *symmetries*.

2. "Acoustic" Spacetimes and Superfluid ^4He

- liquid consisting of many ^4He atoms, all phases allinged
- model ground state as *single* quantum particle: $\varphi_0 = \rho_0 e^{i\theta}$

$$\mathcal{L}_{4He} = i\varphi^\dagger\partial_t\varphi - \frac{1}{2m}\partial_i\varphi^\dagger\partial_i\varphi + \mu\varphi^\dagger\varphi - \alpha^2(\varphi^\dagger\varphi)^2$$

kinetic energy — *conservation of particle#* — *SSB potential*

Non-relativistic Lagrangian for Superfluid ^4He

Low-energy limit:

- Let $\varphi = \rho e^{i\theta}$, $\rho = \rho_0 + \delta\rho$, $\theta = \theta_0 + \delta\theta$
- Integrate out high-energy fluctuations $\delta\rho$

$$\mathcal{L}_{4He} = \mathcal{L}_0[\rho_0, \theta_0] + \mathcal{L}'_{4He}[\delta\theta]$$

ground state — *low-energy fluctuations above ground state (EFT)*

2. "Acoustic" Spacetimes and Superfluid ^4He

$$\mathcal{L}'_{4He} = \frac{1}{4\alpha^2}(\partial_t\theta + v_i\partial_i\theta)^2 - \frac{\rho_0}{2m}(\partial_i\theta)^2 \qquad i = 1, 2, 3$$

EFT for Superfluid ^4He
To 2nd order in $\delta\theta$:
$v_i = (1/m)\partial_i\theta$

... identical to...

⇕

$$\mathcal{L}'_{4He} = \frac{1}{2}\sqrt{g}\, g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta \qquad \mu, \nu = 0, 1, 2, 4$$

Massless scalar field in curved spacetime!

$$g_{\mu\nu}dx^\mu dx^\nu = \frac{\rho}{cm}\{-c^2dt^2 + \delta_{ij}(dx^i - v^i dt)(dx^j - v^j dt)\} \qquad c^2 \equiv 2\alpha^2\rho/m$$

Can now model black hole physics:

speed of light = speed of low-energy ocillations (*i.e.*, "sound" modes)

Hence: "acoustic" spacetimes and "acoustic" black holes

2. "Acoustic" Spacetimes and Superfluid ^4He

(i) *What is the kinematic background of acoustic spacetimes?*

Option #1: Minkowski spacetime

$$g_{\mu\nu}dx^\mu dx^\nu = (\rho/cm)\{-c^2dt^2 + \delta_{ij}(dx^i - v^i dt)(dx^j - v^j dt)\}$$

$$= \eta_{\mu\nu}dx^\mu dx^\nu + g'_{\mu\nu}dx^\mu dx^\nu$$

acoustic metric ($g_{\mu\nu}dx^\mu dx^\nu$)

background ($\eta_{\mu\nu}dx^\mu dx^\nu$)

low-energy fluctuations ($g'_{\mu\nu}dx^\mu dx^\nu$)

✓ **Option #2: Neo-Newtonian spacetime**

Superfluid ^4He in Neo-Newtonian ST → (*1st order $\delta\theta$*) → Massless scalar field in Minkowski ST

Superfluid ^4He in Neo-Newtonian ST → (*2st order $\delta\theta$*) → Massless scalar field in acoustic ST

2. "Acoustic" Spacetimes and Superfluid ^4He

(ii) *To what extent are "acoustic" spacetimes analogues of GR spacetimes?*

Einstein equations cannot be derived from ^4He EFT. $\Rightarrow$ *Not dynamical analogues!*

Kinematic analogues of GR?

... the features of general relativity that one typically captures in an "analogue model" are the *kinematic* features that have to do with how fields (classical or quantum) are defined on curved spacetime, and the *sine qua non* of any analogue model is the existence of some "effective metric" that captures the notion of the curved spacetimes that arise in general relativity. (Barceló, Liberati, Visser 2005, pg. 10.)

2. "Acoustic" Spacetimes and Superfluid ^4He

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Kinematic analogues of GR?

> The acoustic analogue for black-hole physics accurately reflects half of general relativity -- the kinematics due to the fact that general relativity takes place in a Lorentzian spacetime. The aspect of general relativity that does not carry over to the acoustic model is the dynamics -- the Einstein equations. *Thus the acoustic model provides a very concrete and specific model for separating the kinematic aspects of general relativity from the dynamic aspects.* (Visser 1998, pg. 1790.)

2. "Acoustic" Spacetimes and Superfluid ^4He

(ii) *To what extent are "acoustic" spacetimes analogues of GR spacetimes?*

(Einstein equations cannot be derived from ^4He EFT.) $\Rightarrow$ (*Not dynamical analogues!*)

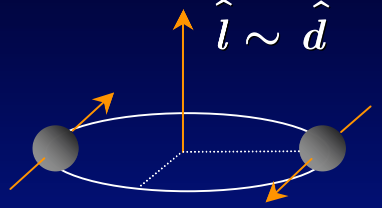
Kinematic analogues of GR?

- If kinematics of GR = Minkowski ST, then *No!*
- Kinematics of GR = *?*

Some features that one normally thinks of as intrinsically aspects of gravity, both at the classical and semiclassical levels (such as horizons and Hawking radiation), can in the context of acoustic manifolds be instead seen to be rather generic features of curved spacetimes and quantum field theory in curved spacetimes, that have nothing to do with gravity *per se*. (Barceló, Liberati, Sonego, Visser 2004, pg. 2.)

2. Standard Model and Superfluid ^3He-A

(Volovik 2003)



- Liquid of many ^3He Cooper Pairs, all phases alligned
- Degrees of freedom: $S_z = 0, \pm 1$; $l_z = 0, \pm 1$
- A-phase: no $S_z = 0$ substates, $\hat{d} \,||\, \hat{l}$,

$$H_{3He\text{-}A} = \chi^{\dagger}{}_{\alpha\beta}\{(\varepsilon - \mu)\sigma_3 + \vec{V}_{\alpha\beta}(\hat{l}, \hat{d}, \vec{k})\}\chi_{\alpha\beta}$$

$E(\vec{k}) = 0$, for 2 values of $\vec{k}$

Low-energy limit

- Expand $E(\vec{k})$ to 2nd order about zero points:

$$E^2(\vec{k}) \approx g_{ij}(k_i - qA_i)(k_j - qA_j) \qquad g_{ij} \sim l_i l_j\,,\ A_i \sim l_i$$

- Effective Lagrangian:

$$\mathcal{L}'_{3He\text{-}A} = \overline{\Psi} g_{\mu\nu}\gamma^{\nu}(\partial_\mu - qA_\mu)\Psi$$

Potential field A_μ interacting with matter field Ψ in curved spacetime

$g_{\mu\nu} \sim (l_i\,,\ v_i)$, $A_0 \sim l_i v_i$

2. Standard Model and Superfluid ^3He-A

- "Induced QED" (Zeldovich 1967): expand to 2nd order in fluctuations in A_μ

$$\mathcal{L}'_{3He\text{-}A} = \overline{\Psi} g_{\mu\nu}\gamma^\nu(\partial_\mu - qA_\mu)\Psi + \boxed{\frac{1}{4\kappa^2}\sqrt{-g}\, g^{\mu\nu}F_{\mu\alpha}F_{\nu\beta}}$$

(3+1)-dim QED in curved spacetime

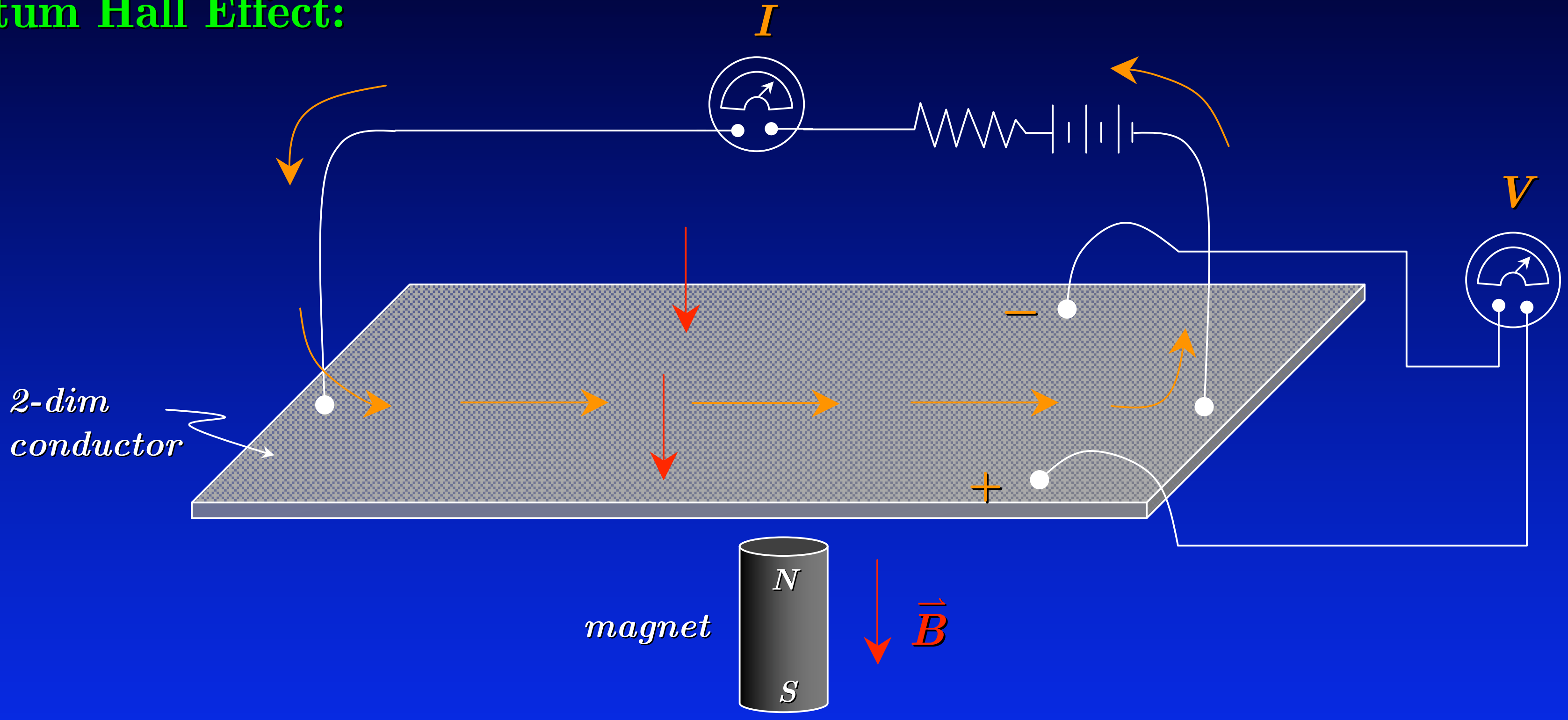
- In-principle extension to $SU(n)$ gauge fields $\Rightarrow$ Standard Model
- Similar treatment of effective metric ("induced gravity" Sakharov 1967) fails to reproduce Einstein-Hilbert term

Interpretation

ground state of ^3He-A	$\Leftrightarrow$	spacetime ($g_{\mu\nu}$)
low-energy fluctuations (quasiparticles, collective modes)	$\Leftrightarrow$	matter fields, potential fields (Ψ, A_μ)
"induced" vaccum corrections to interactions between Ψ and A_μ	$\Leftrightarrow$	gauge fields ($F_{\mu\nu}$)

2. Twistors and Quantum Hall Liquids

Quantum Hall Effect:



For strong magnetic field:

$$I/V = (1/R_H) = \nu \times (e^2/h), \qquad \nu = \text{const.}$$

- **Integer Quantum Hall Effect (IQHE):** $\nu = \text{integer}$
- **Fractional Quantum Hall Effect (FQHE):** $\nu = 1/p, \;\; p = \text{odd integer}$

2. Twistors and Quantum Hall Liquids

$$\mathcal{L}_{QHE} = \psi^\dagger(i\partial_t - eA_0{}^{ext}\psi) - \frac{1}{2m}\psi^\dagger(-i\partial_i + eA_i{}^{ext})^2\psi + V[\psi^\dagger\psi] + \mathcal{L}_{EM}$$

2-dim Electrons (ψ) in presence of mag. ($A_0{}^{ext}$) and elec. ($A_i{}^{ext}$) fields

$\Updownarrow$

... identical to...

$$\mathcal{L}_{QHE} = \varphi^\dagger[i\partial_t - e(a_0 + A_0{}^{ext})\varphi] - \frac{1}{2m}\varphi^\dagger[-i\partial_i + e(a_i + A_i{}^{ext})]^2\varphi + V[\varphi^\dagger\varphi] + \mathcal{L}_{CS}$$

2-dim Bosons (φ) in presence of mag., elec., and Chern-Simons (a_0, a_i) fields

$\mathcal{L}_{CS} = (p/4\pi)\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$
p = odd integer

- CS field = external mag. field for values of $\nu = 1/p$.
- QH Condensate particle content: Electrically charged bosons.
- Excitations: Vortices with fractional charges and "anyonic" statistics.
- Incompressible: *Low energy limit of QH liquid cannot be taken.*
- *Low energy EFT of 1-dim edge states does exist (Wenn 1990):*

$$\mathcal{L}_{edge} = i\psi^\dagger(\partial_t - v\partial_x)\psi$$

massless fermion field in (1+1)-dim Mink. spacetime

What about (3+1)-dim?

2. Twistors and Quantum Hall Liquids

- **2001 - Zhang and Hu: 4-dim QHE**
  - Replace 2-spatial-dim quantum liquid with 4-spatial-dim quantum liquid.
  - Low-energy EFT of 3-spatial-dim edge now describes massless fields in (3+1)-dim Minkowski spacetime.

- **2002 - Sparling: 4-dim QHE and twistors**
  - Edge of 4-dim QH liquid = region of *twistor space.*

- **1960's - Penrose and Twistor Theory:**
  - Twistor $\approx$ light ray; twistor space $\approx$ space of light rays.
  - Minkowski spacetime can be reconstructed from twistors.
  - *Goal*: To reconstruct GR and QFT using twistors... Not yet *completely* doable.

Interpretation

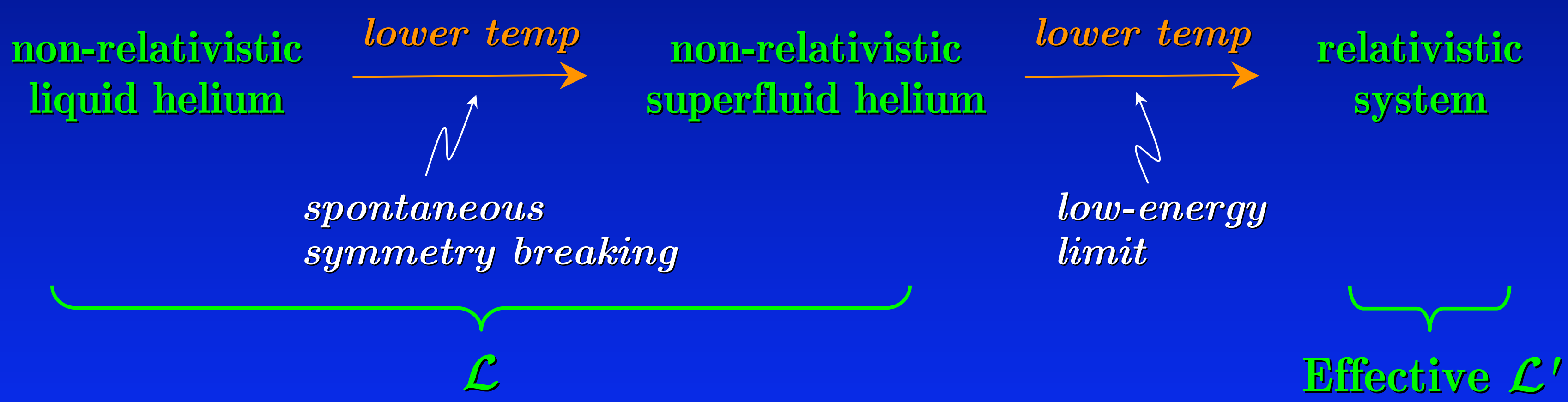
1. Twistors emerge in the low-energy limit of the edge of a 4-dim QH liquid.
2. Spacetime is derivative of twistors.

3. Emergence...

Claim: **Novel phenomena (fields, particles, symmetries, spacetime, *etc*.) *emerge* in low-energy limit of certain condensed matter systems**

Emergence in the low-energy limit

(1) ***Distinct from emergence via symmetry breaking.***



(2) ***Epistemological Emergence***

- **Unpredictability**
- **Irreducibility (and/or unexplainability)**
- **Causal efficacy**

3. … and Emergent Spacetime

Spacetime as emergent…

(a) *… in superfluid ^4He*

- *Motive*: Spacetime as (some aspect of) solutions to Einstein equations in GR.
- *Prospects*: Limited. ^4He EFT lacks GR dynamics/kinematics.

(b) *… in superfluid ^3He-A*

- *Motive*: Spacetime as ground state for QFTs of matter, gauge, and metric fields.
- *Prospects*: Still limited. ^3He-A EFT does not fully recover GR.

(b) *… in 4-dim QH liquids*

- *Motive*: Spacetime as derivative of twistors.
- *Prospects*: Limited. Twistor theory cannot fully recover QFT and GR.

Research programme:

Determine appropriate condensed matter system that produces relevant matter, gauge and metric fields in low-energy limit. (Volovik 2003, Wen 2004.)

4. Universality, Dynamical Structure and Structural Realism

Why does ^{3}He-A reproduce the Standard Model?

- *Universal property* = property stable under perturbations

- *Universality class* = class of systems/states characterized by common universal properties

- Universality class of the ground state of a system determines it's low-energy dynamics (*i.e.*, the corresponding EFT).

- Universal properties of ground state are "generic" properties of EFT:
  - *how system behaves at a phase transition*
  - *whether theory is renormalizable*
  - *symmetries of low-energy fluctuations (i.e., symmetries of EFT)*

4. Universality, Dynamical Structure and Structural Realism

Why does ^{3}He-A reproduce the Standard Model?

- **Universality classes of fermionic ground states are characterized by momentum space topology.**

  - *stable regions in k-space where quasiparticle energies $\rightarrow$ 0*

- **^{3}*He-A* and the Standard Model have ground states characterized by the *same* momentum space topology.**

  - *stable point defects ("Fermi points")*

Thus: **^{3}*He-A* and the Standard Model possess the same *universal properties*.**

4. Universality, Dynamical Structure and Structural Realism

(Same k-space topology) $\Rightarrow$ (Same low-energy dynamics)

Irrespective of microscopic details:

- Standard Model
- ^3He-A
- any condensed matter system with "Fermi points"

} Same low-energy dynamical structure

(Epistemological) Structural Realism:

1. The phenomena of experience are low-energy emergent.
2. Theories of such phenomena are EFTs of a "fundamental" theory T.
3. As EFTs, such theories only provide us with knowledge of the low-energy dynamical structure of T (*i.e.*, the universality class of which T is a member).

4. Universality, Dynamical Structure and Structural Realism

Structural Realist interpretation of spacetime:

1. The spatiotemporal aspects of the phenomena of experience are low-energy emergent.

   - *spacetime symmetries of QFT and GR*

2. The spatiotemporal aspects of the fundamental theory are structural.

   - *Ontologically, spacetime is the ground state of the fundamental condensate, which can only be known structurally.*

Qualification: Universality class that best describes spacetime structure still unknown.