RANDOM VARIABLES

A quote from the Scheinerman textbook:

"A random variable is neither random nor variable"

We have been using random variables, implicitly.

ex: roll 2 dice, examine probability that sum = k, or = even.

↳ define random variable X : sum of two dice rolls.

So, $X[(1,2)] = 3$

$X[(5,5)] = 10$

↳ define random variable Y : parity of two dice rolls.

So, $Y[(1,2)] = 1$

$Y[(5,5)] = 0$

Think of a r.v. as a function,

mapping sample space to $\underbrace{\text{whatever you like}}_{\text{usually a number}}$

Then we can express questions neatly: $P(X<3) = \frac{1}{36}$ ← sum

$P(Y=1) = \frac{1}{2}$ ← parity

2 dice

We can also eliminate absurd events, e.g., $P(X=13)=0$

EXPECTATION : the very basics

As mentioned, a r.v. X can have several values.

It is based on outcomes that result from a random process.

So we don't know what value it will have.

But we can expect it to have some value

Expected value = weighted average

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$$E(X) = \sum_{*} y \cdot P(X=y)$$ see example 34.3 in book (mainly p.237)
(Scheinerman)

$*$ over all possible values y, compatible with X.
(however we only care about finitely many)

$$E(X) = \underbrace{\sum_{s \in S}}_{} \left[X(s) \cdot P(s)\right]$$

over all samples that define X. a finite number

e.g. roll 1 die. X = number observed.

$$E(X) = \sum_{i=1}^{6} X(i) \cdot P(i)$$

$$= 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$$

$$= \frac{1}{6} \cdot (1+2+3+4+5+6) = 3.5$$

$$E(X) = \sum y \cdot P(X=y)$$

example: roll 2 dice. $X = |\text{difference between the } 2|$

possible values of X →	0	1	2	3	4	5
# outcomes supporting value →	6	$5 \cdot 2$	$4 \cdot 2$	$3 \cdot 2$	$2 \cdot 2$	$1 \cdot 2$

(for probability, divide by 36)

$$E(x) = \frac{0 + 10 + 16 + 18 + 16 + 10}{36}$$

(~ 1.944)

EXPECTATION : PROPERTIES

LINEARITY OF EXPECTATION (important)

$c_1, c_2 \in \mathbb{R}$ $\quad E(c_1 X + c_2 Y) = c_1 \cdot E(X) + c_2 \cdot E(Y)$

Generally, $E(c_1 X_1 + c_2 X_2 + \cdots + c_n X_n) = c_1 E(X_1) + c_2 E(X_2) + \cdots c_n E(X_n)$

$E\left(\sum c_i X_i\right) = \sum c_i E(X_i)$ → Does NOT assume independence

Independence: $P(X=a \;\&\; Y=b) = P(X=a) \cdot P(Y=b)$

for all a,b ...

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2 dice, A, B. $\quad X =$ result of A. $\quad Y =$ result of B. $\quad Z = X+Y$

$E(Z) = E(X+Y) = E(X) + E(Y) = 2 \cdot 3.5 = 7$

EXPECTATION : PROPERTIES

$$E(X+Y) = E(X) + E(Y)$$

Linearity of expectation doesn't assume independence

but $E(X \cdot Y) \neq E(X) \cdot E(Y)$ in general.

If X & Y are independent, then $E(X \cdot Y) = E(X) \cdot E(Y)$

However, $E(X \cdot Y) = E(X) \cdot E(Y)$ does NOT imply X & Y are independent.

(see example 34.15)
(Scheinerman)

- We are skipping the proofs of most statements in this section.
- You are not required to study these, but it would probably be beneficial.
- We are also skipping <u>**variance**</u>, which is an important concept to learn independently.