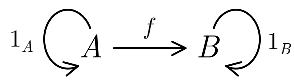
Homework #9. Due: Thurs 11/16

1. Consider a category $\mathcal{C}$ with just two objects A, B, and a single map $A \xrightarrow{f} B$, in addition to the identity maps 1_A, 1_B.



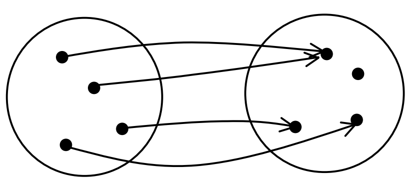
(a) Is f a ***monic***? In other words, is it the case that for *any* maps h, k in $\mathcal{C}$, if $f \circ h = f \circ k$, then $h = k$? If so, what are they? (*Hint*: there are only three maps in $\mathcal{C}$ to choose from!)

(b) Does f have a ***retraction***? In other words, is there a map r in $\mathcal{C}$ such that $r \circ f = 1_A$? If so, what is it?

(c) Is f an ***epic***? In other words, is it the case that for *any* maps h, k in $\mathcal{C}$, if $h \circ f = k \circ f$, then $h = k$? If so, what are they?

(d) Does f have a ***section***? In other words, is there a map s in $\mathcal{C}$ such that $f \circ s = 1_B$? If so, what is it?

2. Monics are also called ***injective*** maps. Epics are also called ***surjective*** maps. In the category $\mathcal{S}$ of sets, an injective map is "into" -- it takes each element in its domain to exactly one element in its codomain. In $\mathcal{S}$, a surjective map is "onto" -- it *exhausts* its codomain in so far as each element in its codomain is assigned at least one element in its domain. Here is an example in $\mathcal{S}$ of a map that is *neither* injective *nor* surjective:



Not injective: There are two elements in the domain that get mapped to the same element in the codomain.
Not surjective: There's an element in the codomain that is not assigned at least one element in the domain.

(a) Give an example (*i.e.*, draw a diagram like the one above) of a map that is *surjective* but not *injective.*

(b) Give an example of a map that is *injective* but not *surjective.*

(c) Give an example of a map that is both *surjective* and *injective.* What are such maps called?