Homework #3. Due: Thurs 9/28

1. In class we discussed the method that Newton and Leibniz introduced for calculating the slope of a tangent to a point on a curve. As a particular example, we chose the curve given by $f(x) = x^2$. The result can be thought of as another function $f'(x)$ that we called the *derivative* of $f(x)$. It was given by $f'(x) = 2x$. Let's now do the same thing for a different curve; namely, the curve $g(x) = 2x^3$.

   (a) Suppose the coordinates of a point P on the curve $g(x) = 2x^3$ are given by the pair of numbers $(x, 2x^3)$, where x is P's "run" coordinate" and $2x^3$ is P's "rise" coordinate. Suppose Q is a point that is "infinitesimally close" to P. If Q's "run" coordinate is given by $x + \delta x$, where δx is an "infinitely small" quantity, what is Q's "rise" coordinate?

   (b) Recall that the slope of a line connecting two points is the difference in their rise coordinates divided by the difference in their run coordinates. What is the slope of a line connecting the points P and Q in (a). Simplify your expression as much as you can, assuming *only* that δx can be treated as a number.

   (c) Now assume that δx can be treated as an infinitely small quantity and can be ignored in your simplified expression in (b). What is the result?

2. Using the concept of a limit, the derivative $f'(x)$ of a function $f(x)$ can be defined in such a way that no reference to "infinitely small" quantities occurs. But $f'(x)$ is normally written as df/dx, and interpreted as a "very small" change in f divided by a very small change in x. How are these "very small" changes any different from the infinitesimals that appeared in the pre-limit concept of the derivative?

3. Determine the derivatives of the following functions. (You may need to refer to the four properties of the derivative in the lecture notes.)

   (a) $f(x) = 10x^3$
   (b) $f(x) = 10x^3 + x^2$
   (c) $f(x) = (2x^3)(10x^3 + x^2)$
   (d) $f(g) = 4g + 2$, where $g(x) = 10x^3$

4. The *indefinite integral* (or *anti-derivative*) $A(x)$ of a function $f(x)$ is that function, if it exists, whose derivative is the function $f(x)$. Symbolically, $A(x) = \int f(x)dx$ = (*the function whose derivative is* $f(x)$). Determine the following indefinite integrals.

   (a) $\int (30x^2 + 2x)dx$
   (b) $\int (20x^6 + 2x^5)dx$

5. The squiggly "S" symbol in the indefinite integral is meant to represent an *infinite sum*; namely, the infinite sum of the areas of an infinite number of *very thin* rectangles, each with *very small* base given by dx and height given by the value of $f(x)$. (This value depends on the specific function $f(x)$; *i.e.*, the specific shape of the curve, the area under which we're attempting to calculate.) We can give a very precise definition of this infinite sum that relates it to the concept of a limit. How are the very thin rectangles with very small bases dx any different from the infinitesimals that appeared in the pre-limit concept of the indefinite integral? In other words, how does defining the indefinite integral in terms of a certain limit allow us to avoid the conceptual problems associated with infinitesimals?