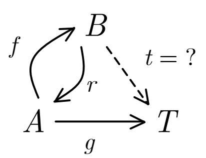
Homework #10. Due: Thurs 11/30

1. Consruct a proof for Proposition 1*: If a map $A \xrightarrow{f} B$ has a *retraction* r, then for any object T and any map $A \xrightarrow{g} T$, there exists a map $B \xrightarrow{t} T$ for which $t \circ f = g$.



$r \circ f = 1_A$

Hint: We're given $r \circ f = 1_A$. Now determine an appropriate form for t (how can you get from B to T in the diagram to the left). Then show that this form for t satisfies $t \circ f = g$. To do this, start with $t \circ f$ on the left and then try to derive g on the right, appealing at each line to only what you're given and/or the rules of category theory. (See the proof of Prop. 1 in the lecture notes for help.) In outline, your proof should look like the following:

$t \circ f = \ldots$ <*justification*>

⋮

$= g$ <*justification*>

2. Construct a proof for Proposition 2*: If a map $A \xrightarrow{f} B$ has a *section* s, then for any object T and any maps $B \xrightarrow{t_1} T$, $B \xrightarrow{t_2} T$, if $t_1 \circ f = t_2 \circ f$, then $t_1 = t_2$.

Hint: We're given two items:
(1) $f \circ s = 1_B$
(2) $t_1 \circ f = t_2 \circ f$
Now show that $t_1 = t_2$. Start with t_1 on the left and try to derive t_2 on the right, using only what we're given and/or the rules of category theory. (See the proof of Prop. 2 in the lecture notes for help.) In outline, your proof should look like the following:

$t_1 = \ldots$ <*justification*>

⋮

$= t_2$ <*justification*>