RANDOM VARIABLES

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odd even

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odd even

Think of these as functions, mapping sample space to a number.

e.g., $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(2,1), (2,2), \ldots$
$\vdots$
$(6,1), (6,2), \ldots \quad \ldots (6,6)\}$

$X \rightarrow 2 \ldots 12$

$Y \rightarrow 0, 1$

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(for probability, divide by 36)

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$$E[X] = \frac{0 + 10 + 16 + 18 + 16 + 10}{36} \sim 1.944$$

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1000 dice, expected value of sum $= 1000 \cdot 3.5$

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However, $E[X \cdot Y] = E[X] \cdot E[Y]$ does NOT imply X & Y are independent.