

RANDOM VARIABLES

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↳ Y : parity of sum. $Y[(1,2)] = ?$ odd $Y[(5,5)] = ?$ even

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↳ Y : parity of sum.

$$Y[(1,2)] = 1$$

odd

$$Y[(5,5)] = 0$$

even

Think of these as functions, mapping sample space to a number.

e.g., $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), \dots\}$

\vdots

$\{ (6,1), (6,2), \dots, (6,6) \}$

$\left. \begin{array}{l} X \rightarrow 2 \dots 12 \\ Y \rightarrow 0, 1 \end{array} \right\}$

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Example usage: $P(X < 3) = \frac{1}{36}$

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$$E[X] = \frac{0 + 10 + 16 + 18 + 16 + 10}{36} \sim 1.944$$

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$$E[c_1X_1 + c_2X_2 + \dots + c_nX_n] = c_1E[X_1] + c_2E[X_2] + \dots + c_nE[X_n]$$

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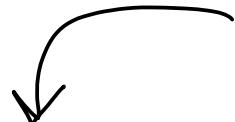
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1000 dice, expected value of sum $= 1000 \cdot 3.5$

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If X & Y are independent, then $E[X \cdot Y] = E[X] \cdot E[Y]$

However, $E[X \cdot Y] = E[X] \cdot E[Y]$ does NOT imply
 X & Y are independent.