SETS

A set is a collection of distinct elements

B = {France, Portugal, Andorra} = countries bordering mainland Spain.

S = {basketball, soccer} = sports that I like.

Natural numbers: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ → Some sources: no zero.

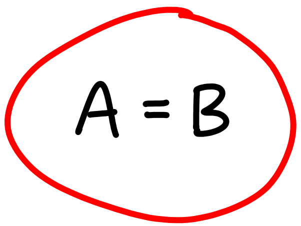
Integers: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Positive integers: $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$

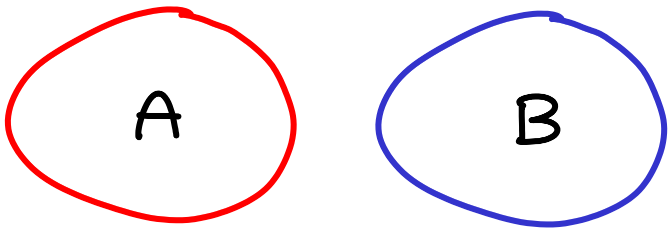
Rational numbers: $\mathbb{Q}$ e.g., $\frac{1}{3}, \frac{5}{7}, \frac{13}{2}, 8$

Irrational numbers e.g., $\pi, e, \sqrt{2}$

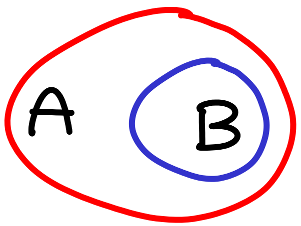
Real numbers: $\mathbb{R}$ all rational & irrational



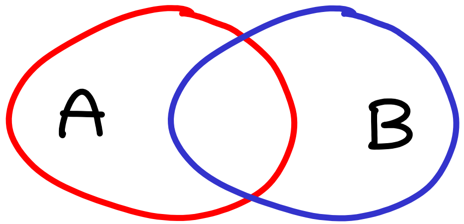
equal sets



disjoint sets



B subset of A



general picture

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If S has n elements, number of subsets of $S = 2^n$.

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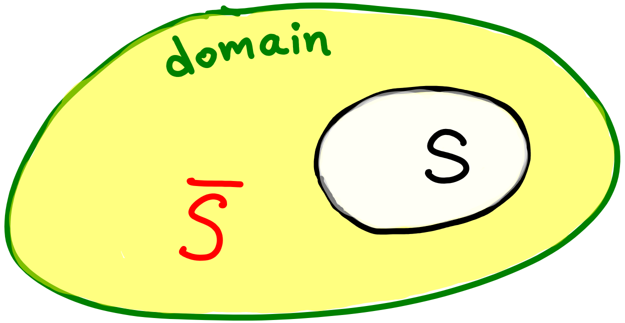

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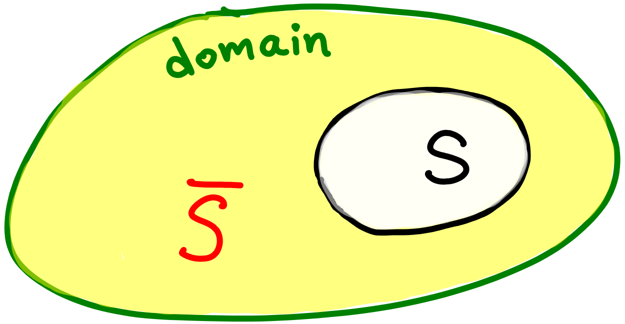

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e.g., domain = $\mathbb{Z}^+$ S = powers of 2, $\overline{S} = \{3, 5, 6, 7, 9, \ldots\}$

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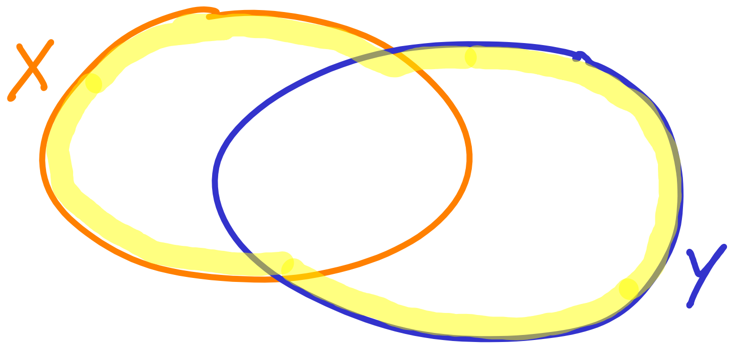

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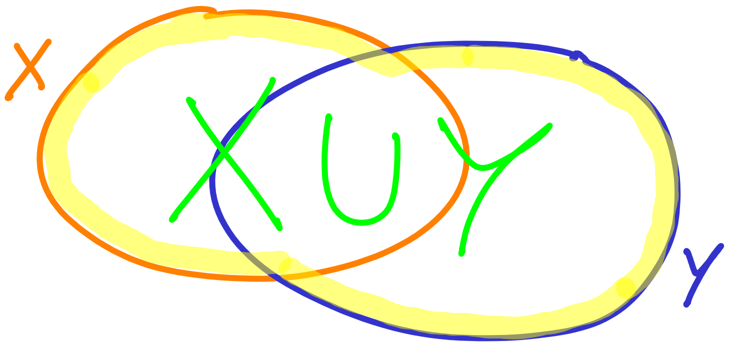

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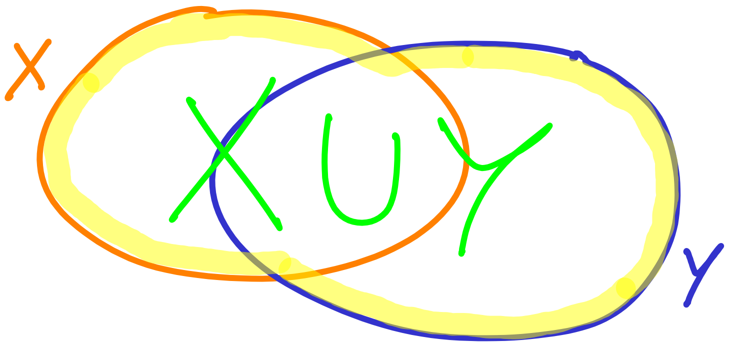

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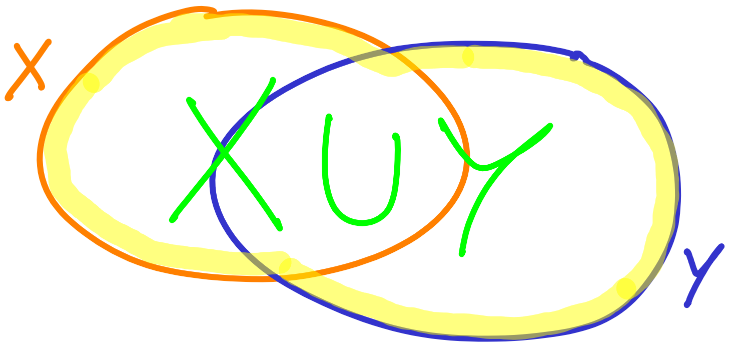
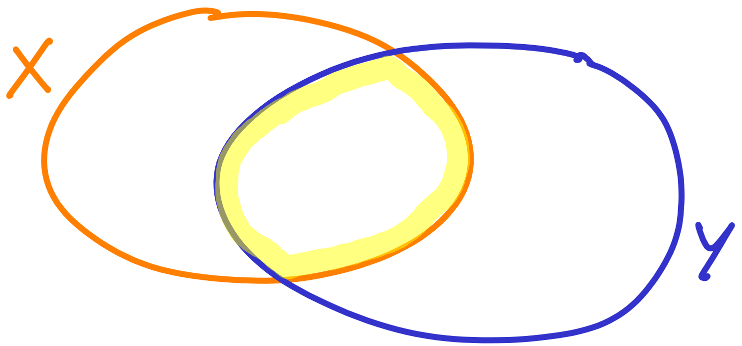


Intersection of sets: include common elements.



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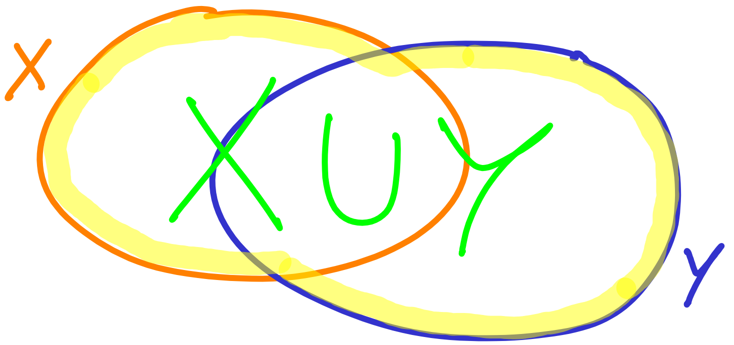

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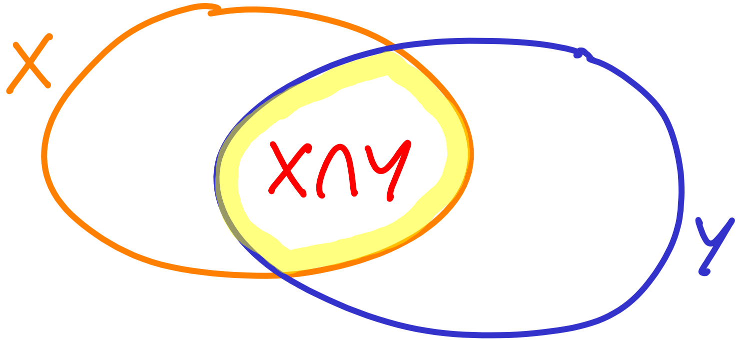

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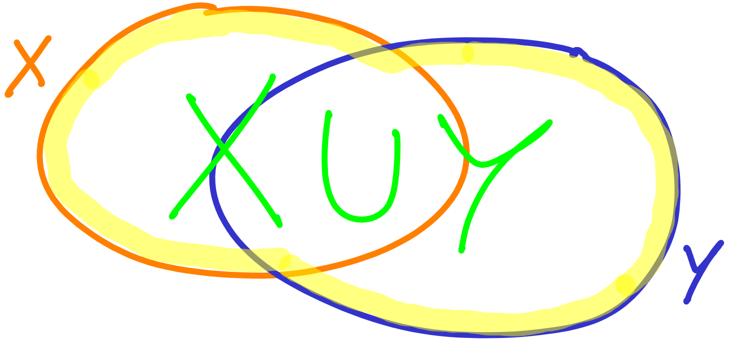

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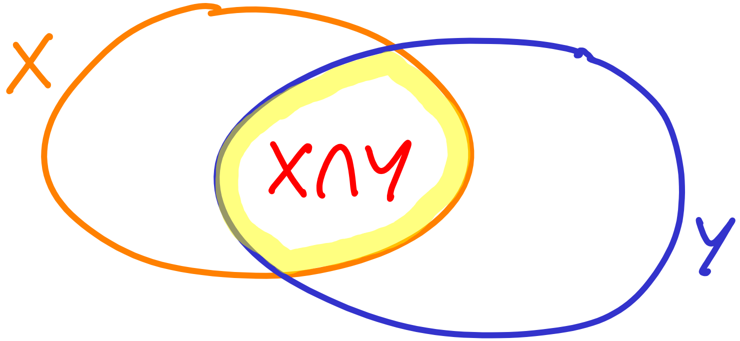
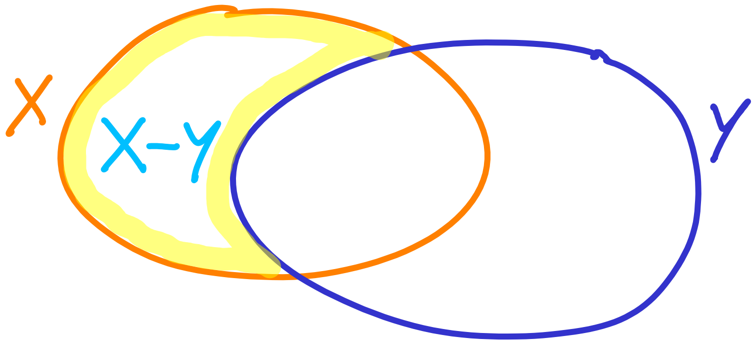


Set difference: $X-Y$

$p \in X-Y$ iff $p \in X$ AND $p \notin Y$



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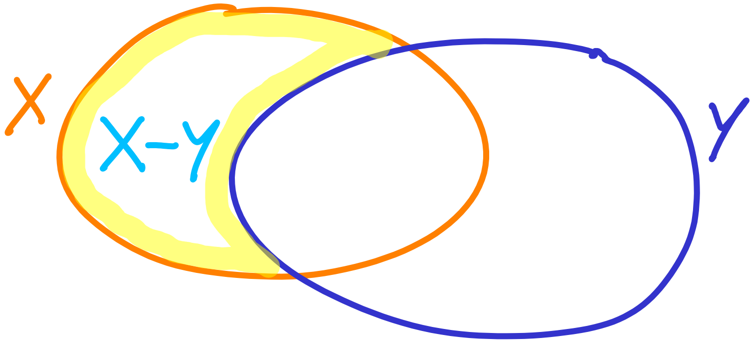
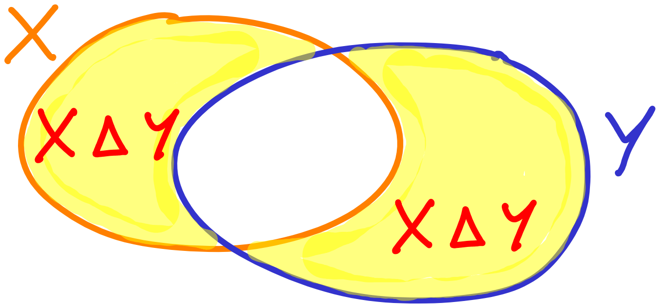


Symmetric difference: $X \Delta Y = (X - Y) \cup (Y - X)$



Set builder notation : use when description of set isn't "basic"

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$$\{5, 14, 19, 23, 28, 32, 37, 41, 46, 50, 55, 64, 69, 73, 78 \ldots\}$$

$S = ?$

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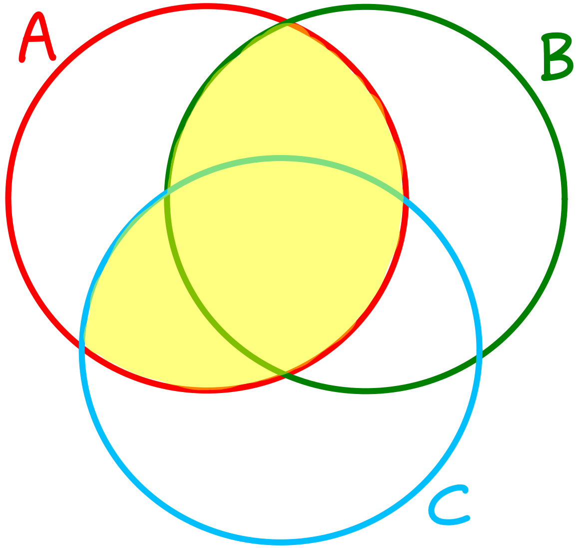
$$S = \{x \in \mathbb{N} \mid \text{sum of digits in } x \text{ is a multiple of } 5\}$$

implied: "all x"

"such that" or "for which"

Distributive Laws for sets:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



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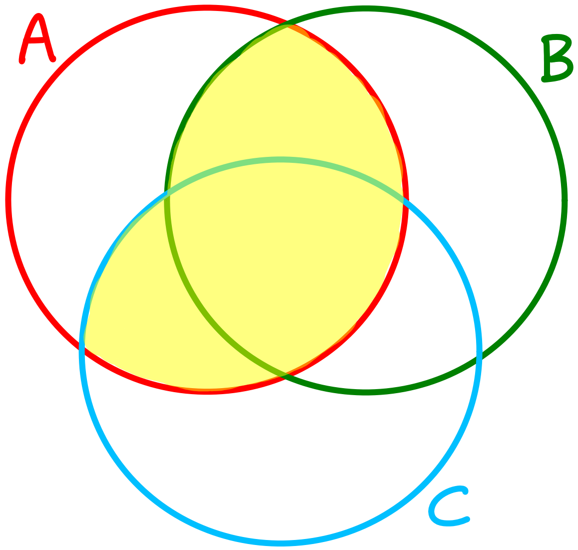

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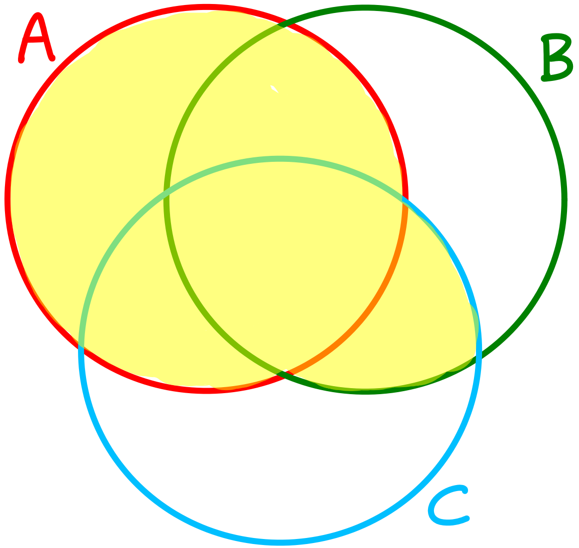
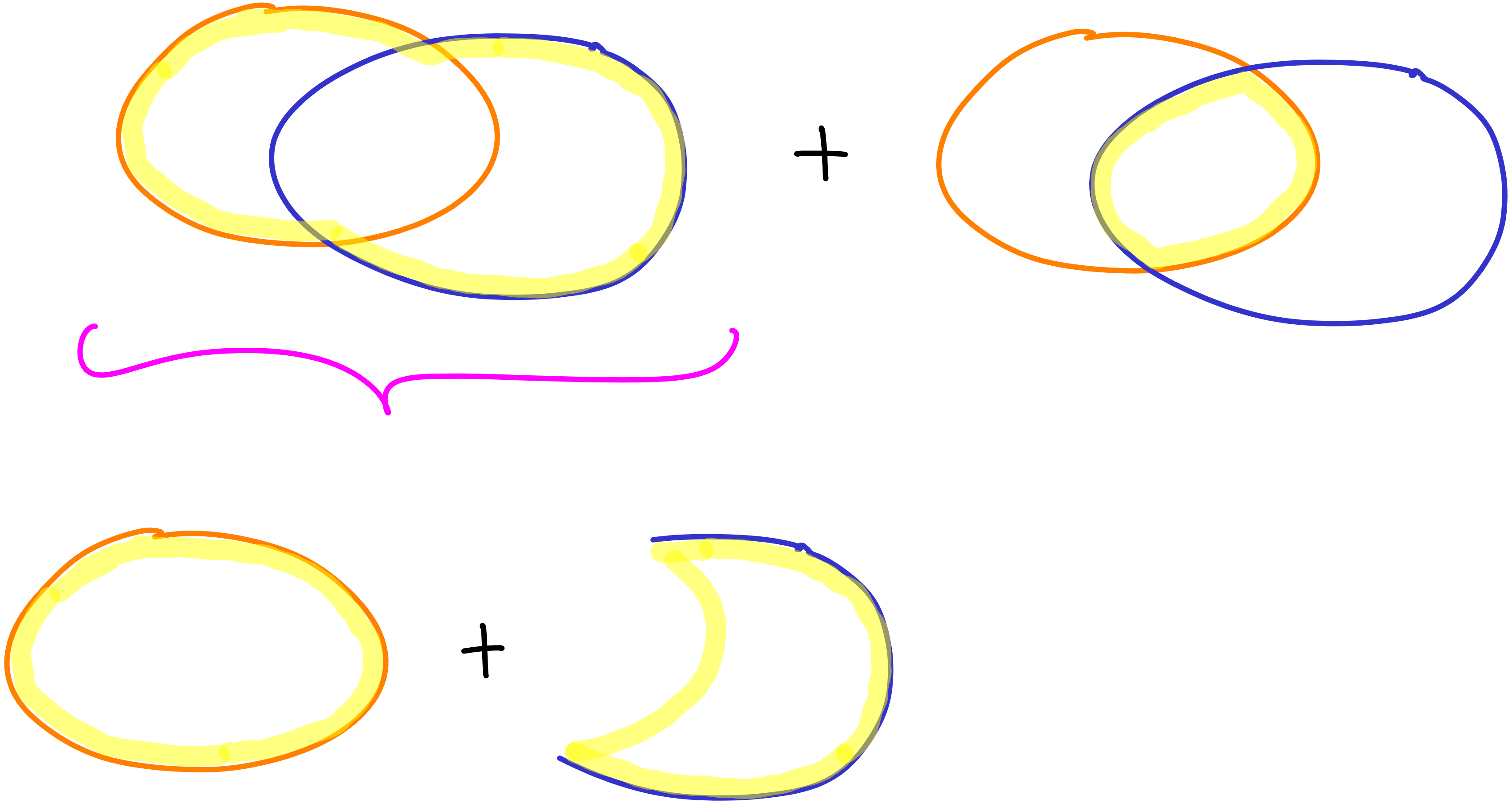


$$|A| + |B| = |A \cup B| + |A \cap B|$$



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$$|A| = 500$$

$$|B| = 200$$

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$|A \cap B| = 100$

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$$|A| + |B| = |A \cup B| + |A \cap B| \longrightarrow |A \cup B| = 600$$

De Morgan's Laws

for propositions: $\neg(A \text{ AND } B) \leftrightarrow \neg A \text{ OR } \neg B$

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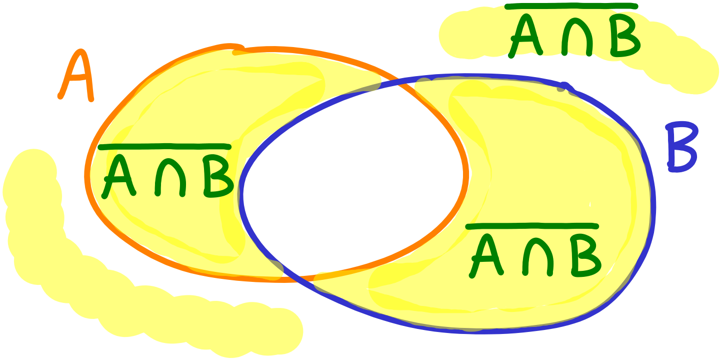

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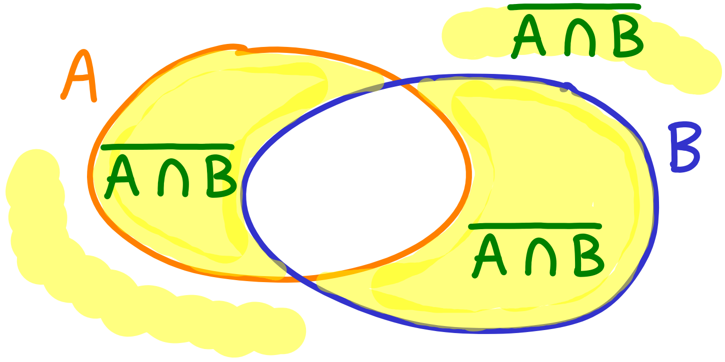

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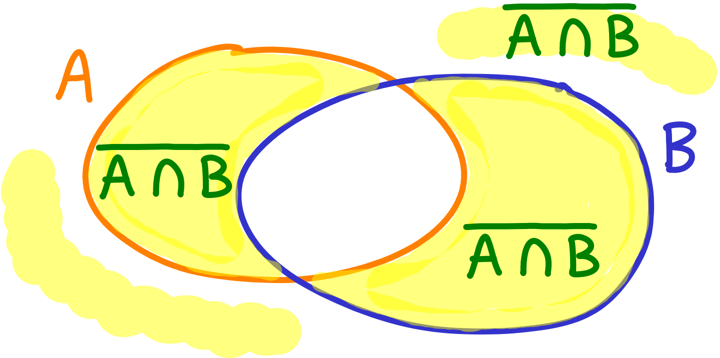

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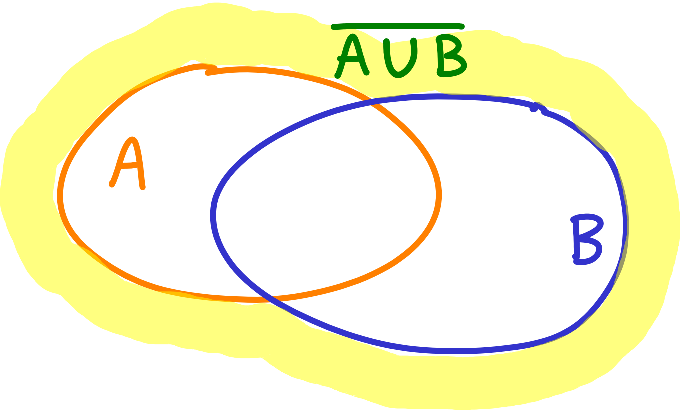

Sequences

Like sets, they are a collection of elements.

2 main differences:

- repeats OK $(a,b,a,b,a,a,b,b,\dots)$
- order matters $(a,b,c) \neq (c,b,a)$

Cartesian product of sets

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For breakfast you can have one of: $B = \{egg, banana\}$

For lunch you can have one of: $L = \{soup, salad\}$

For dinner you can have one of: $D = \{egg, steak, cake\}$

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Cartesian product of sets

produces a set of sequences.

each sequence:

one element per set.

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Cartesian product of sets

produces a set of sequences

If we take the product of n copies of S, we write S^n

e.g., $\{0,1\}^3 = \{(0,0,0),(0,0,1),(0,1,0),(1,0,0),(0,1,1),(1,0,1),(1,1,0),(1,1,1)\}$

set of corners of a cube