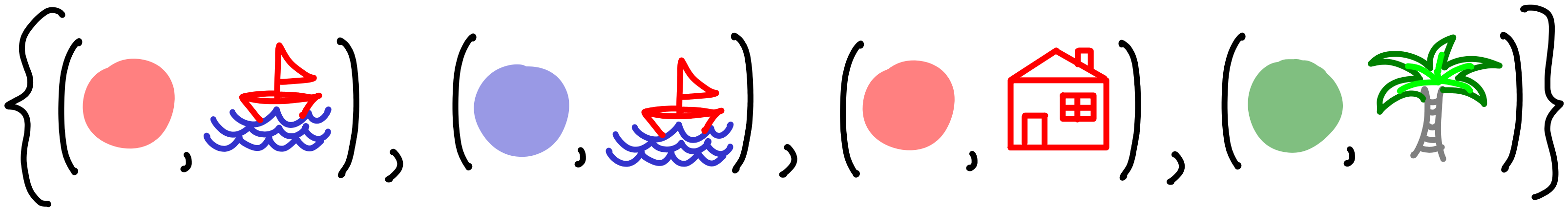
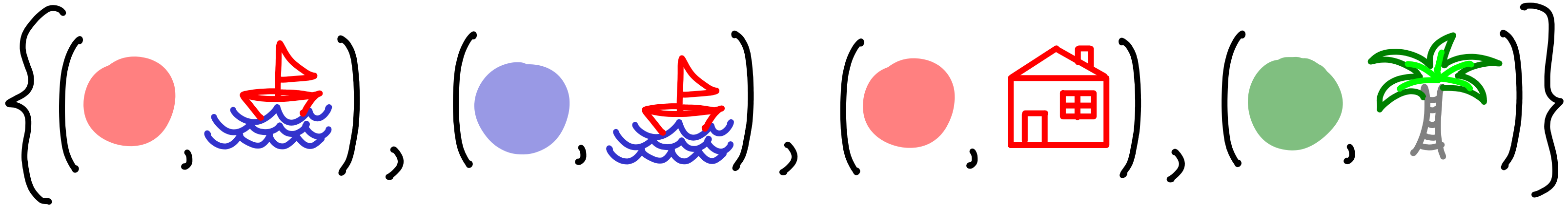
# RELATIONS & FUNCTIONS

just a small note

about some definitions and terminology

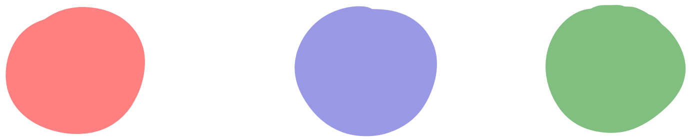


A binary relation is a set of ordered pairs.

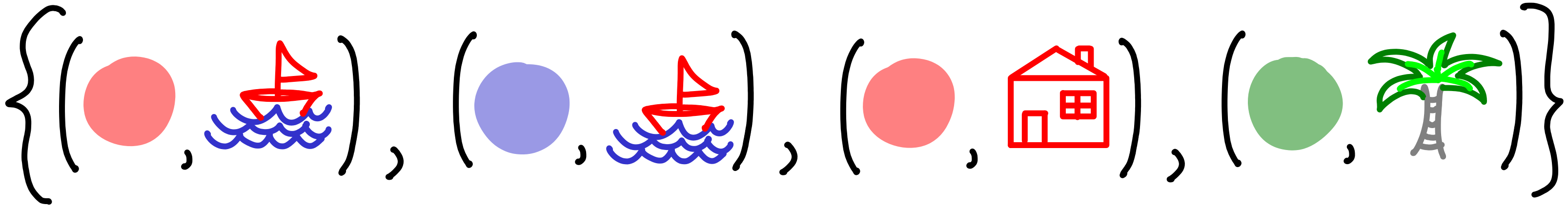


A binary relation is a set of ordered pairs.

In each pair, the first element is from a set called the Domain.



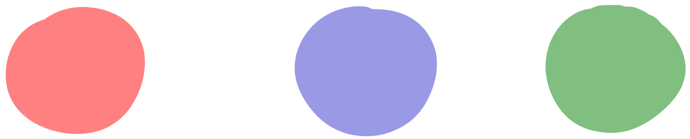
Domain



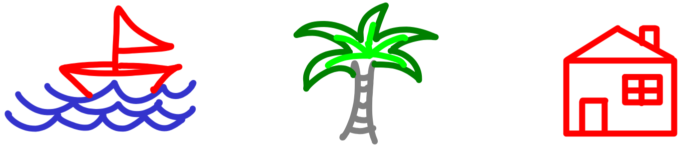
A binary relation is a set of ordered pairs.

In each pair, the first element is from a set called the Domain.

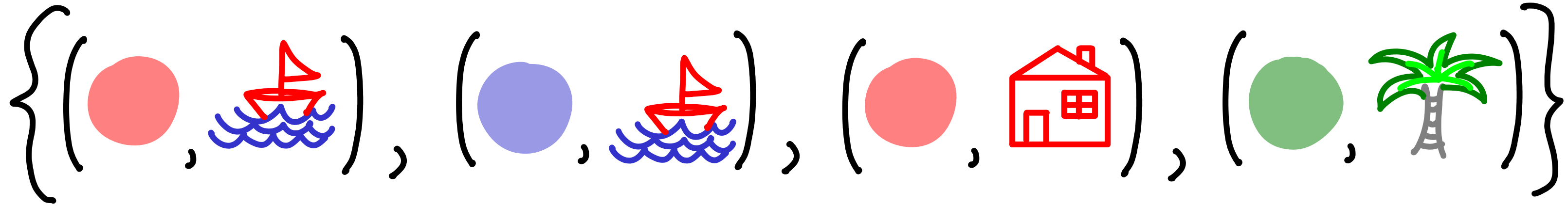
the second element is from a set called the Codomain.



Domain



Codomain



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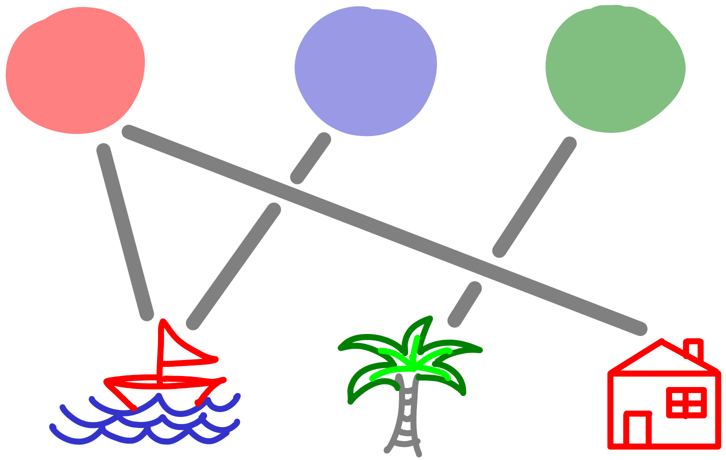


Domain

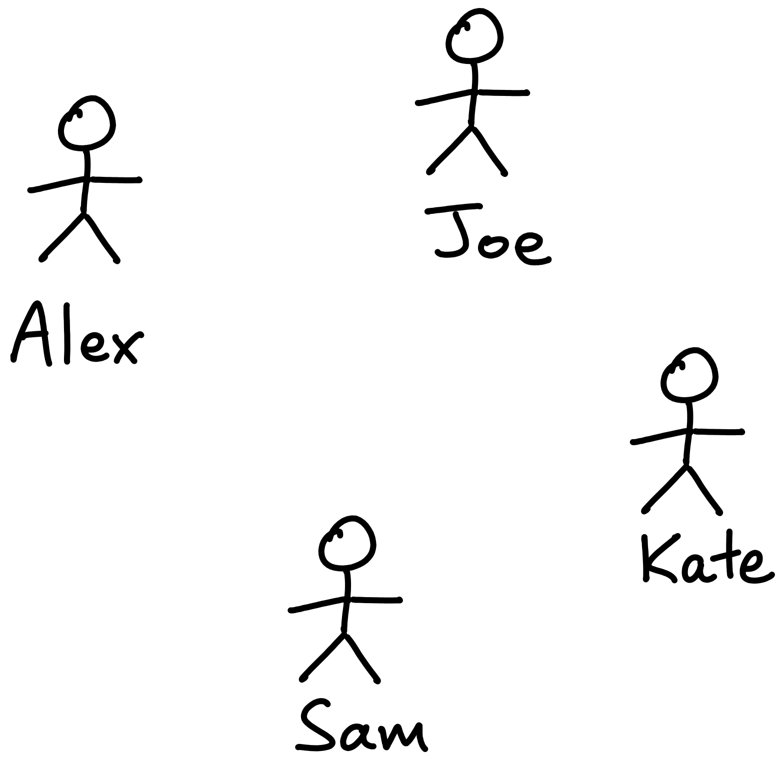
Codomain

The relation is "from" the Domain "to" the Codomain.

IF Domain = Codomain = S

THEN the relation is "on" S.

e.g., $S = \{\text{Alex}, \text{Joe}, \text{Sam}, \text{Kate}\}$



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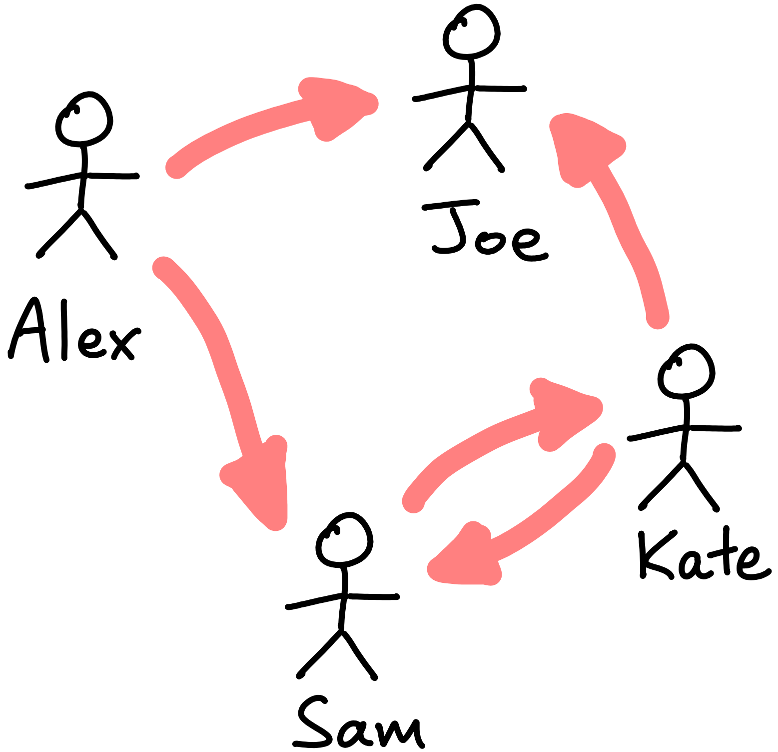


e.g., $S = \{\text{Alex}, \text{Joe}, \text{Sam}, \text{Kate}\}$

$\{(\text{Alex}, \text{Sam}), (\text{Alex}, \text{Joe}), (\text{Kate}, \text{Sam}), (\text{Kate}, \text{Joe}), (\text{Sam}, \text{Kate})\}$

A relation on Integers : $\{(-2,4),(-1,1),(0,0),(1,1),(2,4)\}$

Notice not all integers are present.

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Some sources use "Image" instead (e.g. Scheinerman)

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Notice not all integers are present.

Range = $\{0,1,4\}$ = subset of codomain that is actually present.

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We could have been more specific here:

Domain: $X = \{$integers between $-2$ & $2$, inclusive$\}$

Codomain: $Y = \{$numbers that are the square of some integer$\}$

# The $<$ relation (on $\mathbb{Z}^+$)

$R$: $\{(1,2),(2,3),(1,3),(3,4),(2,4),(1,4),(4,5),(3,5),(2,5),(1,5)\ldots\}$

Given $x, y \in \mathbb{Z}^+$, $(x,y) \in R$ IFF $y - x \in \mathbb{Z}^+$

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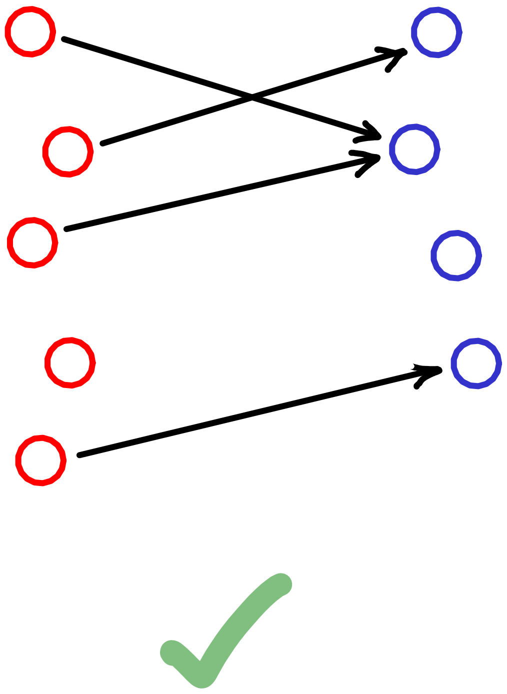
$$R: \{(1,2),(2,3),(1,3),(3,4),(2,4),(1,4),(4,5),(3,5),(2,5),(1,5)\ldots\}$$

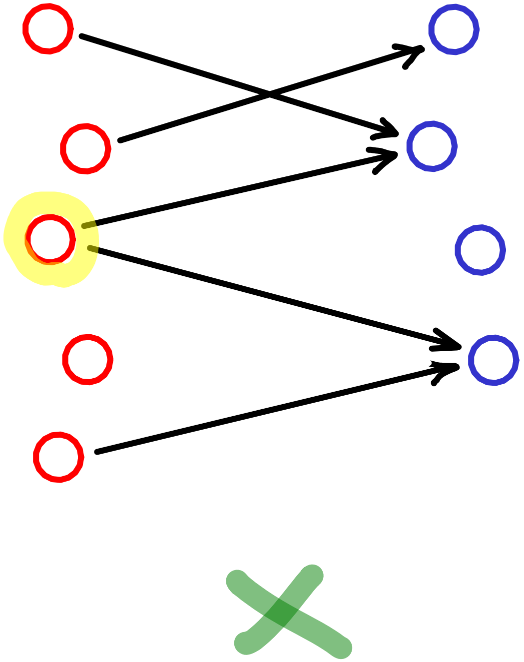
Given $x, y \in \mathbb{Z}^+$, $\underbrace{(x,y) \in R}_{xRy}$ IFF $y - x \in \mathbb{Z}^+$

$x < y$

Let R be a relation from A to B.



R is a function IFF every element in A appears at most once in R.



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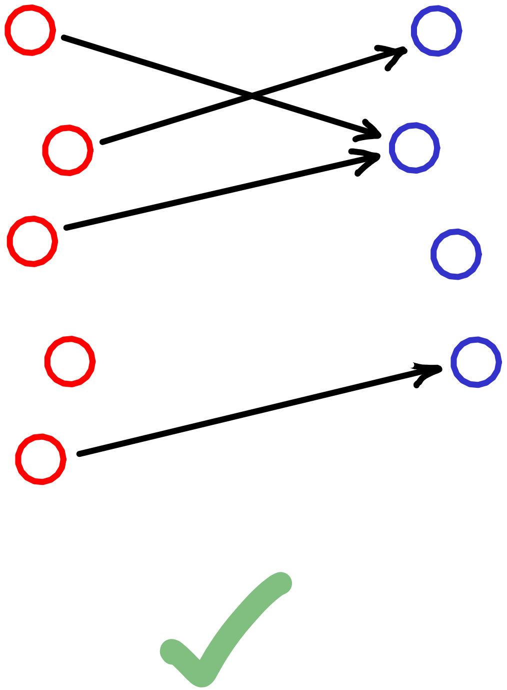

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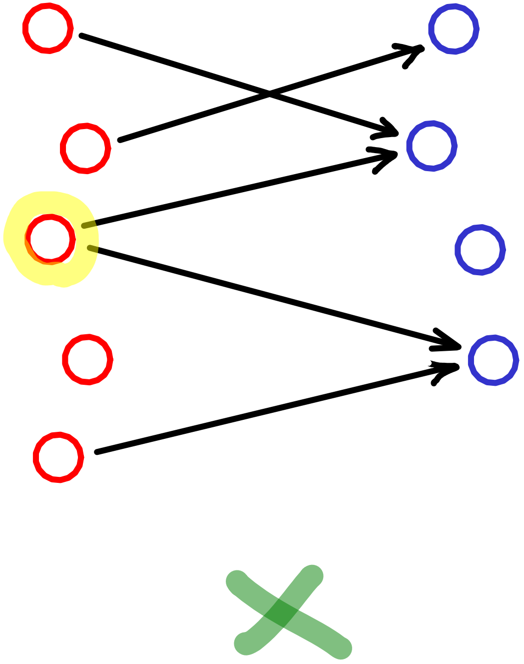
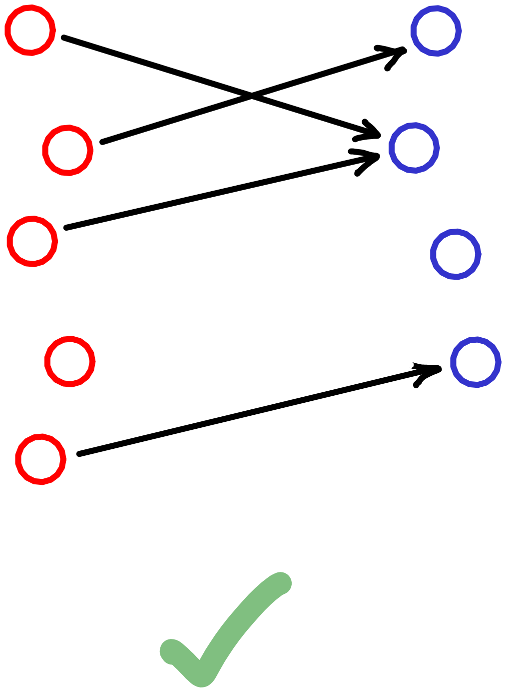


i.e., every ○ has ≤ 1 arrow.

Let R be a relation from A to B.



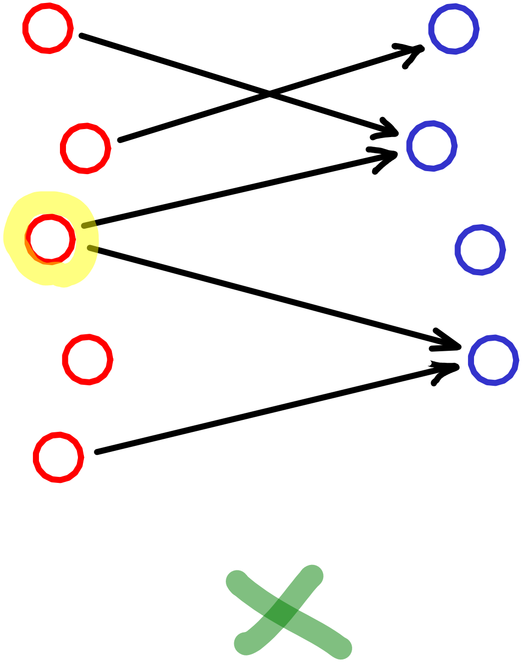
**Definition F1:**

R is a function

IFF every element in A

appears at most once in R.

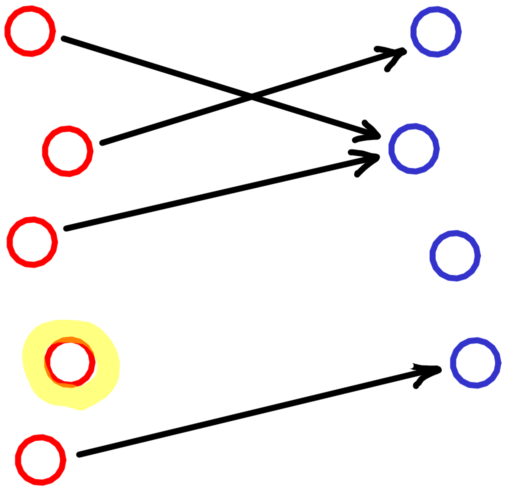
(used in MCS)



i.e., every ○ has $\leq 1$ arrow.

**Definition F2:** "A appears exactly once" (e.g., Scheinerman, Rosen)

Let R be a relation from A to B.



**Definition F2:**

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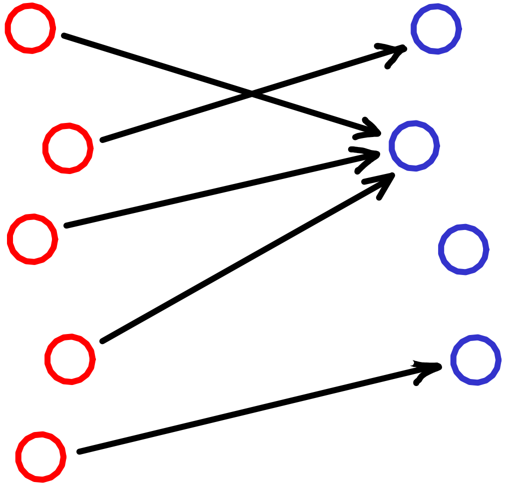
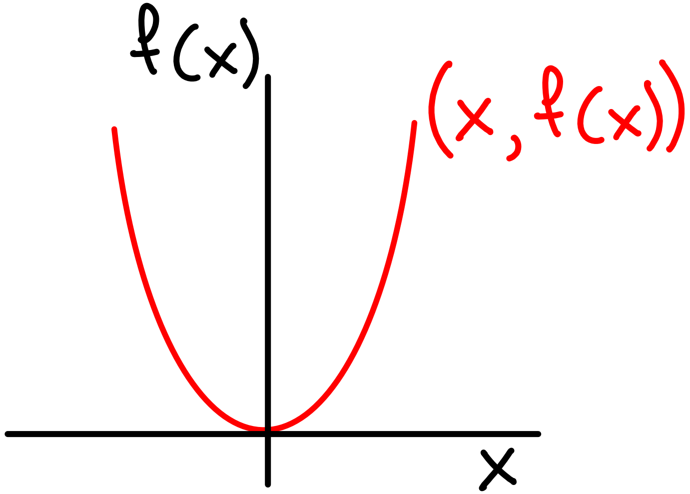


i.e., every ○ has 1 arrow.

$f(x) = x^2$

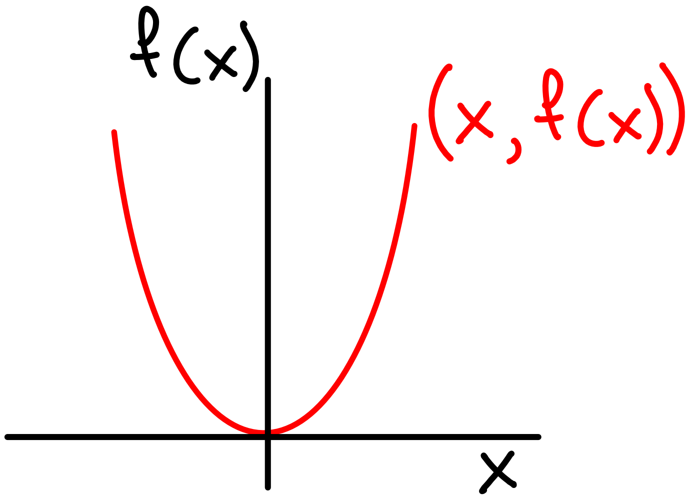



Domain : $\mathbb{R}$

Codomain : $\mathbb{R}$

Range : $\mathbb{R}\,(\geq 0)$

$f(x) = x^2$

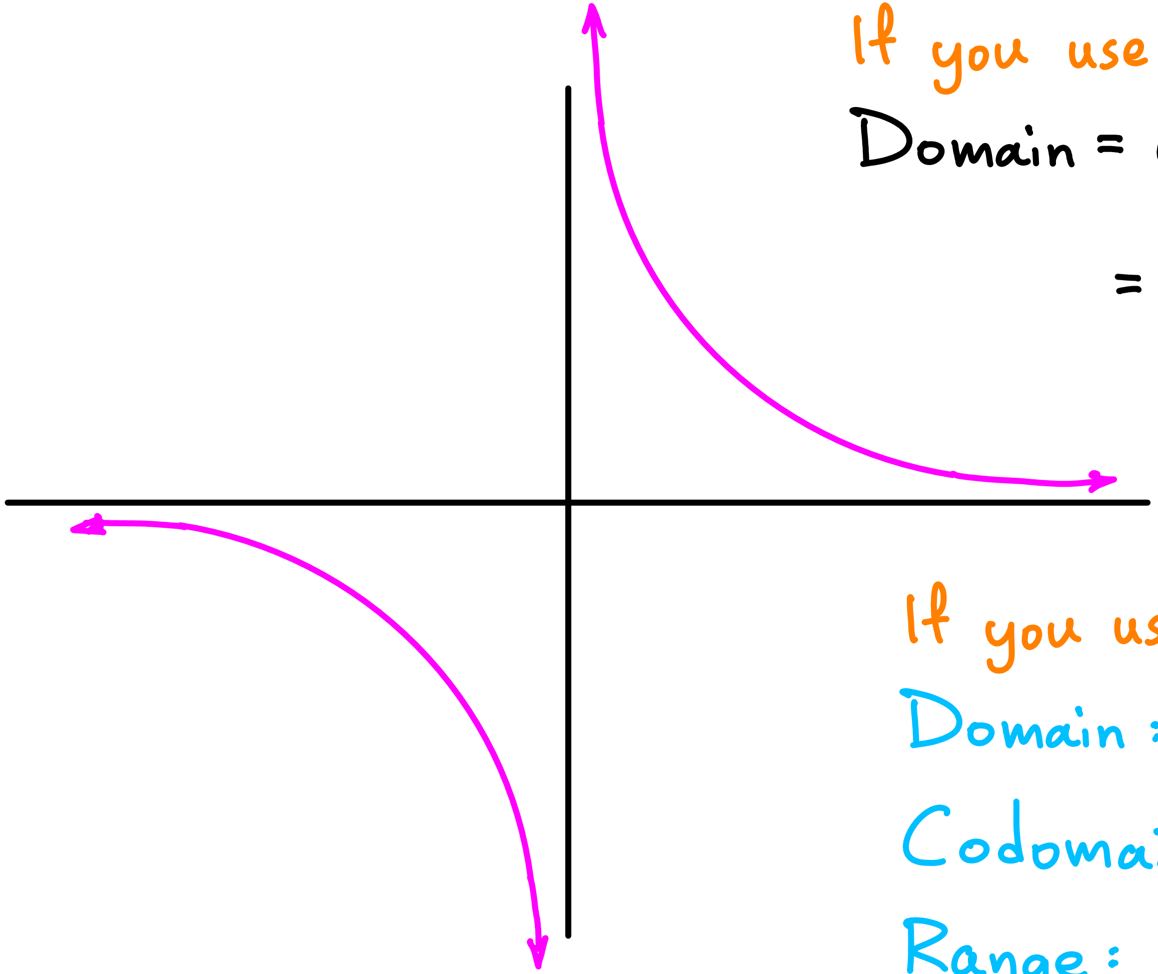



Domain : $\mathbb{R}$

Codomain : $\mathbb{R}$

Range : $\mathbb{R}(\geq 0)$

$g(x) = \frac{1}{x}$



If you use F2:

Domain = Codomain

$= \mathbb{R} - \{0\}$

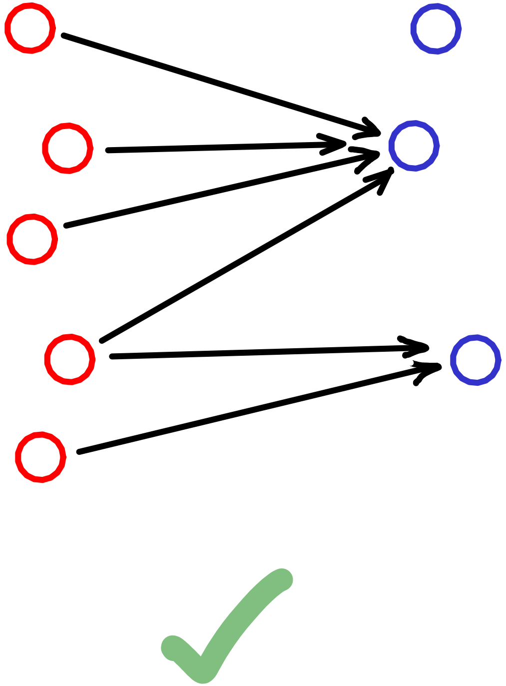
If you use F1:

Domain : $\mathbb{R}$

Codomain : $\mathbb{R}$

Range : $\mathbb{R} - \{0\}$

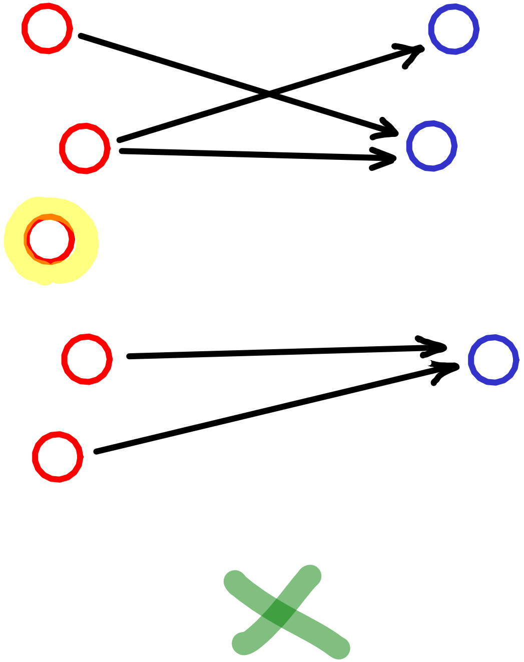
Let R be a relation from A to B.



R is total

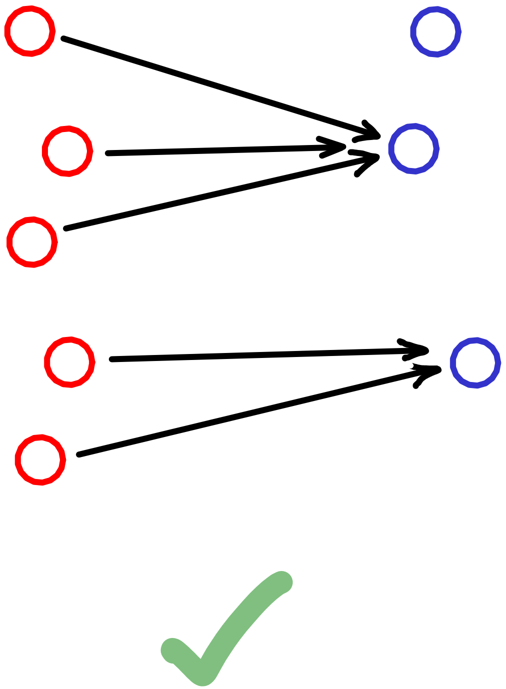
IFF every element in A

appears at least once in R.



i.e., every ○ has $\geq 1$ arrow.

Let R be a ~~relation~~ function from A to B.



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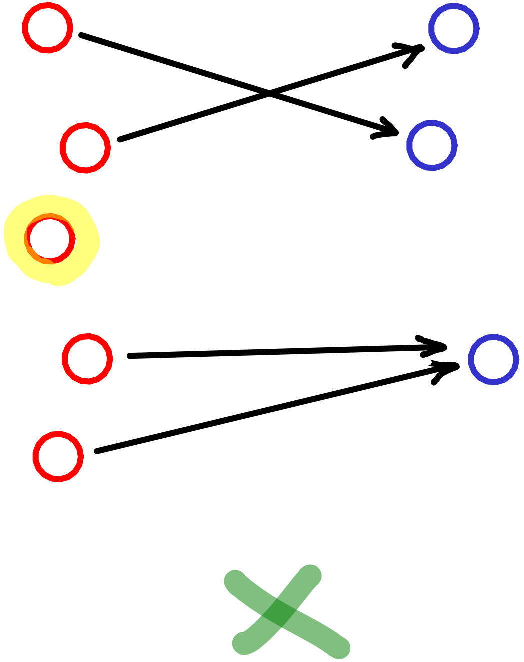
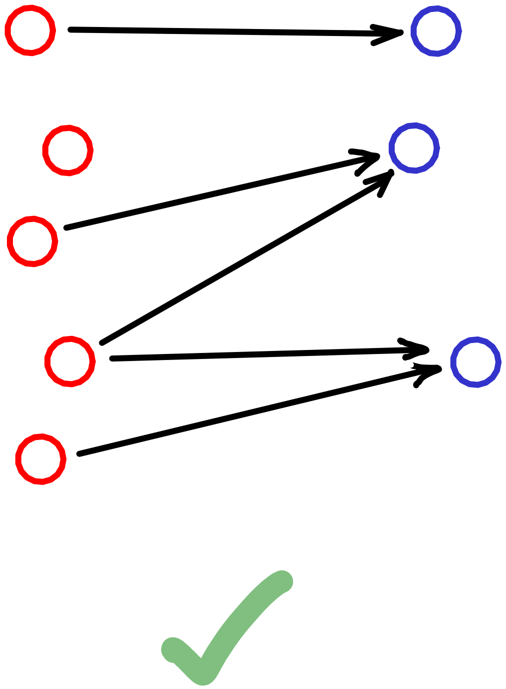


i.e., every ○ has $\geq 1$ arrow.

If you use F2 then all functions are total.

F1 allows the definition of partial functions.

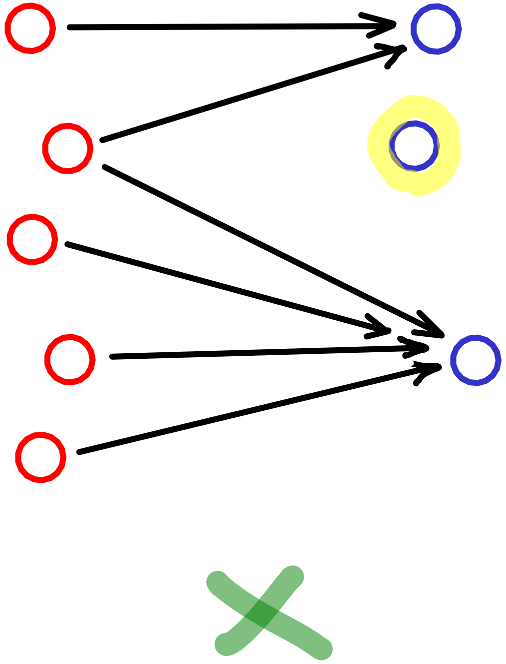
Let R be a relation from A to B.



R is surjective

IFF every element in B

appears at least once in R.



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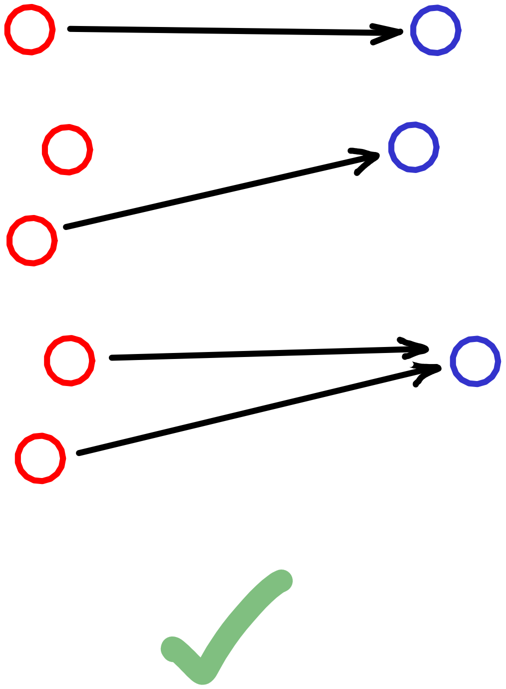

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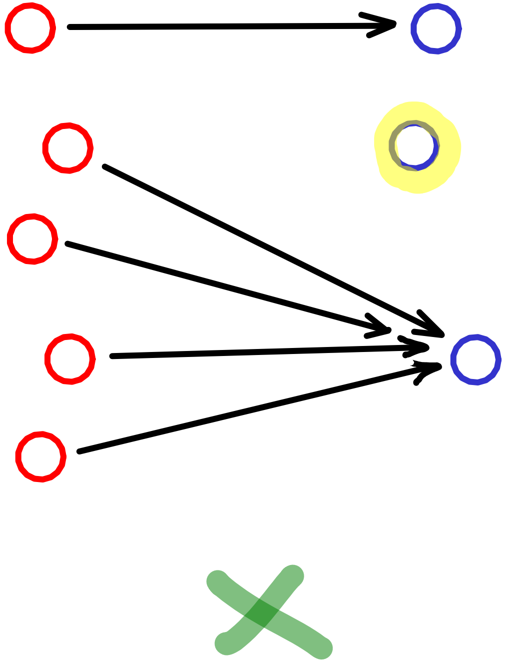
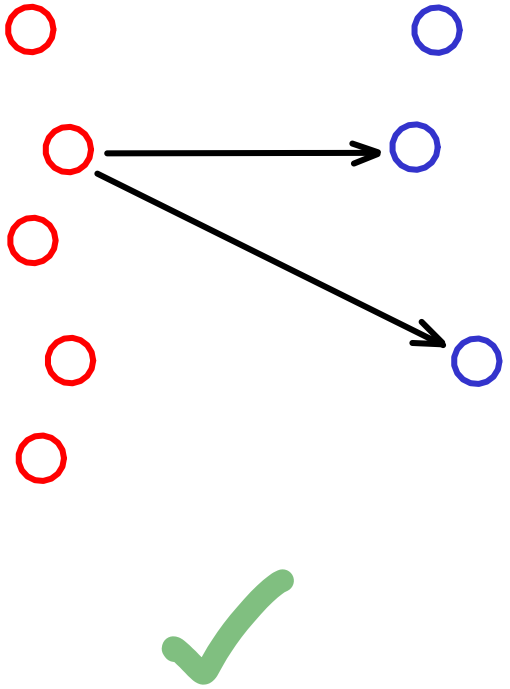


i.e., every ○ has $\geq 1$ arrow.

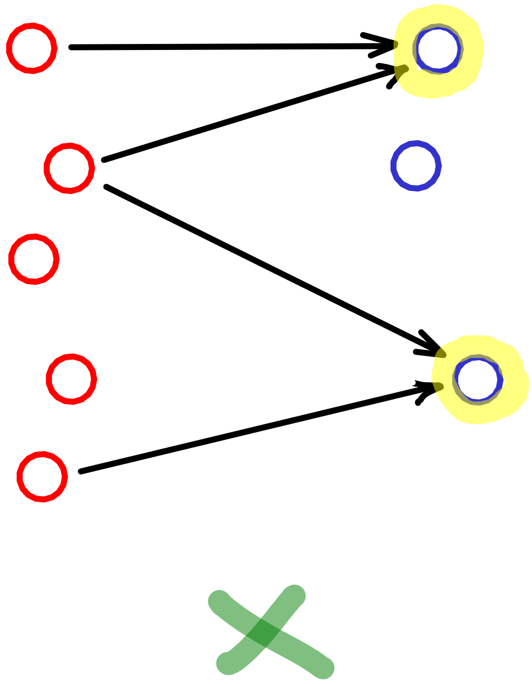
A surjective function is also called an "onto" function

Let R be a relation from A to B.



R is *injective*

IFF every element in B appears at most once in R.



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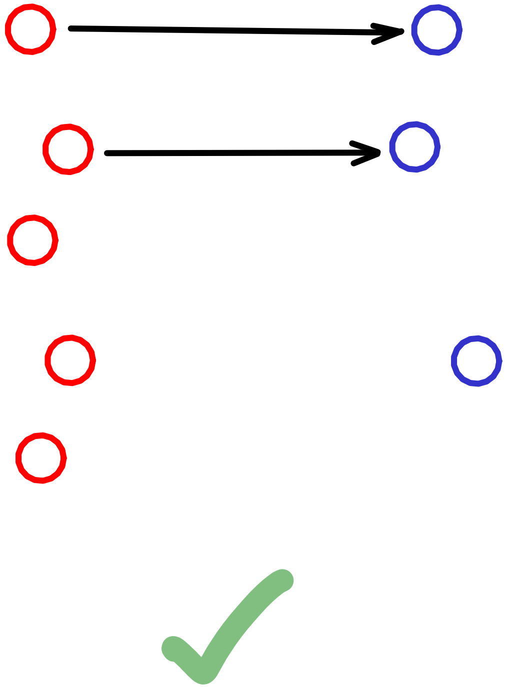
Let R be a ~~relation~~ function from A to B.



R is injective

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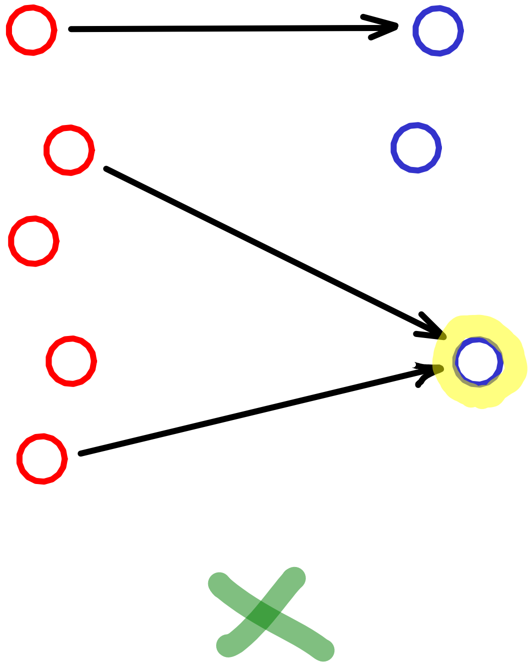

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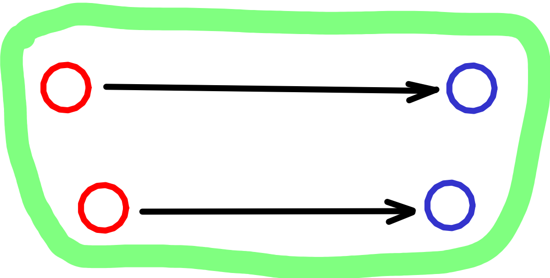

R is injective

IFF every element in B

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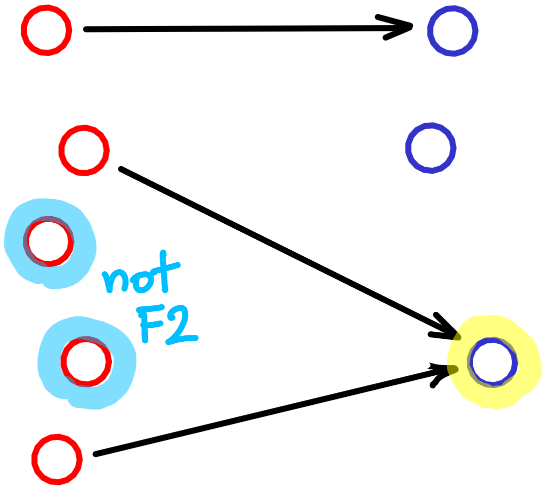
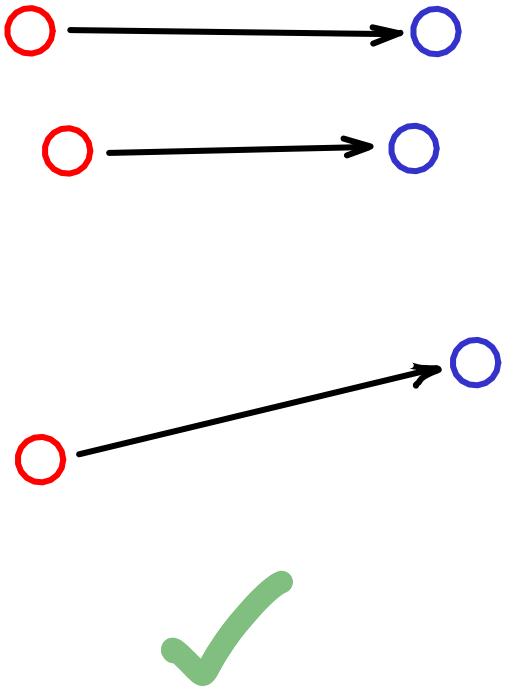



i.e., every ○ has $\leq 1$ arrow.

If R is an injective function then it is "one-to-one".

Definition F2 makes more sense for this terminology.

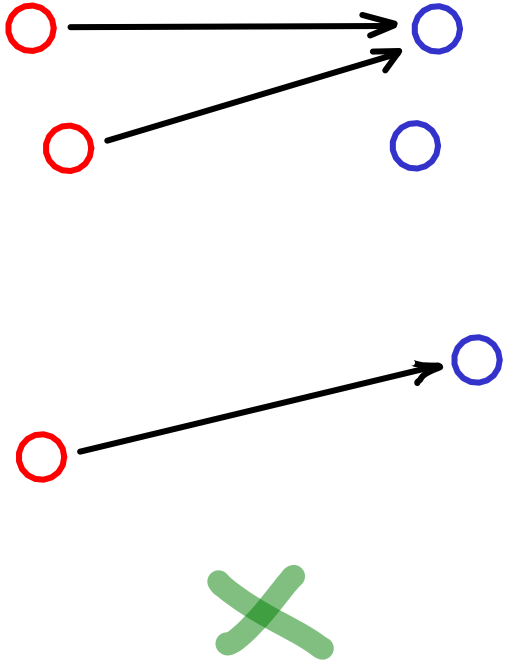
Let R be a relation from A to B.



R is bijective

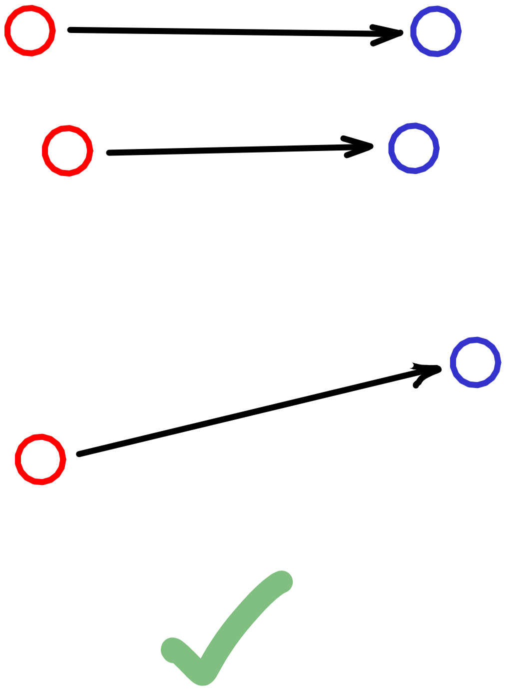
IFF every element

appears exactly once in R.



i.e., every ○ or ○ has 1 arrow.

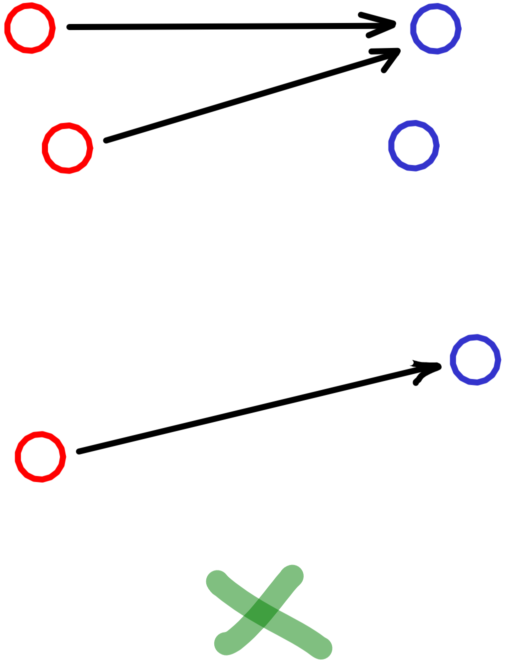
Let $R$ be a relation from $A$ to $B$.



$R$ is **bijective**

IFF every element

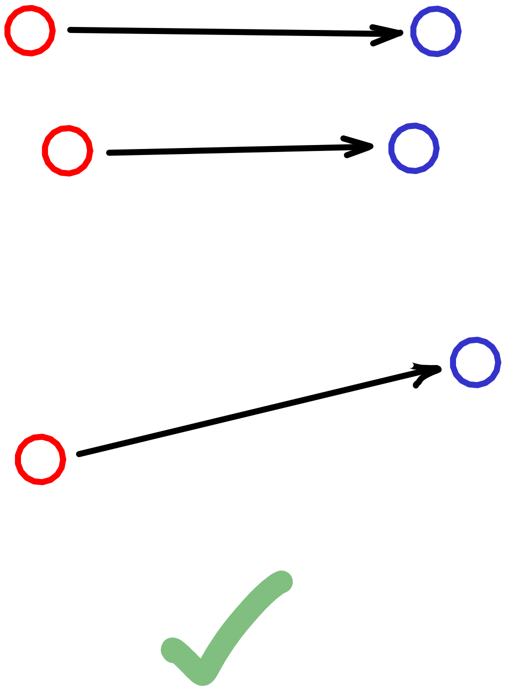
appears exactly once in $R$.



i.e., every ○ or ○ has 1 arrow.

- If every ○ has 1 arrow then $R$ is injective and surjective.

Let R be a relation from A to B.



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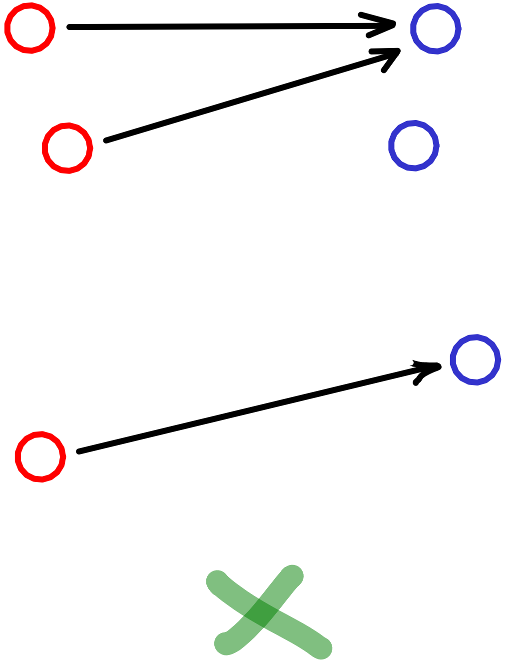
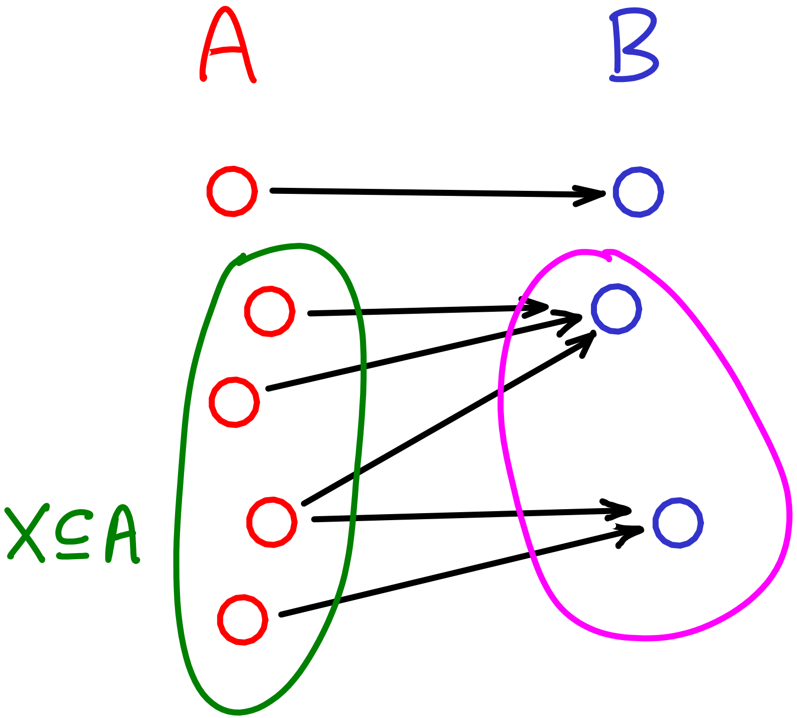


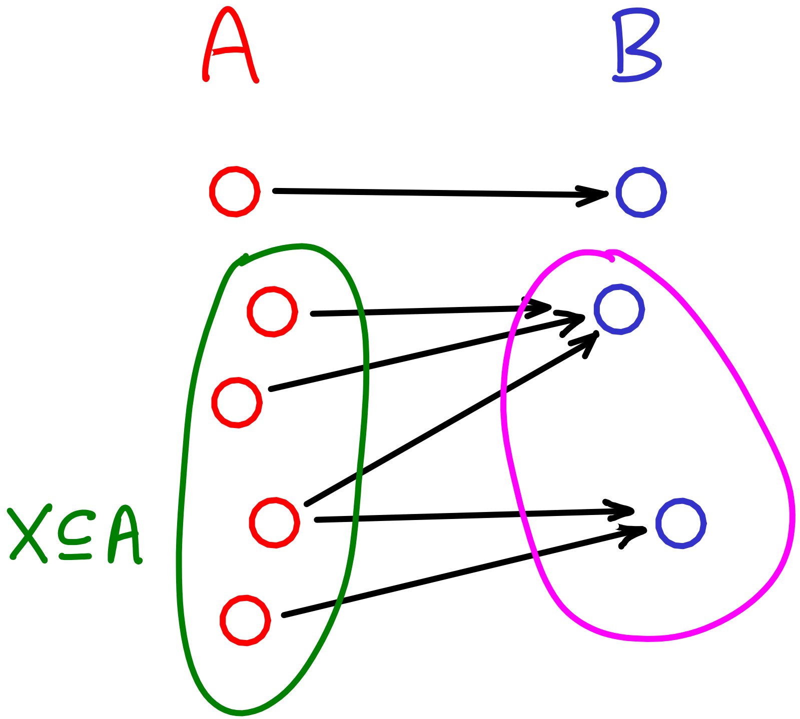
i.e., every ○ or ○ has 1 arrow.

- If every ○ has 1 arrow then R is injective and surjective.
- Requiring 1 arrow for every ○ means F1 & F2 agree here.




"Image" of $X$ = $R(X)$ or $\mathrm{img}(X)$

↪ all $b \in B$ such that $\exists a \in A$ for which $a \to b$




"Image" of $X$ = $R(X)$ or $img(X)$

↳ all $b \in B$ such that $\exists a \in A$ for which $a \to b$

So the Range is the union of all Images.

Recall: Scheinerman uses Image to mean Range.

# Other definitions to look up

- Inverse function $f(a) = b$ , $f^{-1}(b) = a$
- Composition of functions $f(a) = b$ $g(b) = c$ $g(f(a)) = c$
- Equivalence relations
- For relations on a set:
  - reflexive , irreflexive
  - symmetric , antisymmetric
  - transitive

etc