Condensed Matter Physics, Emergent Spacetime, and Structural Realism

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Prospects for modeling spacetime as a phenomenon that emerges in the low-energy limit of a quantum liquid.

1. EFTs in Condensed Matter Systems
2. Superfluid ${}^4He$ and Superfluid ${}^3He\text{-}A$
3. Low-Energy Emergence
4. Universality, Dynamical Structure and Structural Realism

1. EFTs in Condensed Matter Systems

Highly-correlated condensed matter system = *Non-relativistic* many-body quantum system that displays macroscopic quantum effects

- *superfluids*
- *superconductors*
- *Bose-Einstein condensates*
- *quantum Hall liquids*

Effective Field Theory of condensed matter system = Theory of low-energy dynamics of system: describes states with energy close to zero

- *bosonic collective modes of ground state*
- *fermionic excitations above ground state ("quasiparticles")*
- *topological defects ("vortices")*

1. EFTs in Condensed Matter Systems

How to Construct a Condensed Matter EFT

Take Low-energy "limit":

- **Expand Lagrangian in small fluctuations in field variables about ground state and integrate out high-energy fluctuations. (^4He)**

OR

- **Linearize the energy about the values where it vanishes and then construct the corresponding Hamiltonian. (^3He-A)**

For fermionic liquids, the type of EFT that results is ultimately based on *topological* properties of the ground state of system, as opposed to its *symmetries*.

2a. "Acoustic" Spacetimes and Superfluid ^4He

- liquid consisting of many ^4He atoms, all phases alligned
- model ground state as *single* quantum particle: $\varphi_0 = \rho_0 e^{i\theta}$

$$\mathcal{L}_{4He} = i\varphi^\dagger\partial_t\varphi - \frac{1}{2m}\partial_i\varphi^\dagger\partial_i\varphi + \mu\varphi^\dagger\varphi - \alpha^2(\varphi^\dagger\varphi)^2$$

kinetic energy

conservation of particle#

SSB potential

Non-relativistic Lagrangian for Superfluid ^4He

Low-energy limit:

- Let $\varphi = \rho e^{i\theta}$, $\rho = \rho_0 + \delta\rho$, $\theta = \theta_0 + \delta\theta$
- Integrate out high-energy fluctuations $\delta\rho$

$$\mathcal{L}_{4He} = \mathcal{L}_0[\rho_0, \theta_0] + \mathcal{L}'_{4He}[\delta\theta]$$

ground state

low-energy fluctuations above ground state (EFT)

2a. "Acoustic" Spacetimes and Superfluid ^4He

$$\mathcal{L}'_{4He} = \frac{1}{4\alpha^2}\left(\partial_t\theta + v_i\partial_i\theta\right)^2 - \frac{\rho_0}{2m}\left(\partial_i\theta\right)^2 \qquad i = 1, 2, 3$$

EFT for Superfluid ^4He

To 2nd order in $\delta\theta$:
$v_i = (1/m)\partial_i\theta$

... identical to...

⇕

$$\mathcal{L}'_{4He} = \frac{1}{2}\sqrt{g}\; g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta \qquad \mu, \nu = 0, 1, 2, 4$$

Massless scalar field
in curved spacetime!

$$g_{\mu\nu}dx^\mu dx^\nu = \frac{\rho}{cm}\left\{-c^2dt^2 + \delta_{ij}\left(dx^i - v^i dt\right)\left(dx^j - v^j dt\right)\right\} \qquad c^2 \equiv 2\alpha^2\rho/m$$

Can now model black hole physics:

speed of light = speed of low-energy ocillations (*i.e.*, "sound" modes)

Hence: "acoustic" spacetimes and "acoustic" black holes

2a. "Acoustic" Spacetimes and Superfluid ^4He

(i) *What is the background structure of acoustic spacetimes?*

Option #1: Minkowski spacetime

$$g_{\mu\nu}dx^\mu dx^\nu = (\rho/cm)\{-c^2dt^2 + \delta_{ij}(dx^i - v^i dt)(dx^j - v^j dt)\}$$

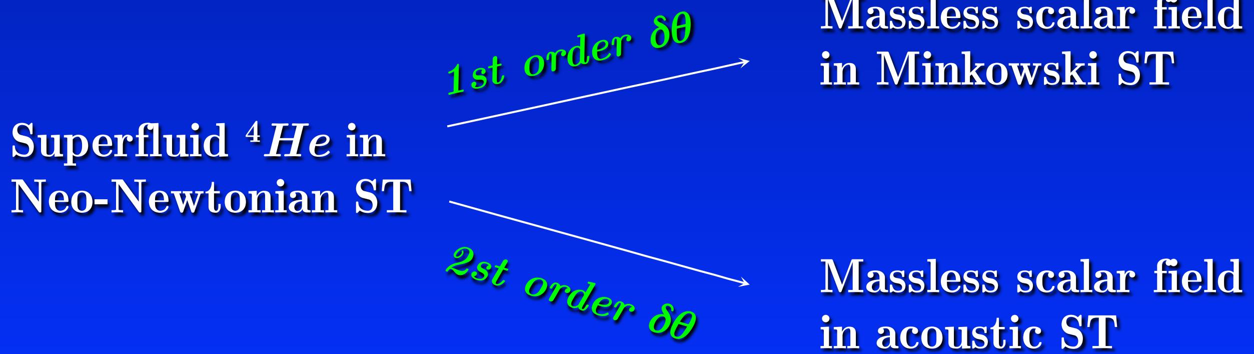
$$= \eta_{\mu\nu}dx^\mu dx^\nu + g'_{\mu\nu}dx^\mu dx^\nu$$

acoustic metric

background

low-energy fluctuations

✓ Option #2: Neo-Newtonian spacetime



Superfluid ^4He in Neo-Newtonian ST

1st order $\delta\theta$

Massless scalar field in Minkowski ST

2st order $\delta\theta$

Massless scalar field in acoustic ST

2a. "Acoustic" Spacetimes and Superfluid ^4He

(ii) *To what extent are "acoustic" spacetimes analogues of GR spacetimes?*

Einstein equations cannot be derived from ^4He EFT. ⟹ *Not dynamical analogues!*

"Kinematic" analogues of GR?

> … the features of general relativity that one typically captures in an "analogue model" are the *kinematic* features that have to do with how fields (classical or quantum) are defined on curved spacetime, and the *sine qua non* of any analogue model is the existence of some "effective metric" that captures the notion of the curved spacetimes that arise in general relativity. (Barceló, Liberati, Visser 2005, pg. 10.)

2a. "Acoustic" Spacetimes and Superfluid ^4He

(ii) *To what extent are "acoustic" spacetimes analogues of GR spacetimes?*

$\left(\text{Einstein equations cannot be derived from } ^4He \text{ EFT.}\right) \Rightarrow \left(\textit{Not dynamical analogues!}\right)$

"Kinematic" analogues of GR?

> The acoustic analogue for black-hole physics accurately reflects half of general relativity -- the kinematics due to the fact that general relativity takes place in a Lorentzian spacetime. The aspect of general relativity that does not carry over to the acoustic model is the dynamics -- the Einstein equations. *Thus the acoustic model provides a very concrete and specific model for separating the kinematic aspects of general relativity from the dynamic aspects.* (Visser 1998, pg. 1790.)

2a. "Acoustic" Spacetimes and Superfluid ${}^4He$

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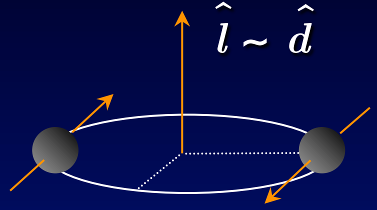
"Kinematic" analogues of GR?

- If kinematics of GR = Minkowski ST, then *No!*
- Kinematics of GR = *?*

> Some features that one normally thinks of as intrinsically aspects of gravity, both at the classical and semiclassical levels (such as horizons and Hawking radiation), can in the context of acoustic manifolds be instead seen to be rather generic features of curved spacetimes and quantum field theory in curved spacetimes, that have nothing to do with gravity *per se*. (Barceló, Liberati, Sonego, Visser 2004, pg. 2.)

2b. Standard Model and Superfluid ^3He-A

(Volovik 2003)



- Liquid of many ^3He Cooper Pairs, all phases alligned
- Degrees of freedom: $S_z = 0, \pm 1;\ l_z = 0, \pm 1$
- A-phase: no $S_z = 0$ substates, $\hat{d} \,||\, \hat{l}$,

$$H_{3He\text{-}A} = \chi^{\dagger}{}_{\alpha\beta}\{(\varepsilon - \mu)\sigma_3 + \vec{V}_{\alpha\beta}(\hat{l}, \hat{d}, \vec{k})\}\chi_{\alpha\beta}$$

$$E(\vec{k}) = 0, \quad \text{for 2 values of } \vec{k}$$

Low-energy limit

- Expand $E(\vec{k})$ to 2nd order about zero points:

$$E^2(\vec{k}) \approx g_{ij}(k_i - qA_i)(k_j - qA_j) \qquad\qquad g_{ij} \sim l_i l_j\,,\ A_i \sim l_i$$

- Effective Lagrangian:

$$\mathcal{L}'_{3He\text{-}A} = \bar{\Psi} g_{\mu\nu}\gamma^{\nu}(\partial_{\mu} - qA_{\mu})\Psi$$

Potential field A_μ
interacting with matter
field Ψ in curved spacetime

$$g_{\mu\nu} \sim (l_i\,,\ v_i),\ A_0 \sim l_i v_i$$

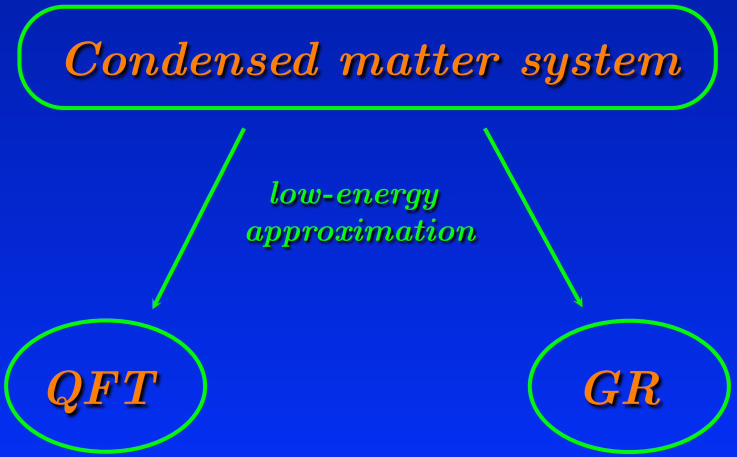
2b. Standard Model and Superfluid $^{3}He\text{-}A$

- "Induced QED" (Zeldovich 1967): expand to 2nd order in fluctuations in A_μ

$$\mathcal{L}'_{3He\text{-}A} = \bar{\Psi} g_{\mu\nu}\gamma^{\nu}(\partial_\mu - qA_\mu)\Psi + \boxed{\frac{1}{4\kappa^2}\sqrt{-g}\; g^{\mu\nu}F_{\mu\alpha}F_{\nu\beta}}$$

(3+1)-dim QED in curved spacetime

- In-principle extension to $SU(n)$ gauge fields ⇒ Standard Model
- Similar treatment of effective metric ("induced gravity" Sakharov 1967) fails to reproduce Einstein-Hilbert term



Condensed matter system

low-energy approximation

QFT

GR

2c. Condensed Matter Approach to Quantum Gravity

Research programme:

- Determine appropriate condensed matter system that produces relevant matter, gauge and metric fields in low-energy limit. (Volovik 2003, Wen 2004.)
- *Background-dependent* approach to GR and Standard Model

Literal Interpretation

Gal-invariant rest frame of condensate	⇔	background structure
low-energy fluctuations (quasiparticles, collective modes)	⇔	matter/potential/metric fields (Ψ, A_μ, $g_{\mu\nu}$)
"induced" vaccum corrections to interactions between Ψ and A_μ	⇔	gauge fields ($F_{\mu\nu}$)

2c. Condensed Matter Approach to Quantum Gravity

Relationalist Option (just condensate)

(1) background structure = high-energy properties of condensate
(2) physical fields = low-energy fluctuations
(3) relativistic ST structure = properties of low-energy fluctuations

Substantivalist Options (condensate vs spacetime)

A. "Conservative"

(1') background structure = properties of substantival Neo-Newt ST
(2), (3)

B. "Intrepid"

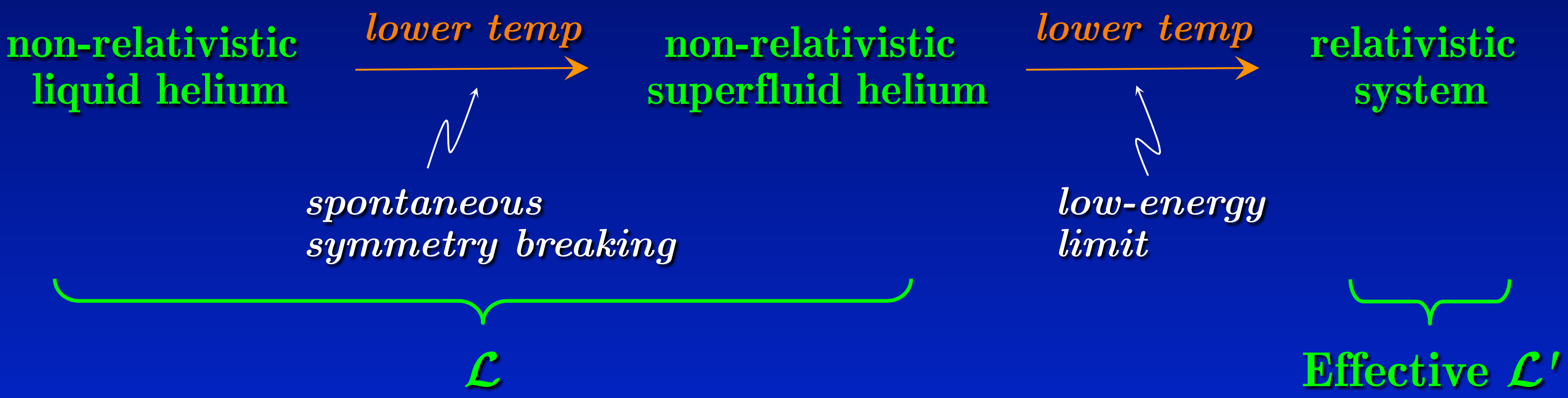
(1'), (2)
(3') relativistic ST structure = properties of low-energy "*emergent*" substantival ST

3. Emergence

Claim: Novel phenomena (fields, particles, symmetries, spacetime, *etc.*) *emerge* in low-energy limit of certain condensed matter systems

Emergence in the low-energy limit

(1) *Distinct from emergence via symmetry breaking.*



non-relativistic liquid helium

lower temp

non-relativistic superfluid helium

lower temp

relativistic system

spontaneous symmetry breaking

low-energy limit

$\mathcal{L}$

Effective $\mathcal{L}'$

(2) *Epistemological Emergence*

- Unpredictability
- Irreducibility
- Unexplainability

4. Universality, Dynamical Structure and Structural Realism

Why does ^{3}He-A reproduce the Standard Model?

- ^{3}*He-A* and Standard Model (sector above electroweak symmetry breaking) belong to same *universality class.*

- *Universality class* = characterized by common low-energy EFT

- Well-defined in Renormalization Group (RG) Theory:
  universality class = fixed point of RG flow

- Common "universal properties" = "generic" properties of EFT:
  - *decay behavior of correlation functions*
  - *gapless energy spectrum*
  - *symmetries of low-energy fluctuations (i.e., symmetries of EFT)*

4. Universality, Dynamical Structure and Structural Realism

- **Universality classes of fermionic ground states are characterized by momentum space topology.**

    - stable regions in k-space where quasiparticle energies → 0

- **^{3}*He-A* and the Standard Model have ground states characterized by the *same* momentum space topology.**

    - stable point defects ("Fermi points")

Same *k*-space topology ⇒ Same low-energy dynamics

Irrespective of microscopic details:

- Standard Model
- ^{3}*He-A*
- any condensed matter system with "Fermi points"

} Same low-energy dynamical structure

4. Universality, Dynamical Structure and Structural Realism

(Epistemological) Structural Realism:

1. **The phenomena of experience are low-energy emergent.**
2. **Theories of such phenomena are EFTs of a "fundamental" theory T.**
3. **As EFTs, such theories only provide us with knowledge of the low-energy dynamical structure of T (*i.e.*, the universality class of which T is a member).**

4. Universality, Dynamical Structure and Structural Realism

Structural Realist interpretation of spacetime:

1. The spatiotemporal aspects of the phenomena of experience are low-energy emergent.

   - *These are the spatiotemporal aspects of QFT and GR.*

2. The spatiotemporal aspects of the fundamental condensate are structural.

   - *These are the spatiotemporal properties of the universality class to which the fundamental condensate belongs.*

Qualification: Universality class that best describes spacetime structure still unknown.