Roll 2 dice. Define random variables X, Y

↳ X: sum of two dice. $X[(1,2)] = 3$ $X[(5,5)] = 10$

↳ Y: parity of sum. $Y[(1,2)] = 1$ $Y[(5,5)] = 0$

odd even

Think of these as functions, mapping sample space to a number.

e.g., $\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),\ldots$
$\vdots$
$(6,1),(6,2),\ldots \qquad \ldots(6,6)\}$

$X \to 2\ldots12$

$Y \to 0,1$

Example usage: $P(X<3) = \frac{1}{36}$ $P(Y=1) = \frac{1}{2}$ $P(X=13) = 0$

Expected value = weighted average

$$E[X] = \sum_{*} y \cdot P(X=y)$$

$*$ over all possible values, y, that X could be.

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example: roll 2 dice. $X = |\text{difference between the two}|$

possible values of X →	0	1	2	3	4	5
# outcomes per value →	6	$5\cdot2$	$4\cdot2$	$3\cdot2$	$2\cdot2$	$1\cdot2$

(for probability, divide by 36)

$$E[X] = \frac{0 + 10 + 16 + 18 + 16 + 10}{36} \sim 1.944$$

LINEARITY OF EXPECTATION $E[X+Y] = E[X] + E[Y]$

Generally, for $c_i \in \mathbb{R}$

$$E[c_1X_1 + c_2X_2 + \cdots + c_nX_n] = c_1E[X_1] + c_2E[X_2] + \cdots c_nE[X_n]$$

$$E\left[\sum X_i\right] = \sum E[X_i]$$

Does NOT assume independence $\longrightarrow$ $P(X=a \;\&\; Y=b) = P(X=a)\cdot P(Y=b)$ for all a,b ...

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2 dice, A, B. X = result of A. Y = result of B. $Z = X+Y$

$$E[Z] = E[X+Y] = E[X] + E[Y] = 2\cdot 3.5 = 7$$

1000 dice, expected value of sum $= 1000 \cdot 3.5$

EXPECTATION : PROPERTIES

$E[X+Y] = E[X] + E[Y]$ → always true

but $E[X \cdot Y] = E[X] \cdot E[Y]$ → NOT always true

If X & Y are independent, then $E[X \cdot Y] = E[X] \cdot E[Y]$

However, $E[X \cdot Y] = E[X] \cdot E[Y]$ does NOT imply X & Y are independent.