Concepts to be familiar with before reading this document

$\neg X$: "not X"

"proposition"

IF-THEN $\rightarrow$

IFF $\leftrightarrow$

Truth Tables

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If P is a proposition then so is $\neg P$.

Any proposition is either true (T) or false (F)

Truth Tables

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Any proposition is either true (T) or false (F)

but P and $\neg P$ can't both be T, or both F.

TRUTH TABLES

If P is a proposition then so is $\neg P$.

Any proposition is either true (T) or false (F)

but P and $\neg P$ can't both be T, or both F.

If we know one, we know the other

P	$\neg P$
T	F
F	T

Recall: "it's raining" and "it's cloudy" are not propositions. ← can vary

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George Boole , 1840's

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P & Q are Boolean variables aka propositional variables

that can be used in other statements, e.g., (P AND Q)

"it is raining and cloudy"

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"it is raining and cloudy"

P	Q	P AND Q
T	T	
T	F	
F	T	
F	F	

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"it is raining and cloudy"

P	Q	P AND Q
T	T	T
T	F	F
F	T	F
F	F	F

only one way for this to happen

P = "it's raining"
Q = "it's cloudy"

P OR Q

"it is raining or cloudy"

P = "it's raining"
Q = "it's cloudy"

$$P \text{ OR } Q$$

$$\text{"it is } \underbrace{\text{raining or cloudy}}_{\text{don't care which}}\text{"}$$

P OR Q

$\underbrace{\hspace{6em}}_{\text{don't care which}}$

P	Q	P OR Q
T	T	?
T	F	?
F	T	?
F	F	?

Let's fill in a truth table
without caring about what P & Q mean

$$\underbrace{P \text{ OR } Q}_{\text{don't care which}}$$

P	Q	P OR Q
T	T	T
T	F	T
F	T	T
F	F	F

P = "it's raining"
Q = "it's cloudy"

P OR Q

"it is $\underbrace{\text{raining or cloudy}}_{\text{don't care which}}$"

CONTEXT RESTORED

P	Q	P OR Q
T	T	T
T	F	T
F	T	T
F	F	F

P = "it's raining"
Q = "it's cloudy"

P OR Q

$\underbrace{\text{"it is raining or cloudy"}}_{\text{don't care which}}$

P	Q	P OR Q
T	T	T
T	F	T
F	T	T
F	F	F

this combination can't happen!
We know that $P \to Q$ in our example.

P = "it's raining"
Q = "it's cloudy"

P OR Q

"it is $\underbrace{\text{raining or cloudy}}_{\text{don't care which}}$"

P	Q	P OR Q
T	T	T
T	F	T
F	T	T
F	F	F

this combination can't happen!
We know that $P \rightarrow Q$ in our example.

Is the truth table wrong or invalid?

Could have asked the same for P AND Q

P = "it's raining"
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P OR Q

"it is $\underbrace{\text{raining or cloudy}}_{\text{don't care which}}$"

P	Q	P OR Q
T	T	T
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No. This is the truth table for OR.

P = "it's raining"
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P OR Q

"it is $\underbrace{\text{raining or cloudy}}_{\text{don't care which}}$"

P	Q	P OR Q
T	T	T
T	F	T
F	T	T
F	F	F

this combination can't happen!
We know that $P \rightarrow Q$ in our example.

Is the truth table wrong or invalid?

Could have asked the same for P AND Q

No. This is the truth table for OR.

All combinations are considered. Context & extra info isn't.

$\underbrace{P \text{ OR } Q}_{\text{don't care which is true}}$

P	Q	P OR Q
T	T	T
T	F	T
F	T	T
F	F	F

$$\underbrace{P \text{ OR } Q}_{\text{don't care which is true}}$$

P	Q	P OR Q
T	T	T
T	F	T
F	T	T
F	F	F

compare this to:

- "either P or Q but not both"
- "precisely one of P, Q"
- "exclusively one of P or Q"

$\underbrace{P \text{ OR } Q}_{\text{don't care which is true}}$

P	Q	P OR Q
T	T	T
T	F	T
F	T	T
F	F	F

compare this to:

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T	T	T
T	F	T
F	T	T
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compare this to:

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- P XOR Q

P	Q	P XOR Q
T	T	F
T	F	T
F	T	T
F	F	F

P IFF Q

$P \leftrightarrow Q$

$P \rightarrow Q$ and $Q \rightarrow P$

P	Q	P IFF Q
T	T	?
T	F	
F	T	
F	F	

P IFF Q

$P \leftrightarrow Q$

$P \to Q$ and $Q \to P$

P	Q	P IFF Q
T	T	T
T	F	?
F	T	?
F	F	

consistent

P IFF Q

$P \leftrightarrow Q$

$P \rightarrow Q$ and $Q \rightarrow P$

P	Q	P IFF Q	
T	T	T	consistent
T	F	F	inconsistent
F	T	F	
F	F	?	

P IFF Q

$P \leftrightarrow Q$

$P \to Q$ and $Q \to P$

P	Q	P IFF Q
T	T	T
T	F	F
F	T	F
F	F	T

consistent

inconsistent

why?

P IFF Q

$P \leftrightarrow Q$

$P \rightarrow Q$ and $Q \rightarrow P$

P	Q	P IFF Q	
T	T	T	consistent
T	F	F	inconsistent
F	T	F	inconsistent
F	F	T	?

Example:

P: pigs can fly (F)

Q: dogs can talk (F)

P IFF Q

$P \leftrightarrow Q$

$P \rightarrow Q$ and $Q \rightarrow P$

P	Q	P IFF Q
T	T	T
T	F	F
F	T	F
F	F	T

consistent

inconsistent

?

Example:

P: pigs can fly (F)

Q: dogs can talk (F)

P IFF Q:

$P \rightarrow Q$

and

$Q \rightarrow P$

P IFF Q

$P \leftrightarrow Q$

$P \rightarrow Q$ and $Q \rightarrow P$

P	Q	P IFF Q
T	T	T
T	F	F
F	T	F
F	F	T

consistent

inconsistent

?

Example:

P: pigs can fly (F)

Q: dogs can talk (F)

P IFF Q:

if pigs can fly then dogs can talk

and

if dogs can talk then pigs can fly

P IFF Q

$P \leftrightarrow Q$

$P \to Q$ and $Q \to P$

P	Q	P IFF Q
T	T	T
T	F	F
F	T	F
F	F	T

consistent

inconsistent

Example:

P: pigs can fly (F)

Q: dogs can talk (F)

P IFF Q: This is true! (when P,F are F)

if pigs can fly then dogs can talk

and

if dogs can talk then pigs can fly

$P \rightarrow Q$

P	Q	$P \rightarrow Q$
T	T	?
T	F	?
F	T	
F	F	

$P \to Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	?
F	F	?

consistent
inconsistent

$P \to Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

consistent

inconsistent

} ?

$P \rightarrow Q$

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

consistent
inconsistent

?

Example:

P: pigs can fly (F)

Q: I like apples F? T?

Maybe I do, maybe I don't

$P \to Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

consistent

inconsistent

?

Example:

P: pigs can fly (F)

Q: I like apples F? T?

$P \to Q$:

if pigs can fly then I like apples

$P \to Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

consistent
inconsistent

Example:

P: pigs can fly (F)

Q: I like apples F? T?

$P \to Q$: This is true! (when P is F)

if pigs can fly then I like apples

It doesn't matter if I like apples or not

vacuous truth

$P \to Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

consistent
inconsistent

Example:

P: pigs can fly (F)

Q: I like apples F? T?

$P \to Q$: This is true! (when P is F)

if pigs can fly then I like apples

It doesn't matter if I like apples or not

If it doesn't matter / apply, why bother considering these cases?

$P \to Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

consistent
inconsistent

Example:

P: pigs can fly (F)

Q: I like apples F? T?

$P \to Q$: This is true! (when P is F)

if pigs can fly then I like apples

It doesn't matter if I like apples or not

If it doesn't matter / apply, why bother considering these cases?

It is often important to simplify statements with Boolean variables

Example coming soon

$P \rightarrow Q$

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

consistent
inconsistent

$P \rightarrow Q$: This is true! (when P is F)

if pigs can fly then I like apples

It doesn't matter if I like apples

Example:

P: pigs can fly (F)

Q: I like apples F? T?

Example 2:

P: $\exists$ life on Mars F? T?

Q: I like basketball (T)

$P \to Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

consistent
inconsistent

Example:

P: pigs can fly (F)

Q: I like apples F? T?

$P \to Q$: This is true! (when P is F)

if pigs can fly then I like apples

It doesn't matter if I like apples

Example 2:

P: $\exists$ life on Mars F? T?

Q: I like basketball (T)

$P \to Q$:

if $\exists$ life on Mars then I like basketball

$P \to Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

consistent
inconsistent

Example:

P: pigs can fly (F)

Q: I like apples F? T?

$P \to Q$: This is true! (when P is F)

if pigs can fly then I like apples

It doesn't matter if I like apples

Example 2:

P: $\exists$ life on Mars F? T?

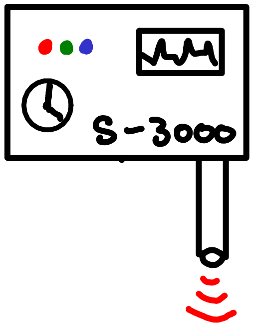
Q: I like basketball (T)

$P \to Q$: This is true! (when Q is T)

if $\exists$ life on Mars then I like basketball

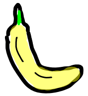
It doesn't matter if there is life on Mars

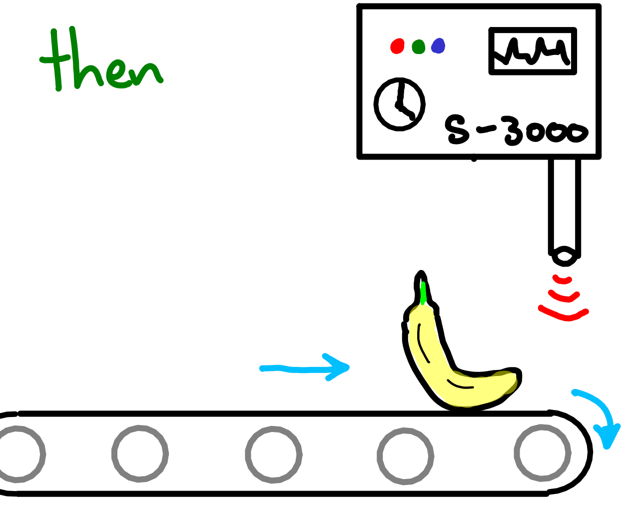
A machine has a sensor that sets 2 variables.



P_1

P_2

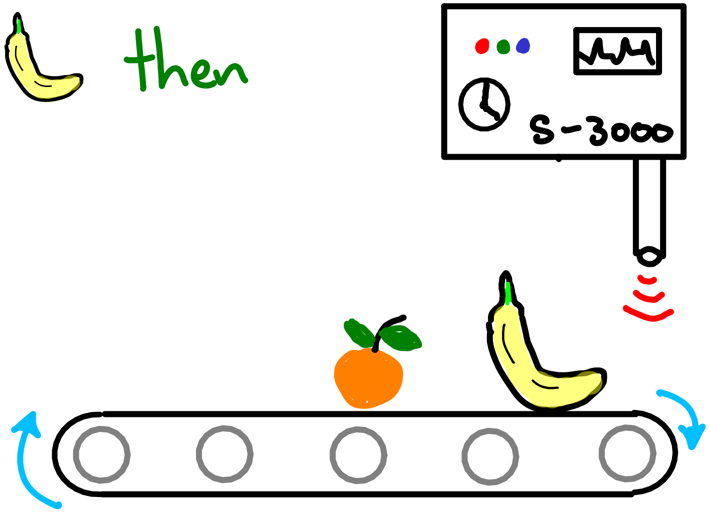
A machine has a sensor that sets 2 variables. If it senses  then $P_1 = T$, otherwise $P_1 = F$.



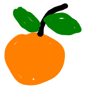
P_2

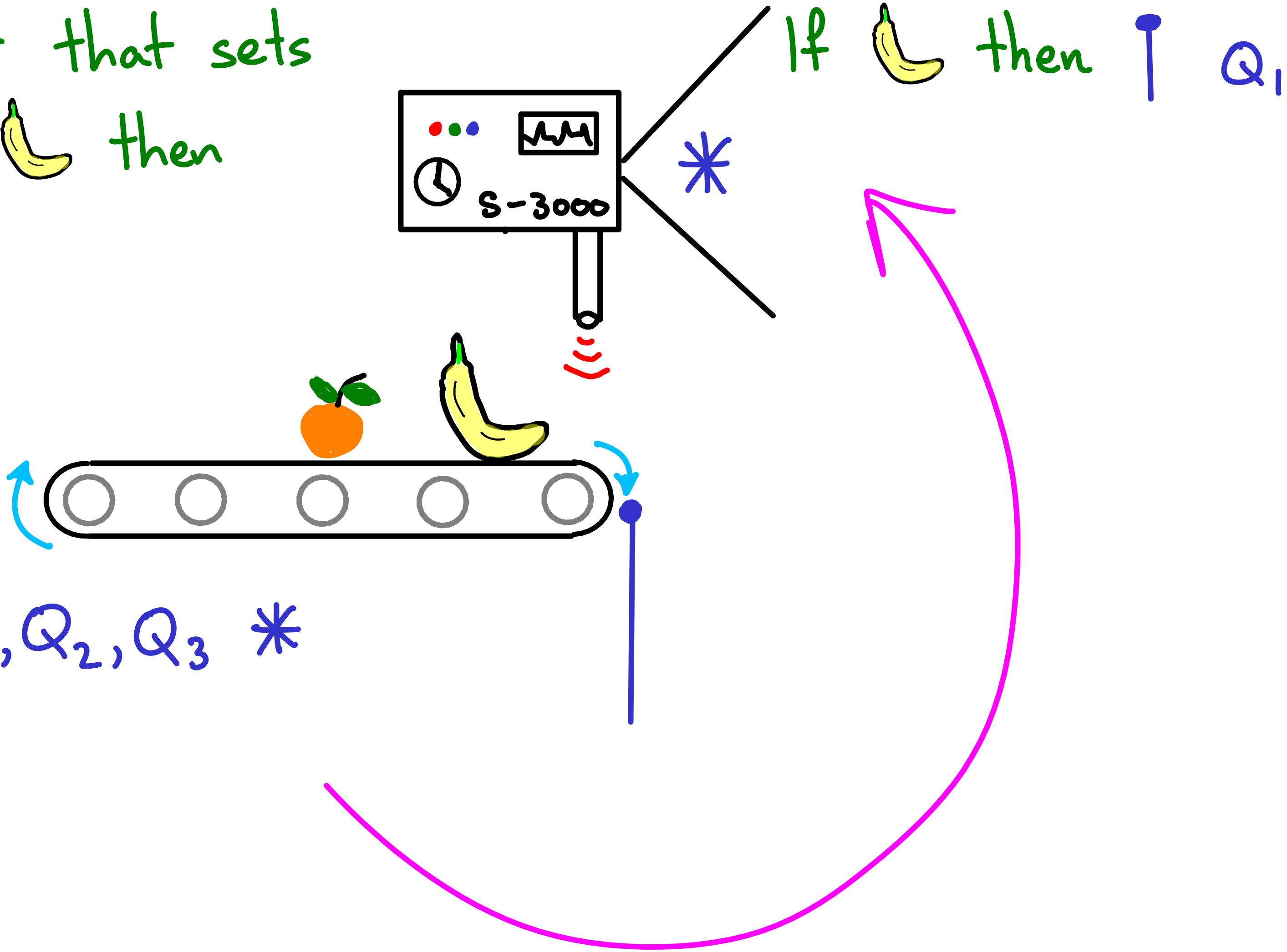
A machine has a sensor that sets 2 variables. If it senses 🍌 then $P_1 = T$, otherwise $P_1 = F$.

If it senses 🍊 then $P_2 = T$, otherwise $P_2 = F$.

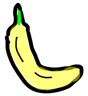


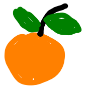
A machine has a sensor that sets 2 variables. If it senses then $P_1 = T$, otherwise $P_1 = F$.

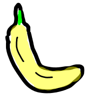
If it senses  then $P_2 = T$, otherwise $P_2 = F$.

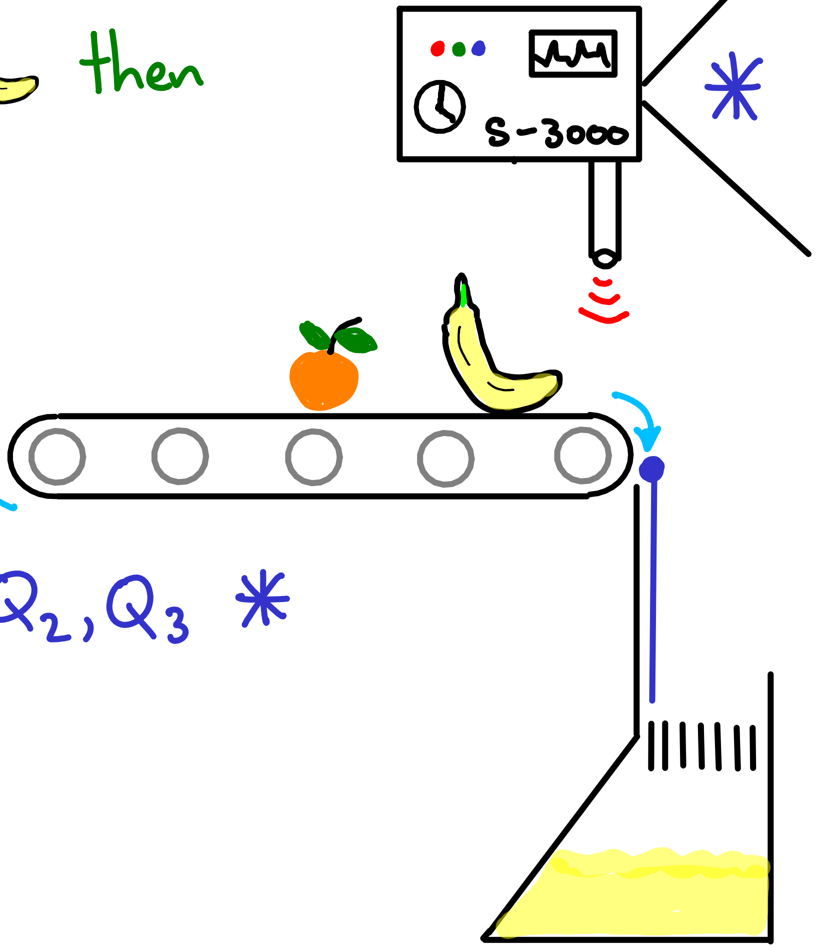


Also, 3 desired actions, Q_1, Q_2, Q_3 *

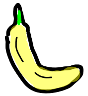
A machine has a sensor that sets 2 variables. If it senses  then $P_1 = T$, otherwise $P_1 = F$.

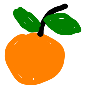
If it senses  then $P_2 = T$, otherwise $P_2 = F$.

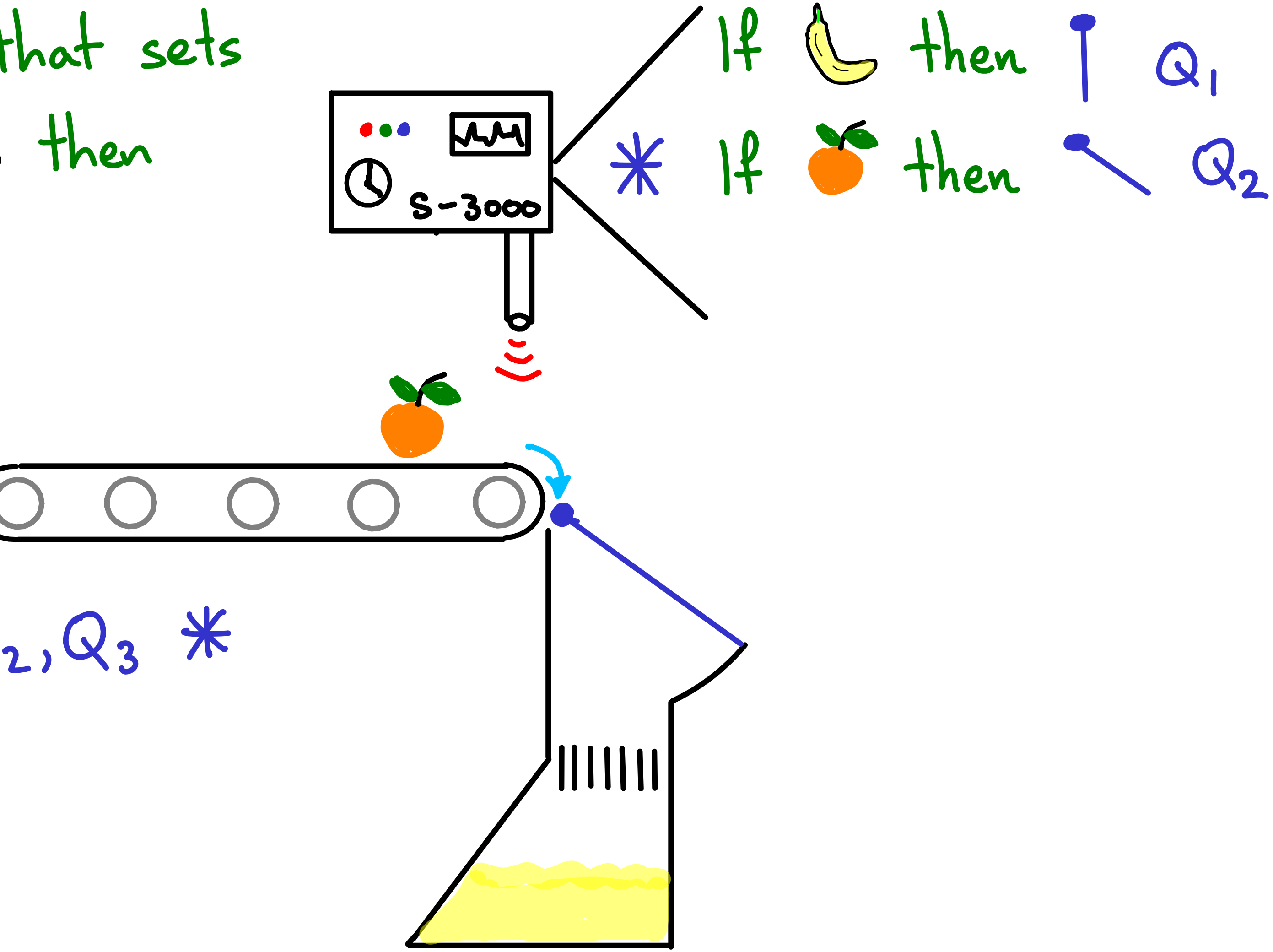
If  then  Q_1



Also, 3 desired actions, Q_1, Q_2, Q_3 *

A machine has a sensor that sets 2 variables. If it senses  then $P_1 = T$, otherwise $P_1 = F$.

If it senses  then $P_2 = T$, otherwise $P_2 = F$.

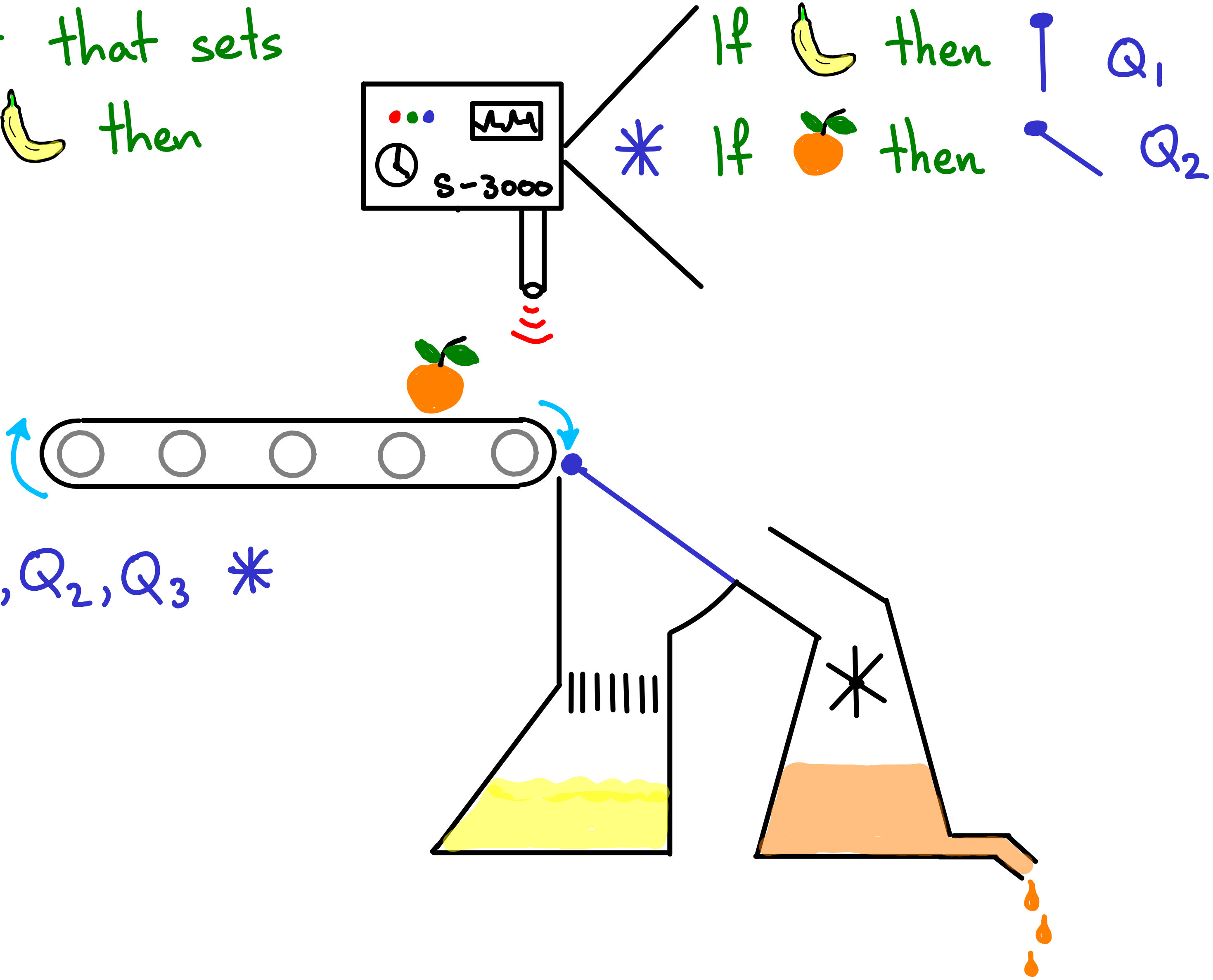


Also, 3 desired actions, Q_1, Q_2, Q_3 *

A machine has a sensor that sets 2 variables. If it senses [banana] then $P_1 = T$, otherwise $P_1 = F$.

If it senses [orange] then $P_2 = T$, otherwise $P_2 = F$.

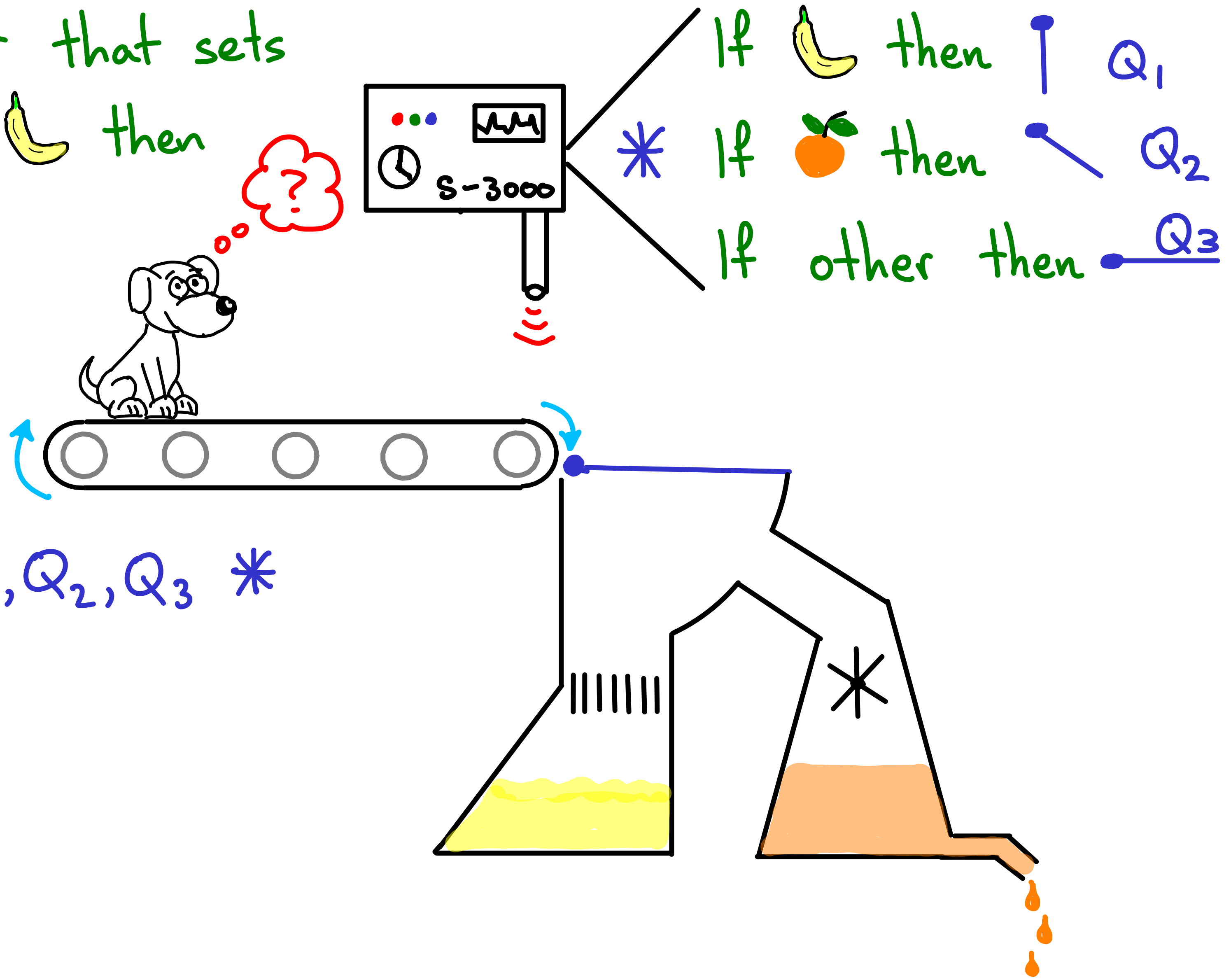
Also, 3 desired actions, Q_1, Q_2, Q_3 *



A machine has a sensor that sets 2 variables. If it senses then $P_1 = T$, otherwise $P_1 = F$.

If it senses then $P_2 = T$, otherwise $P_2 = F$.

Also, 3 desired actions, Q_1, Q_2, Q_3 *



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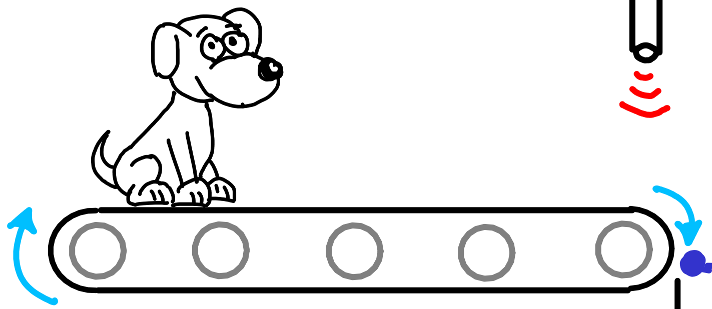

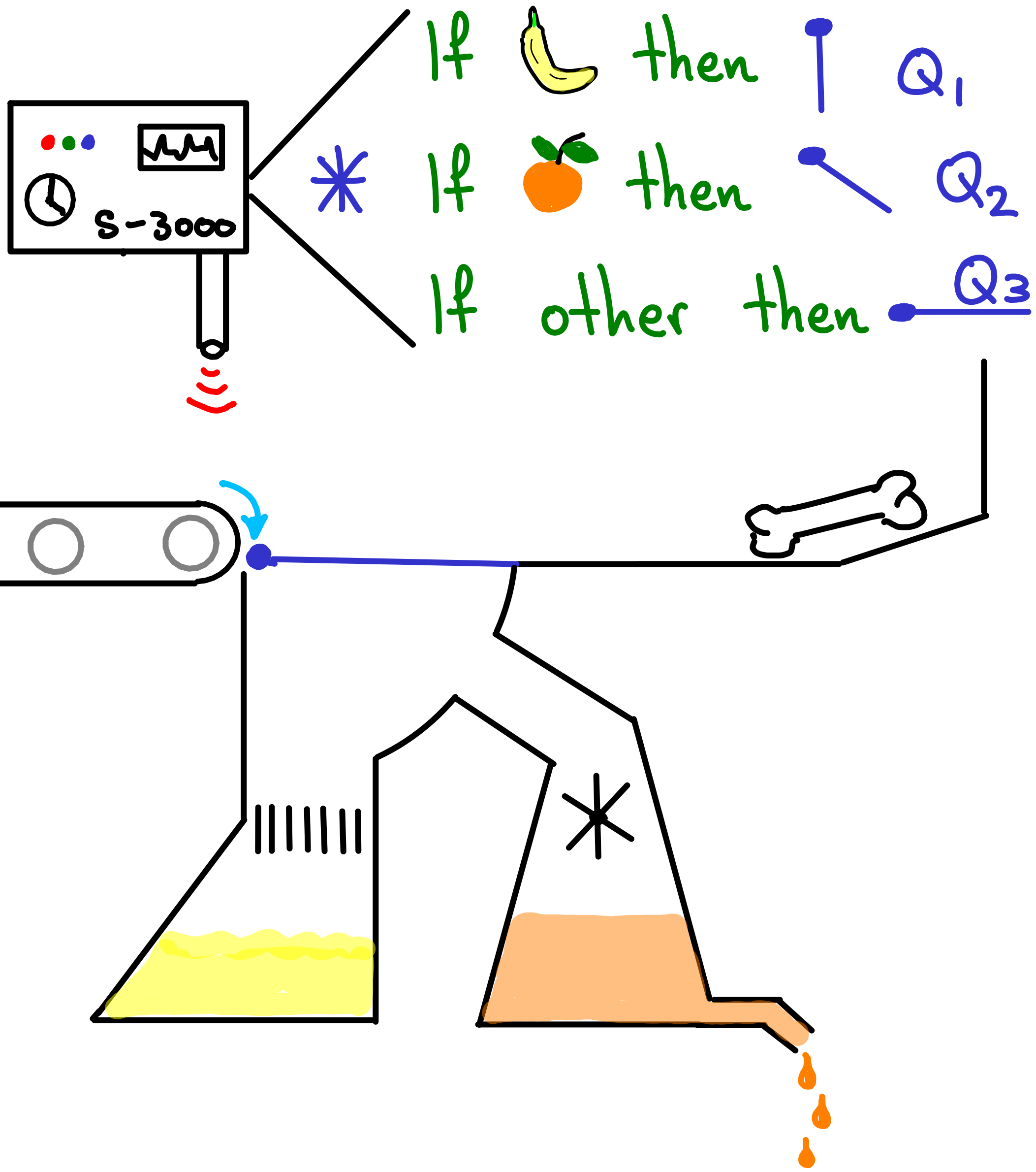

Also, 3 desired actions, Q_1, Q_2, Q_3 *

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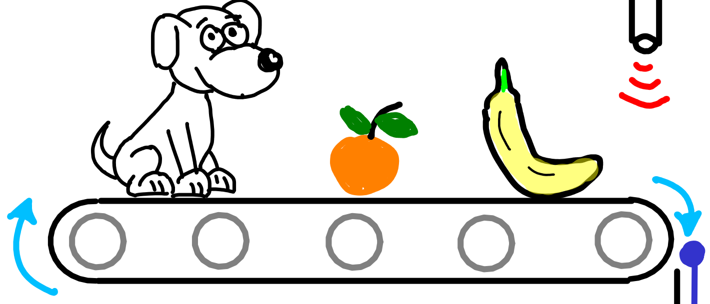
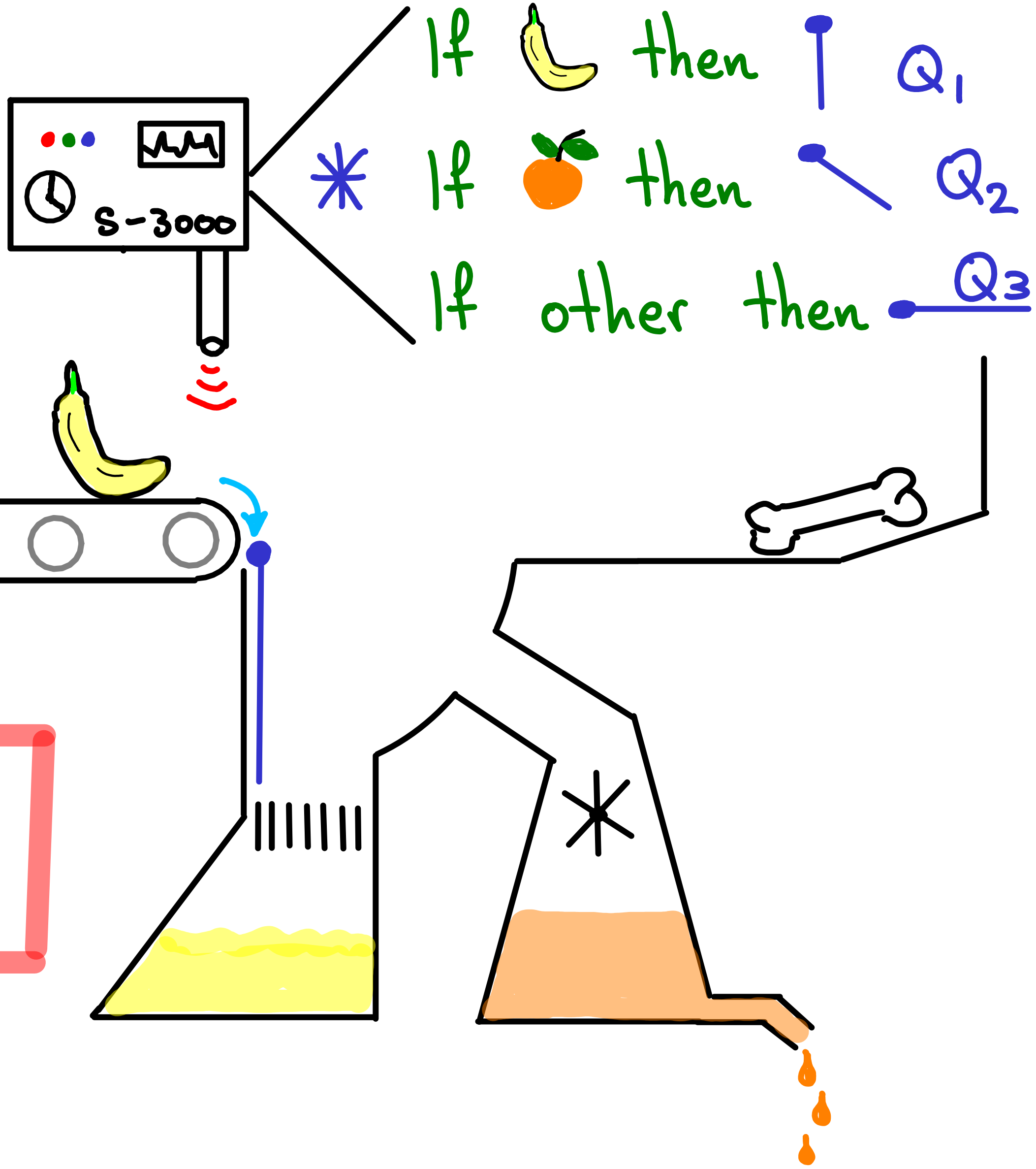
If it senses then $P_2 = T$, otherwise $P_2 = F$.



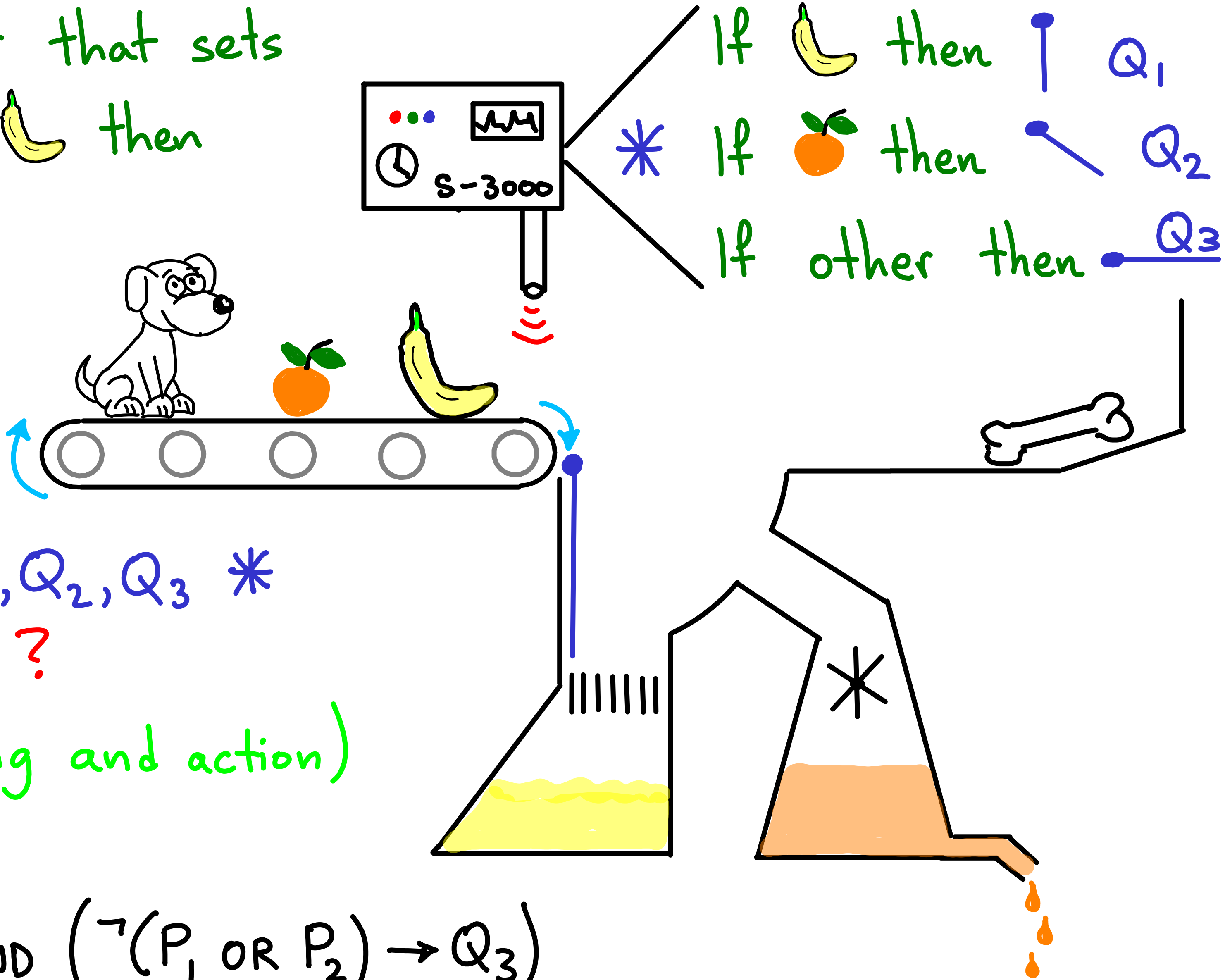


Also, 3 desired actions, Q_1, Q_2, Q_3 *

Is the machine working?
(given a sensor reading and action)

A machine has a sensor that sets 2 variables. If it senses 🍌 then $P_1 = T$, otherwise $P_1 = F$.

If it senses 🍊 then $P_2 = T$, otherwise $P_2 = F$.



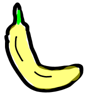
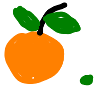
Also, 3 desired actions, Q_1, Q_2, Q_3 *

Is the machine working?

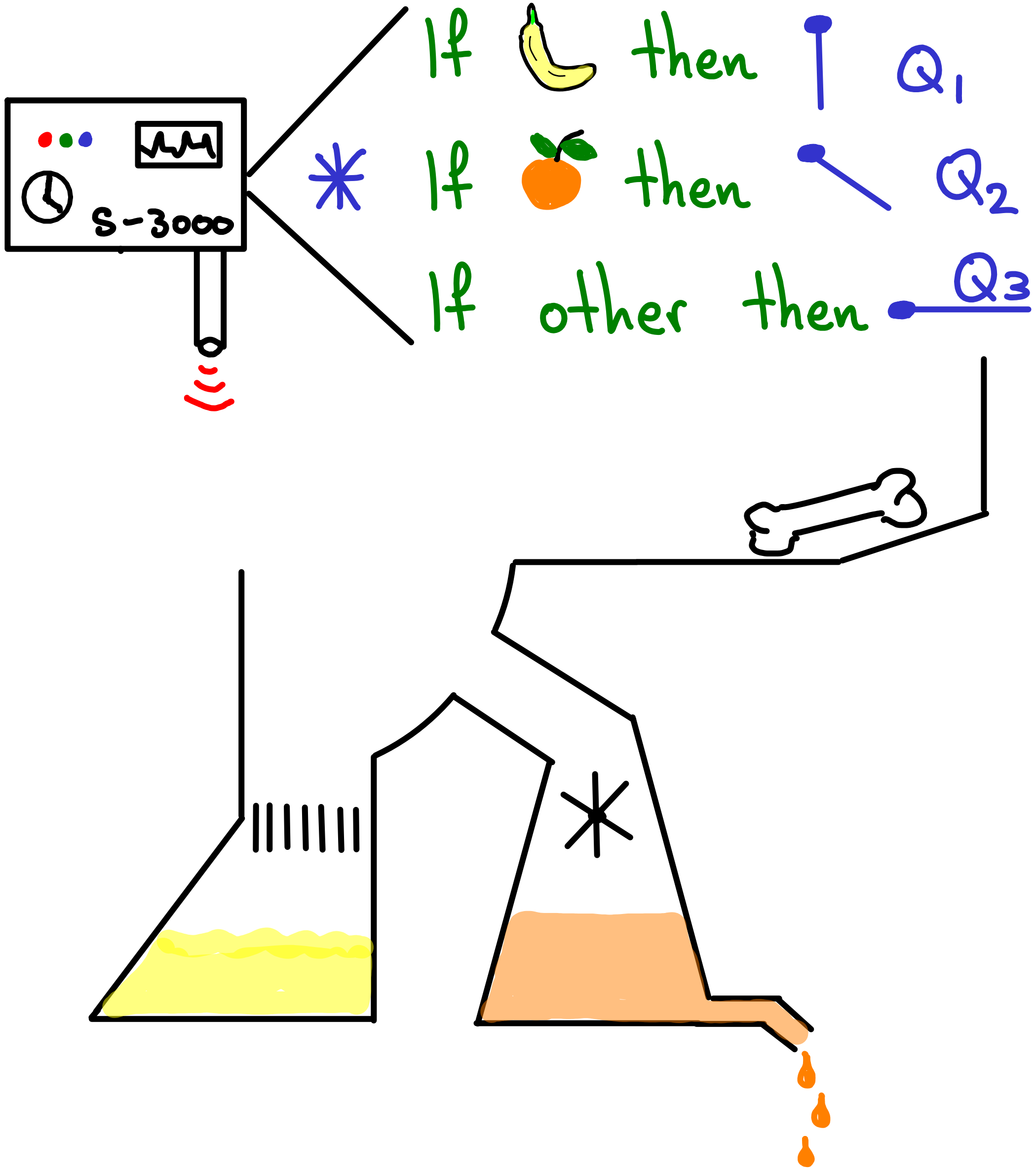
(given a sensor reading and action)

Evaluate:

$$(P_1 \to Q_1) \text{ AND } (P_2 \to Q_2) \text{ AND } (\neg(P_1 \text{ OR } P_2) \to Q_3)$$

P_1 IFF . P_2 IFF .

Is the machine working?



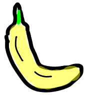
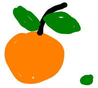
$$(P_1 \to Q_1)$$

AND

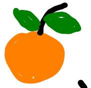
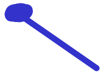
$$(P_2 \to Q_2)$$

AND

$$(\neg(P_1 \text{ OR } P_2) \to Q_3)$$

P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action = 

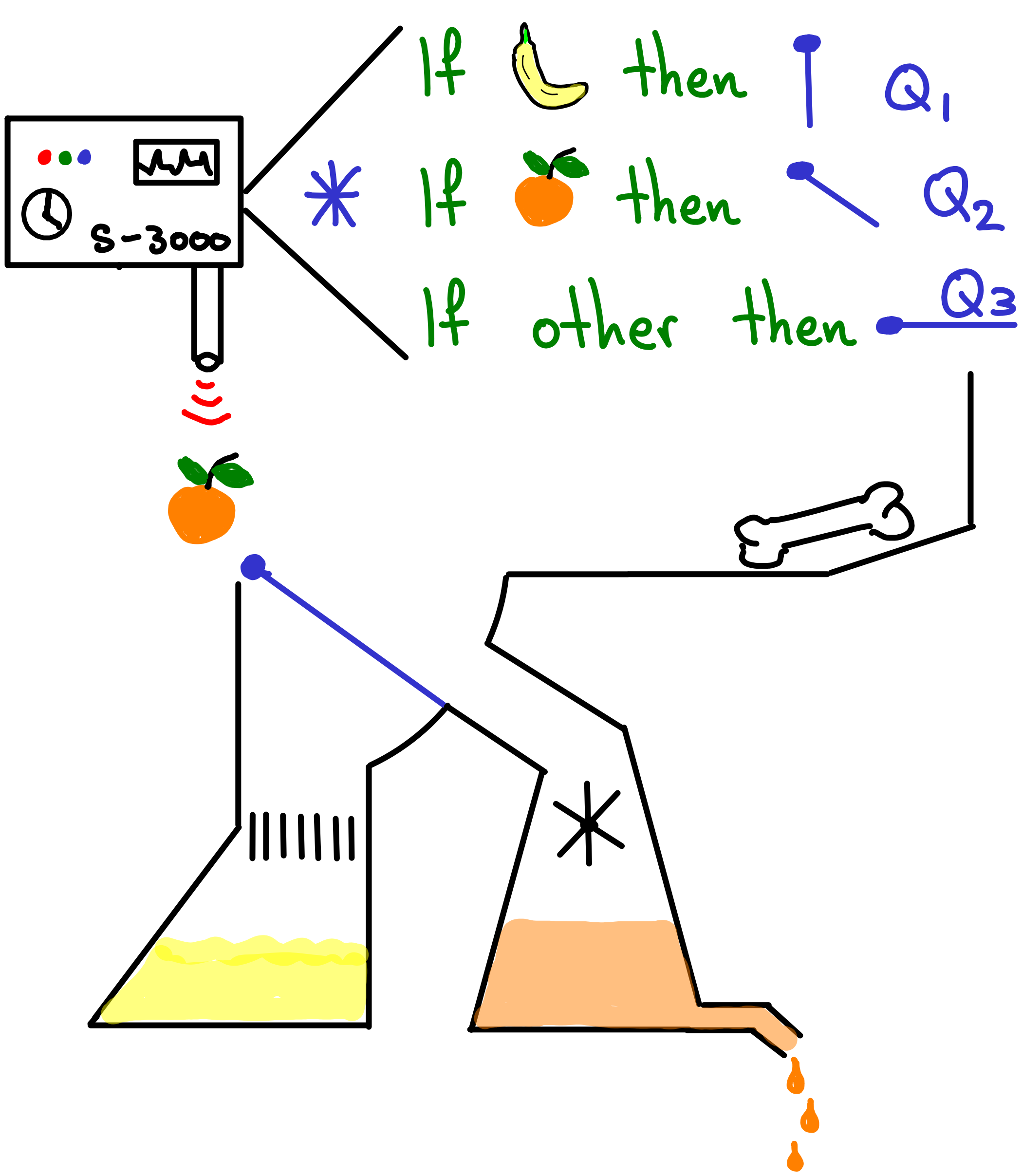
$(P_1 \to Q_1)$

AND

$(P_2 \to Q_2)$

AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

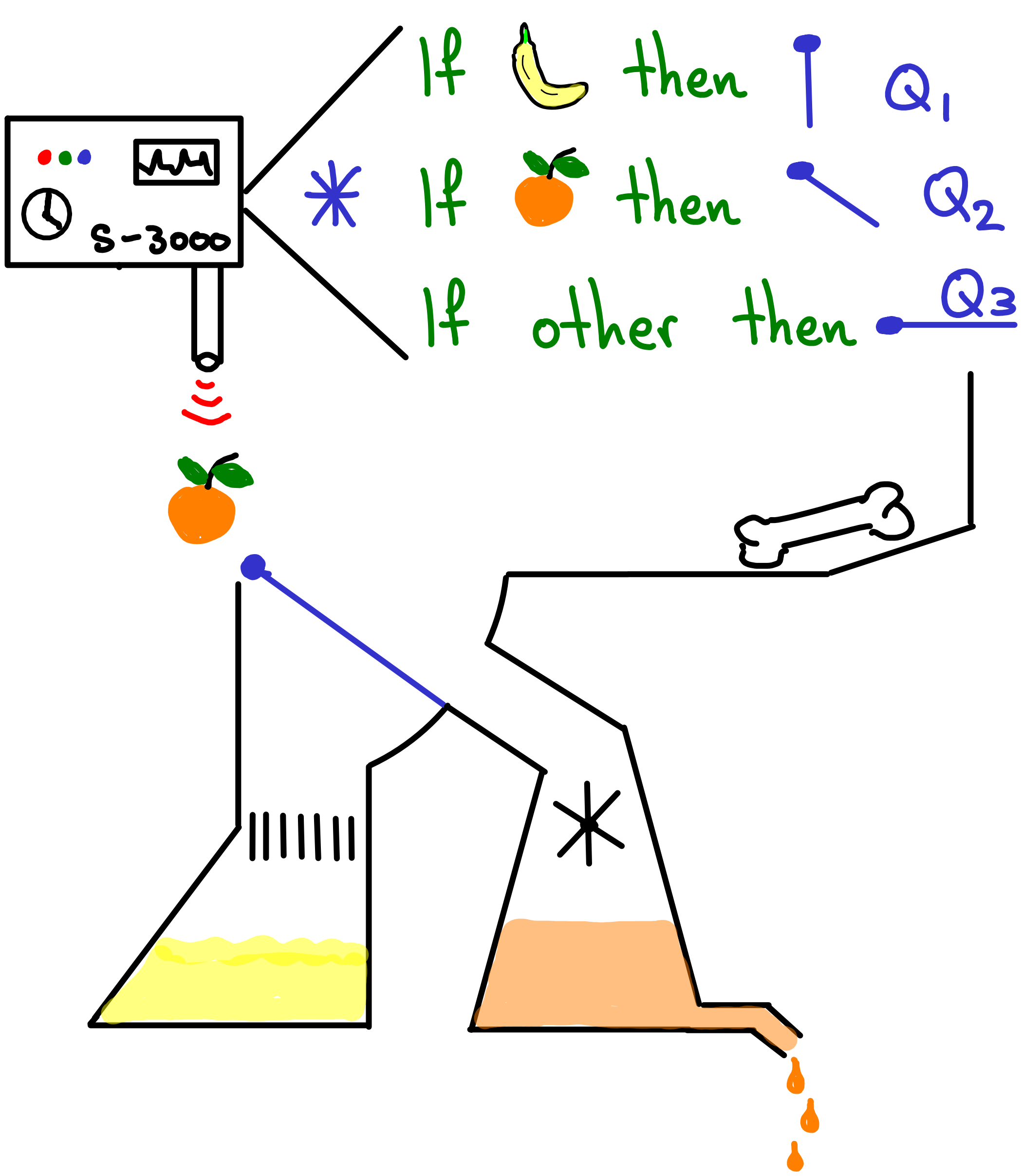
$(P_1 \to Q_1)$

AND

$(P_2 \to Q_2)$

AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \rightarrow Q_1)$

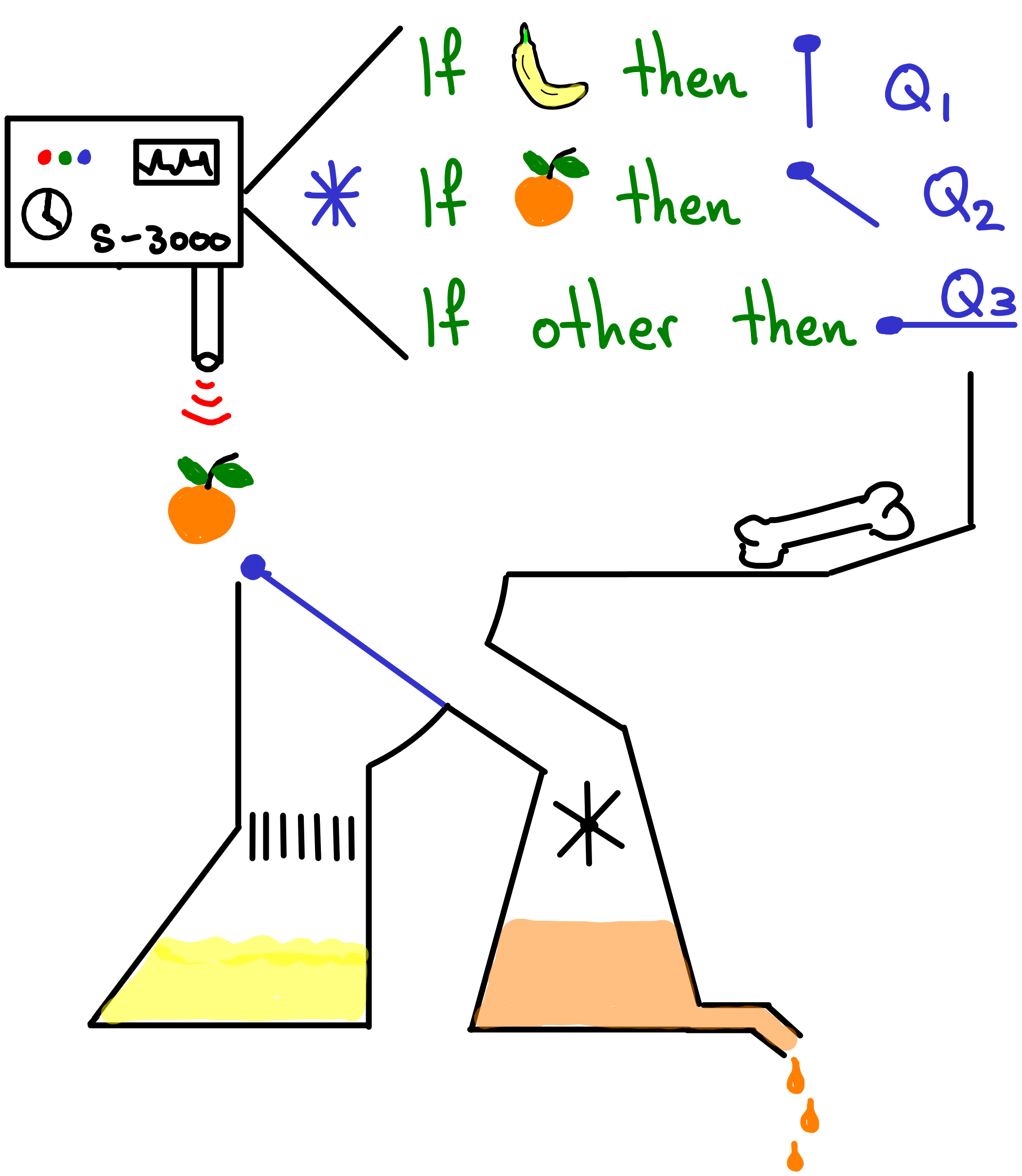
AND

$(P_2 \rightarrow Q_2)$

AND

$(\neg(P_1 \text{ OR } P_2) \rightarrow Q_3)$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$

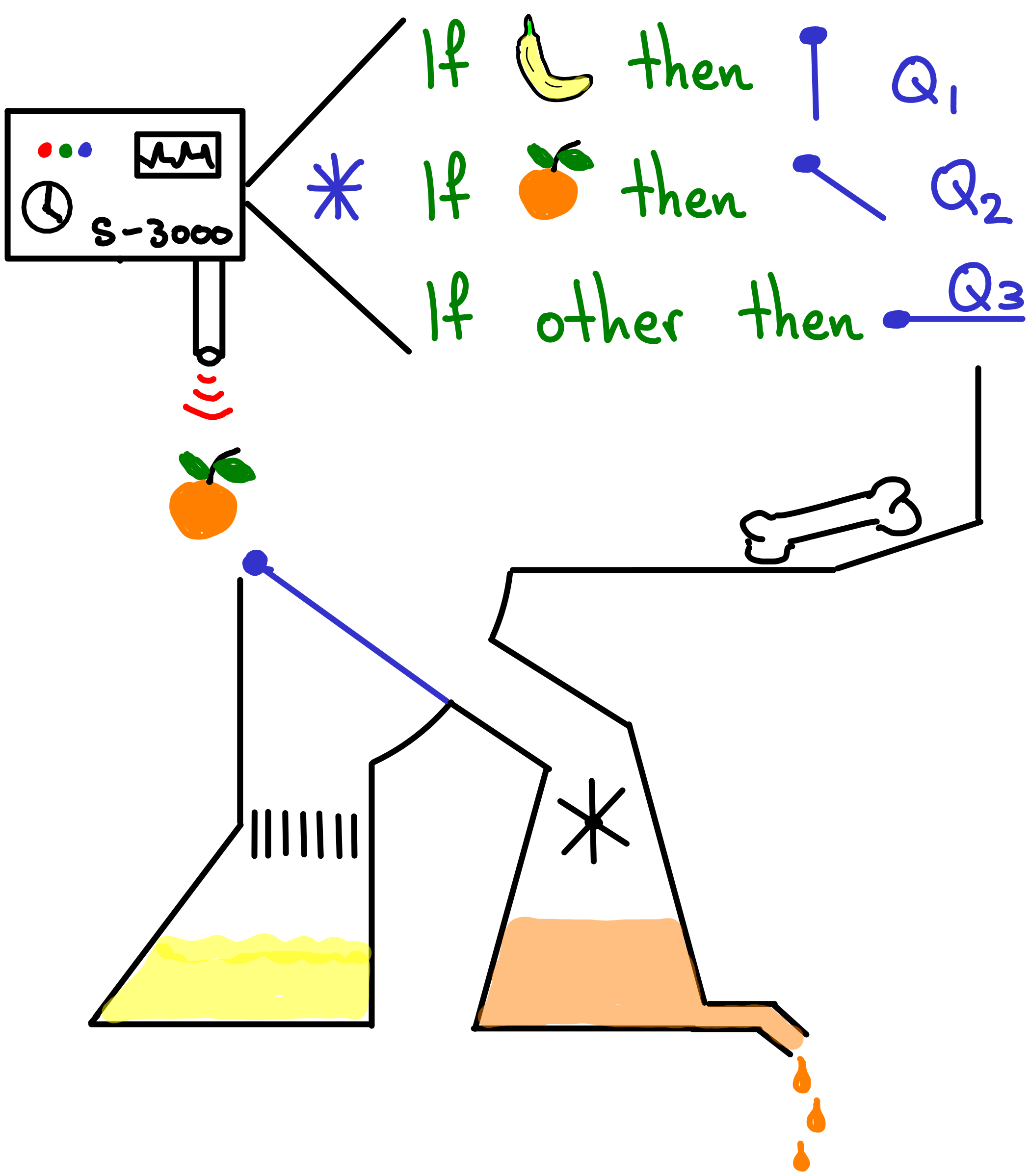
AND

$(P_2 \to Q_2)$ } $T \to T$

AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$

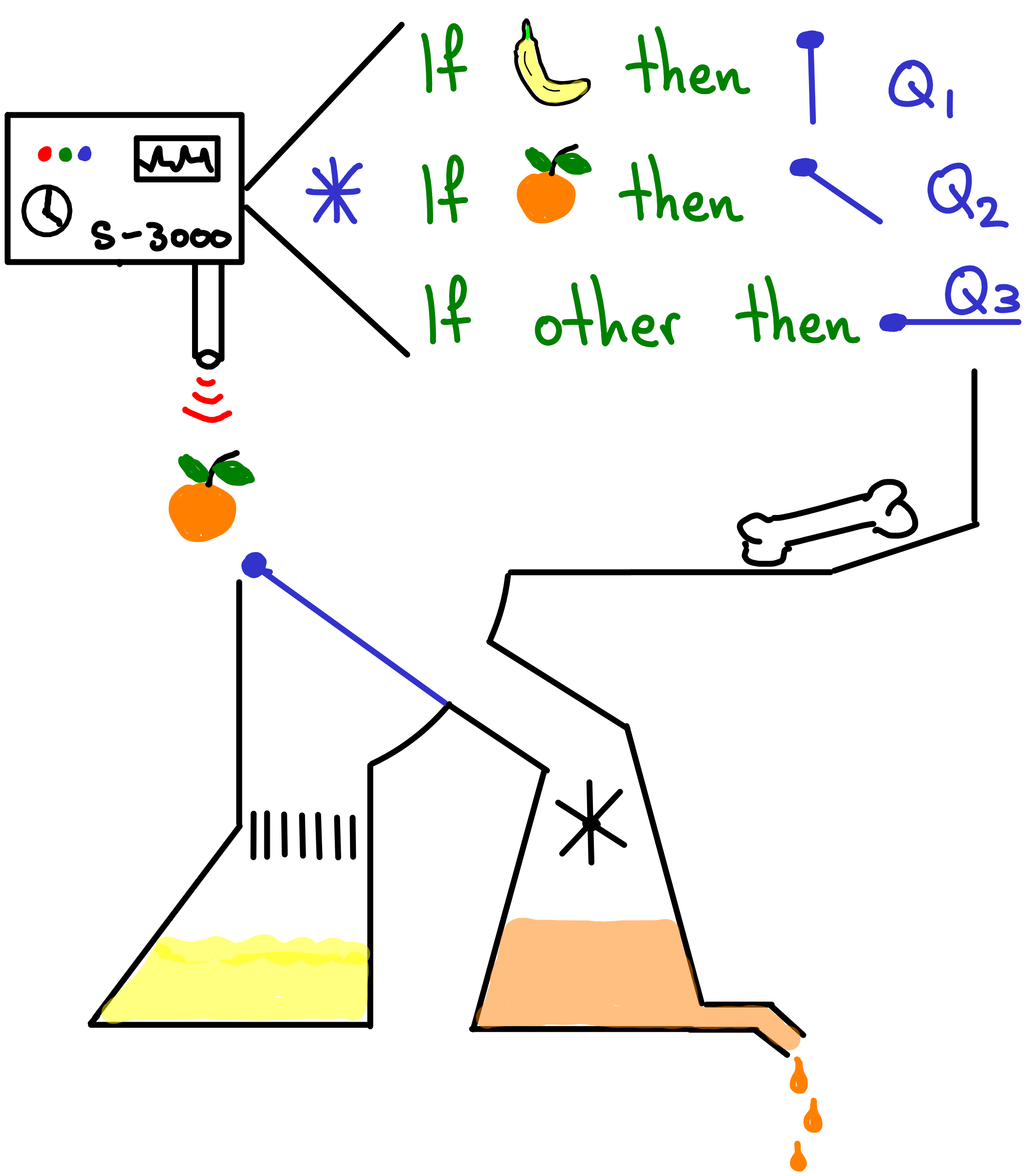
AND

$(P_2 \to Q_2)$ } $T \to T$: T

AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \rightarrow Q_1)$ } $F \rightarrow F$

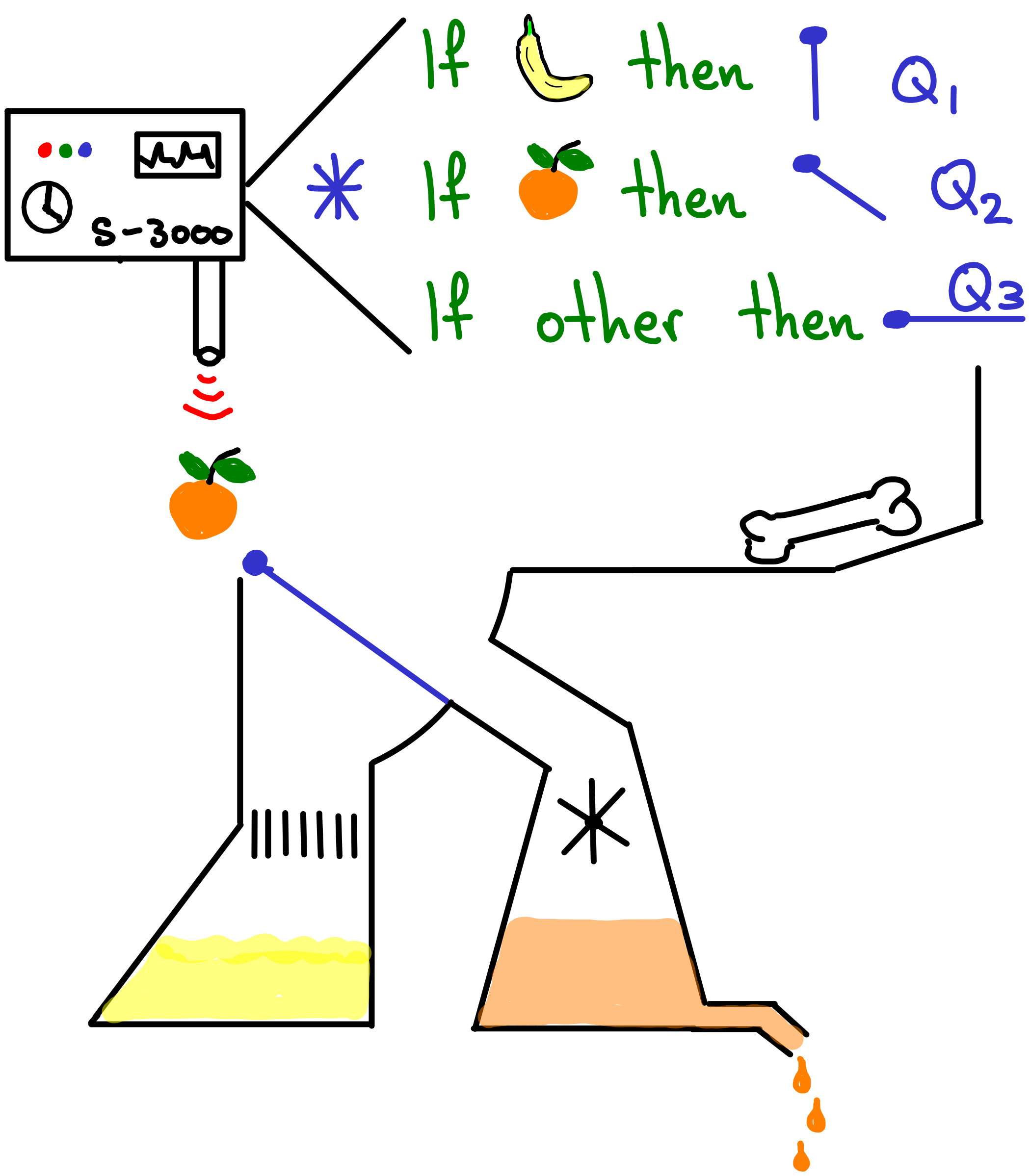
AND

$(P_2 \rightarrow Q_2)$ } $T \rightarrow T$: T

AND

$(\neg(P_1 \text{ OR } P_2) \rightarrow Q_3)$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$ } F → F : T

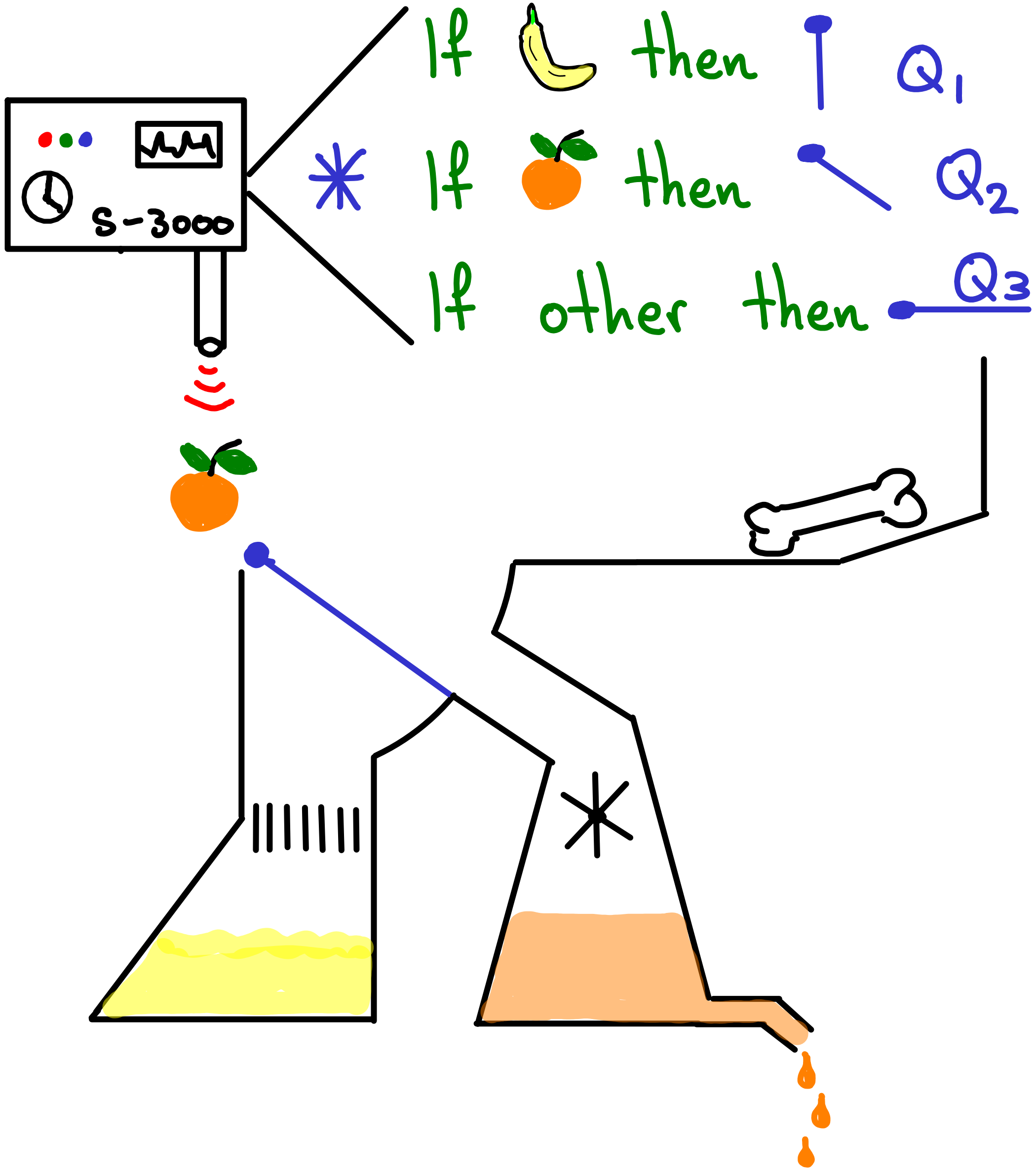
AND

$(P_2 \to Q_2)$ } T → T : T

AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \rightarrow Q_1)$ } F → F : T

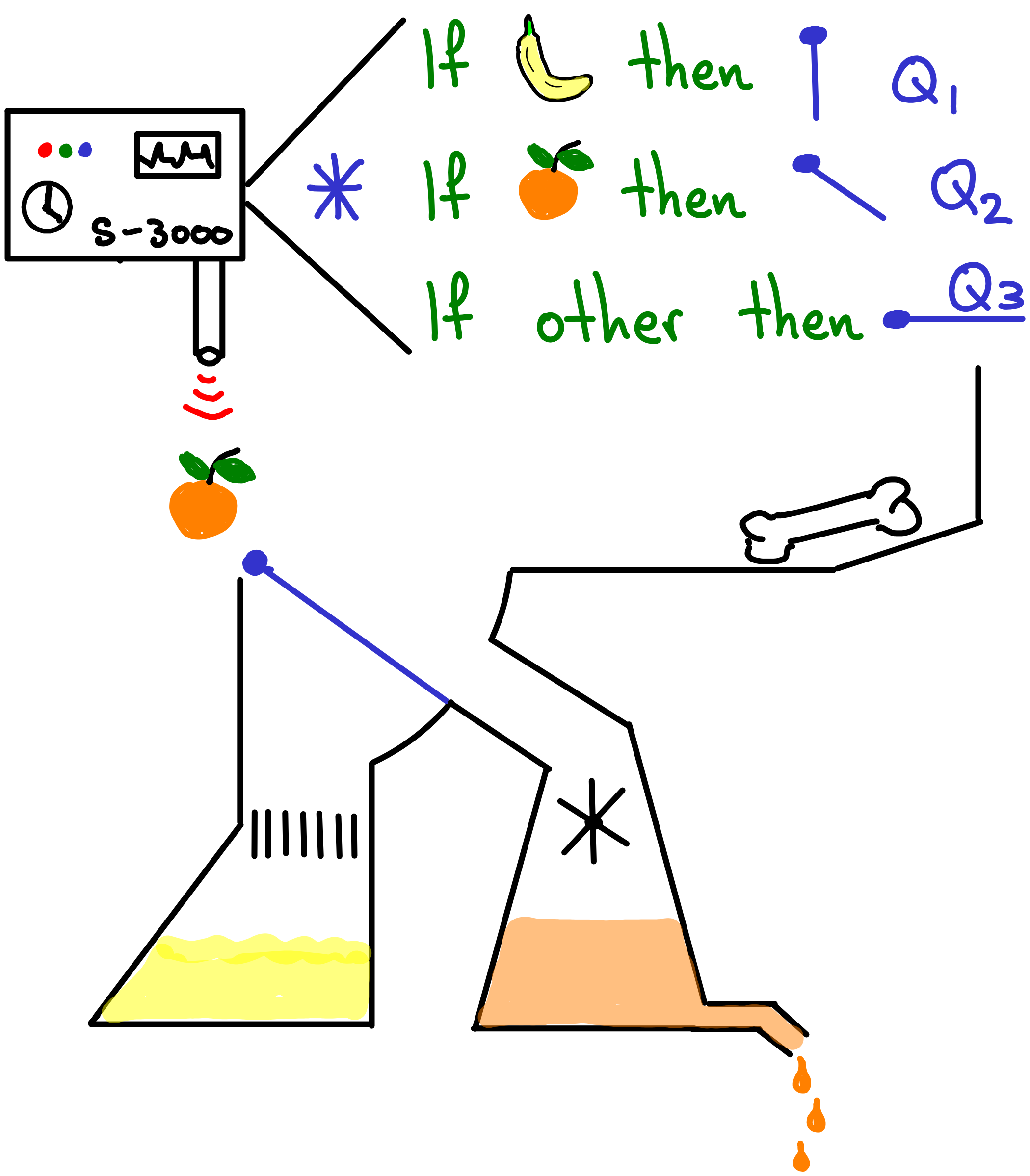
AND

$(P_2 \rightarrow Q_2)$ } T → T : T

AND

$(\neg(P_1 \text{ OR } P_2) \rightarrow Q_3)$ } ¬(F OR T) → F

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$ } $F \to F$: T

AND

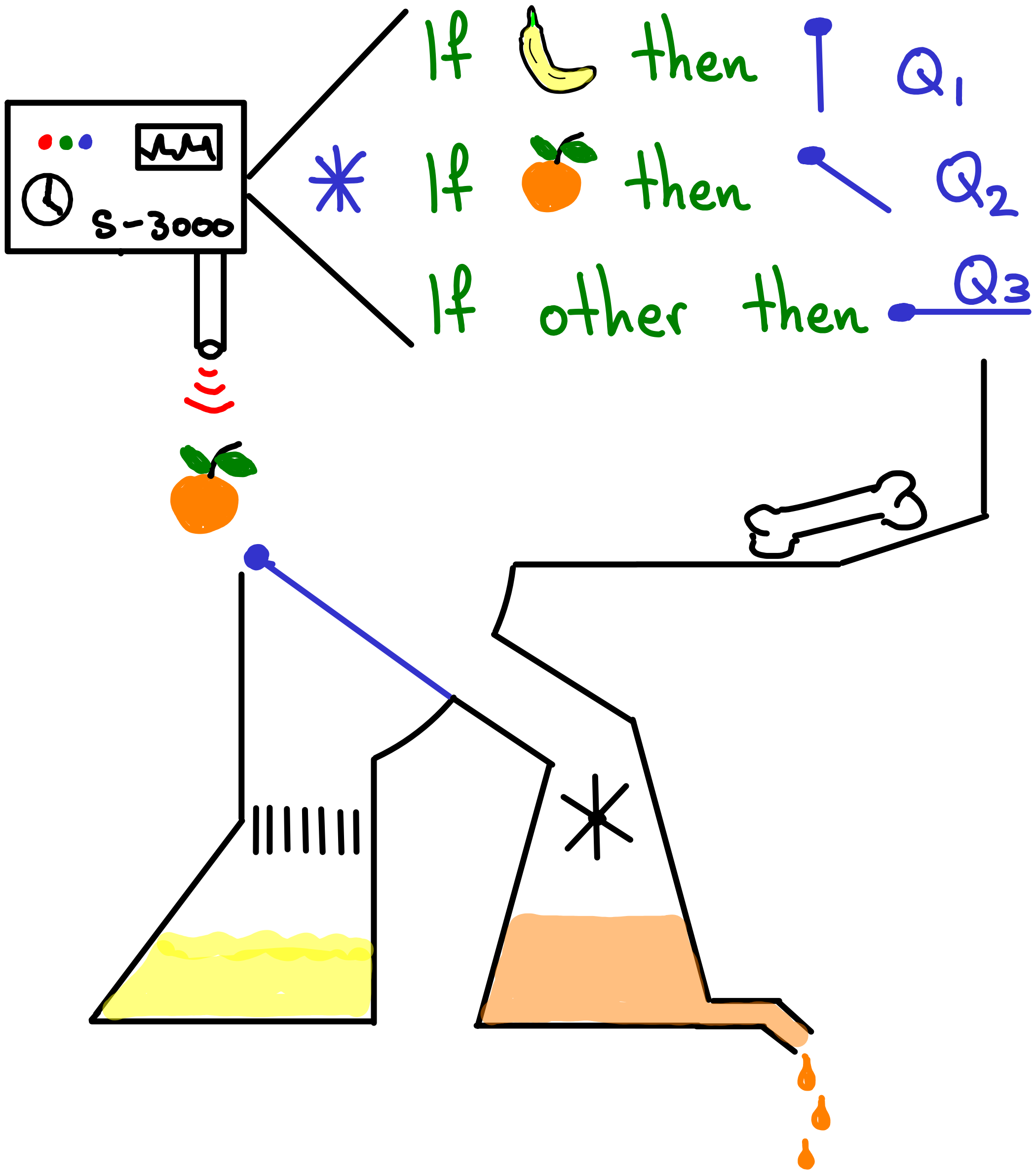
$(P_2 \to Q_2)$ } $T \to T$: T

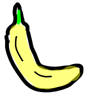
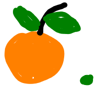
AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$ } $\neg(F \text{ OR } T) \to F$

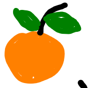
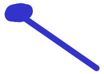
$\neg T \to F$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action = 

$(P_1 \to Q_1)$ } F → F : T

AND

$(P_2 \to Q_2)$ } T → T : T

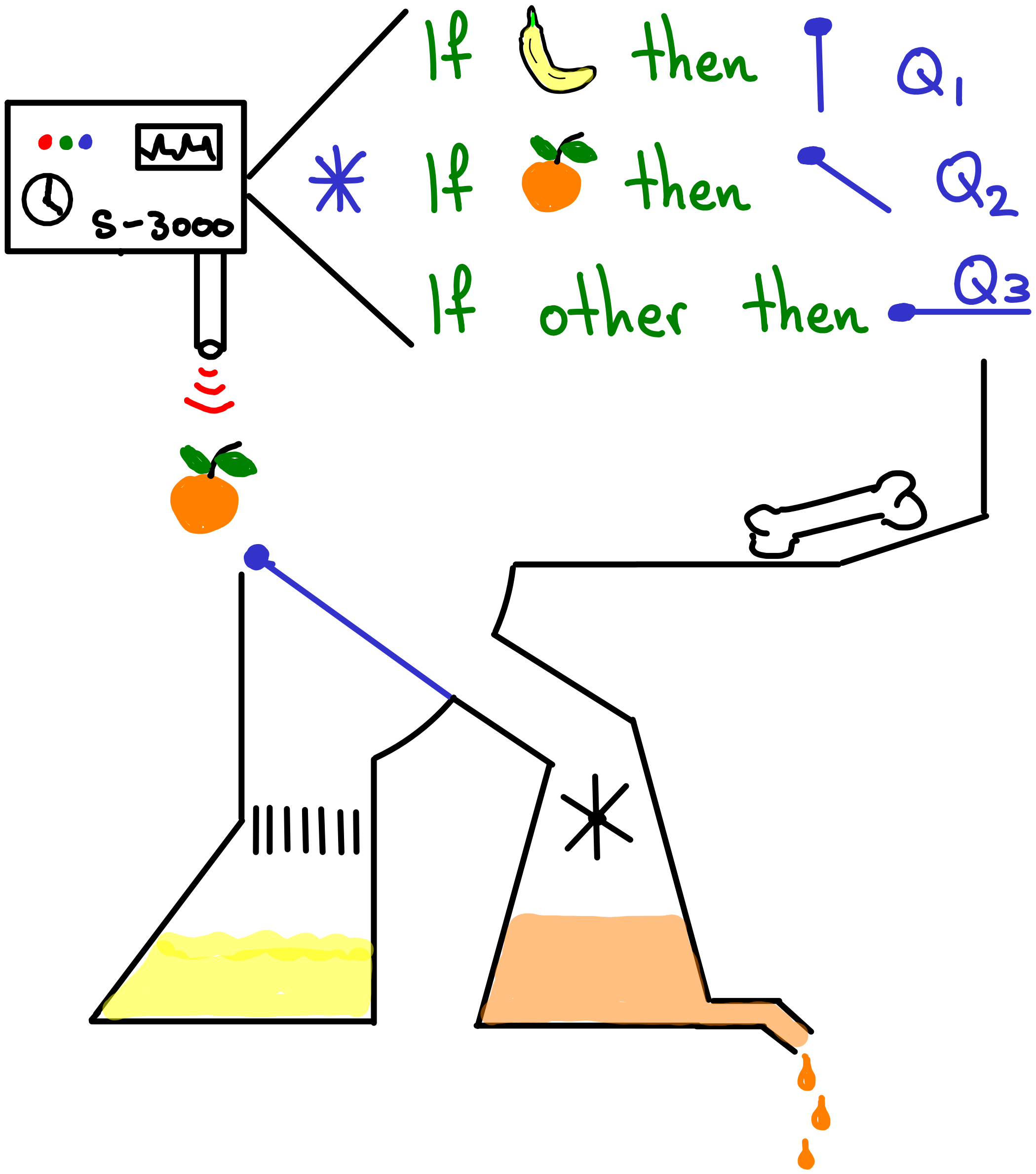
AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$ } $\neg$(F OR T) → F

$\neg$T → F

F → F

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \rightarrow Q_1)$ } $F \rightarrow F$: T

AND

$(P_2 \rightarrow Q_2)$ } $T \rightarrow T$: T

AND

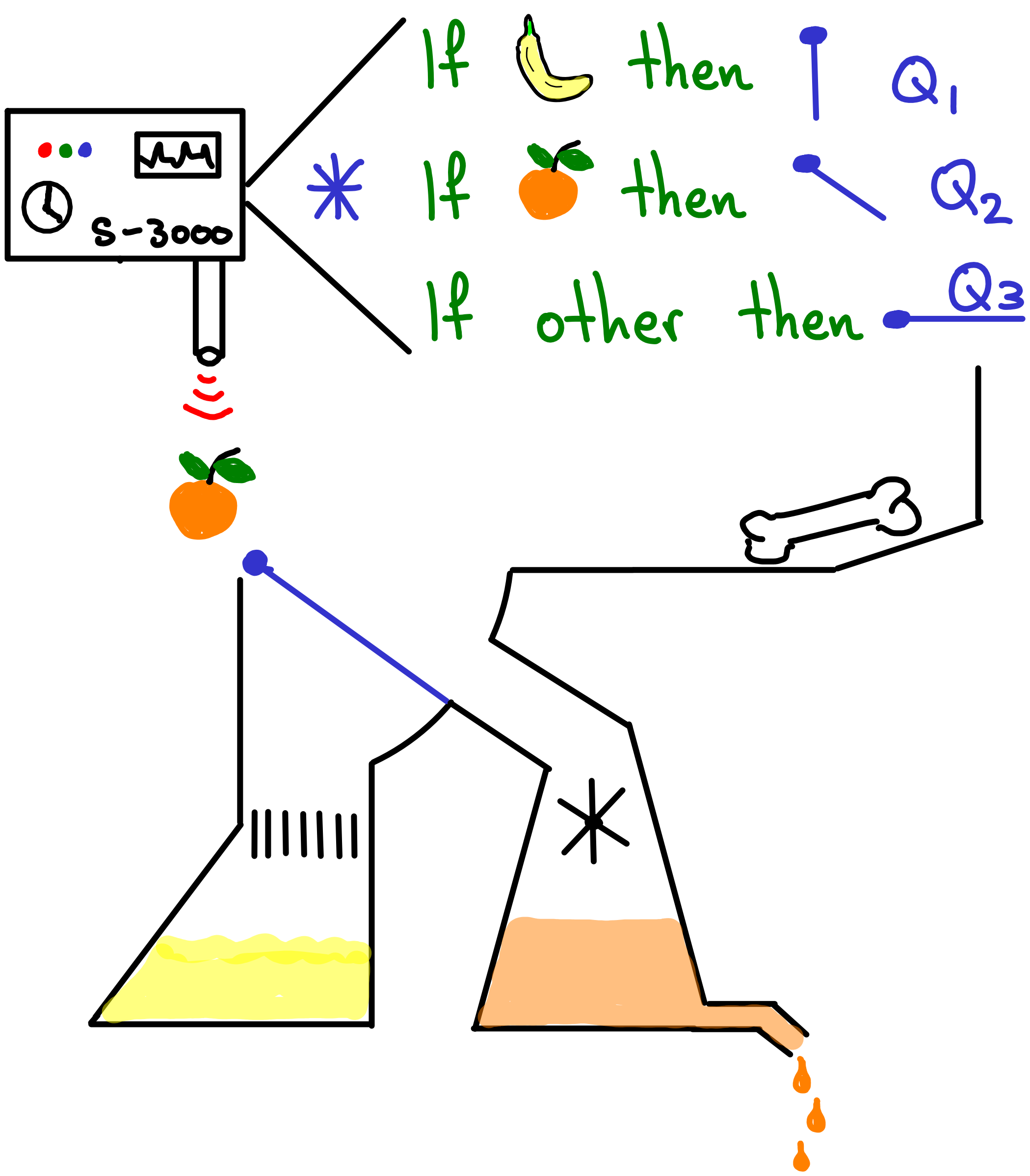
$(\neg(P_1 \text{ OR } P_2) \rightarrow Q_3)$ } $\neg(F \text{ OR } T) \rightarrow F$

$\neg T \rightarrow F$

$F \rightarrow F$

T

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$ } $F \to F$: T

AND

$(P_2 \to Q_2)$ } $T \to T$: T

AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$ } $\neg(F \text{ OR } T) \to F$

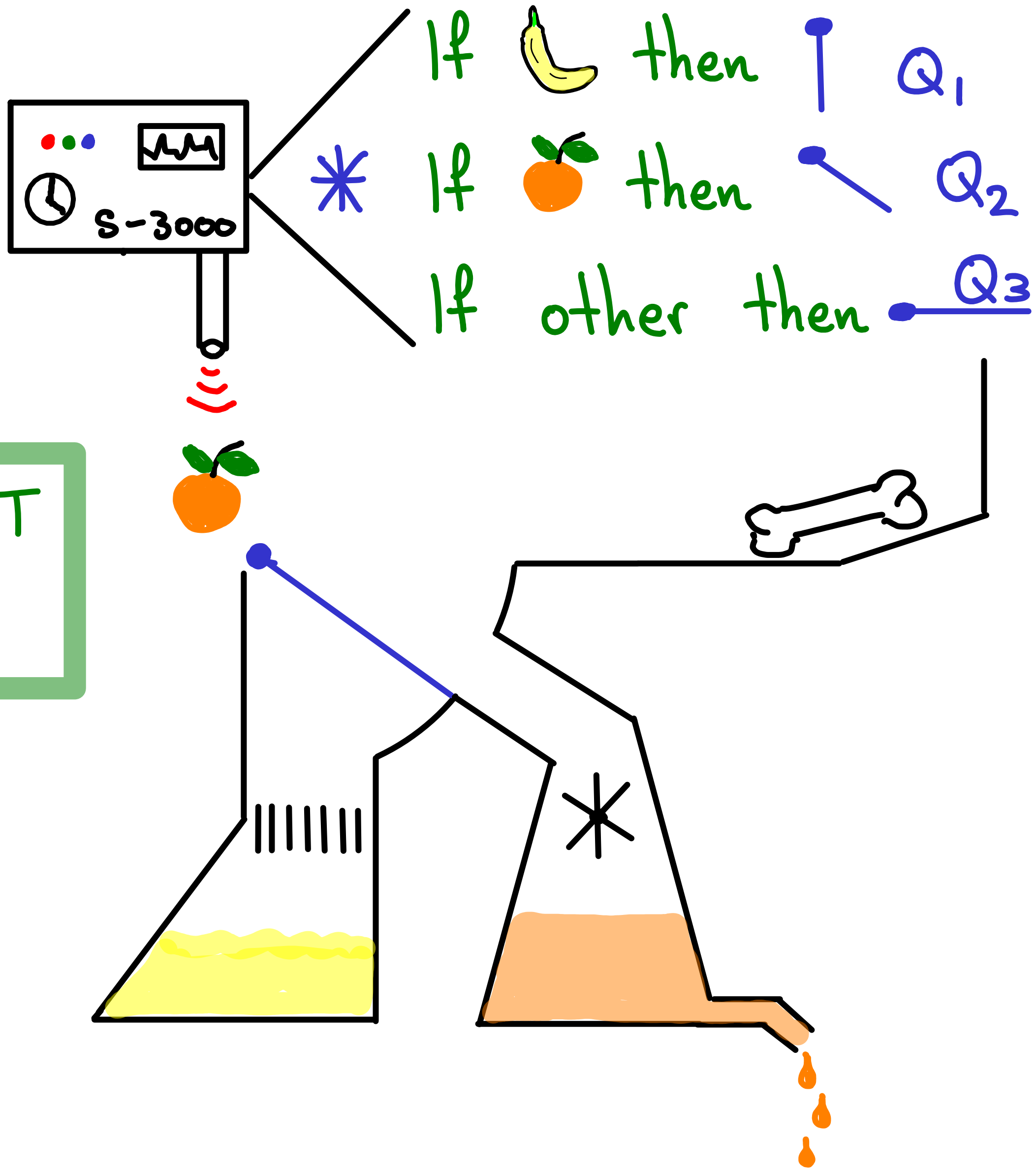
$\neg T \to F$

$F \to F$

T

$\boxed{\text{T AND T AND T}}$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working? Yes.

e.g., sense , action =

$(P_1 \to Q_1)$ } F → F : T

AND

$(P_2 \to Q_2)$ } T → T : T

AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$ } $\neg$(F OR T) → F

$\neg$T → F

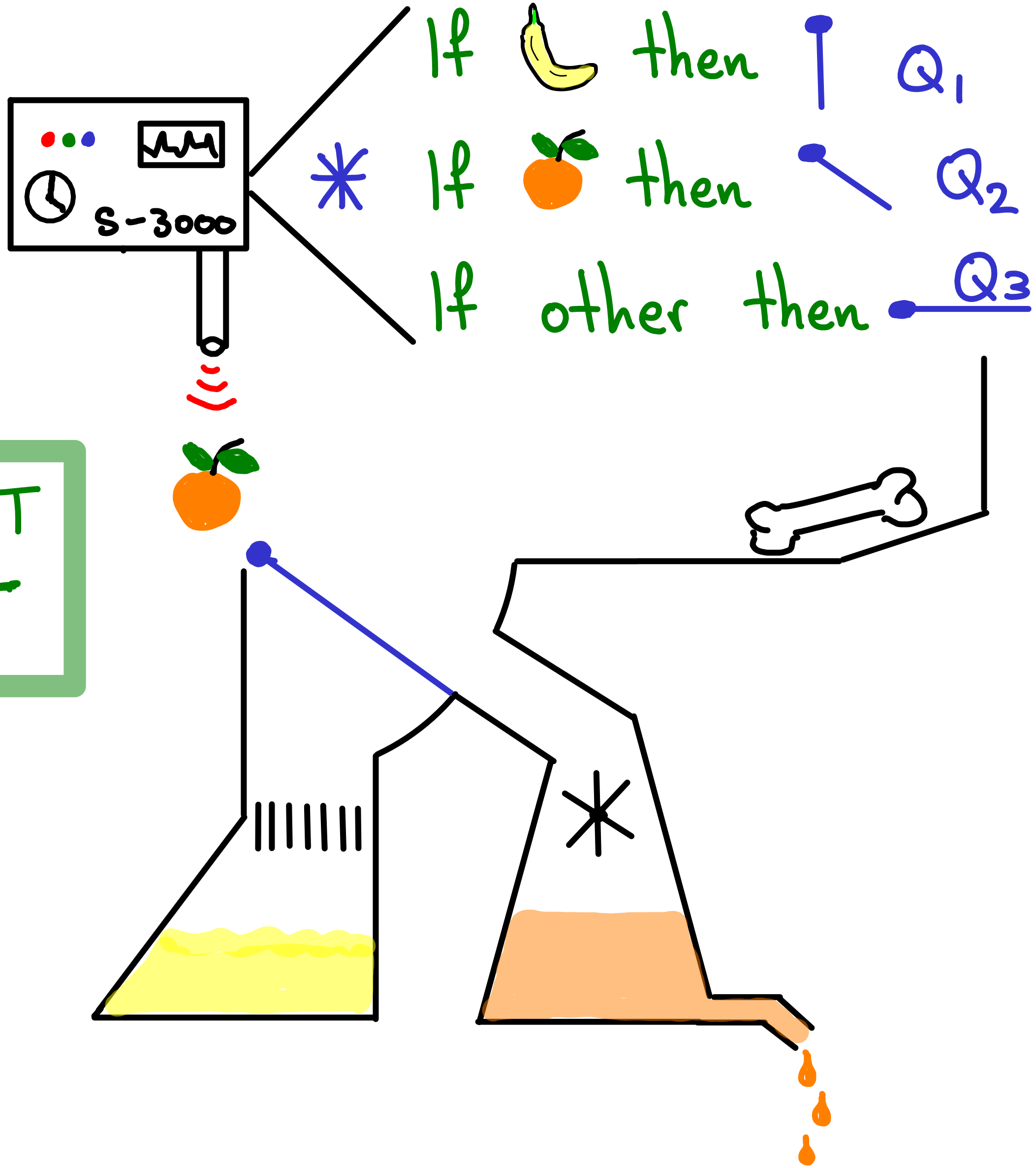
F → F

T

T AND T AND T

conclusion: T

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \rightarrow Q_1)$

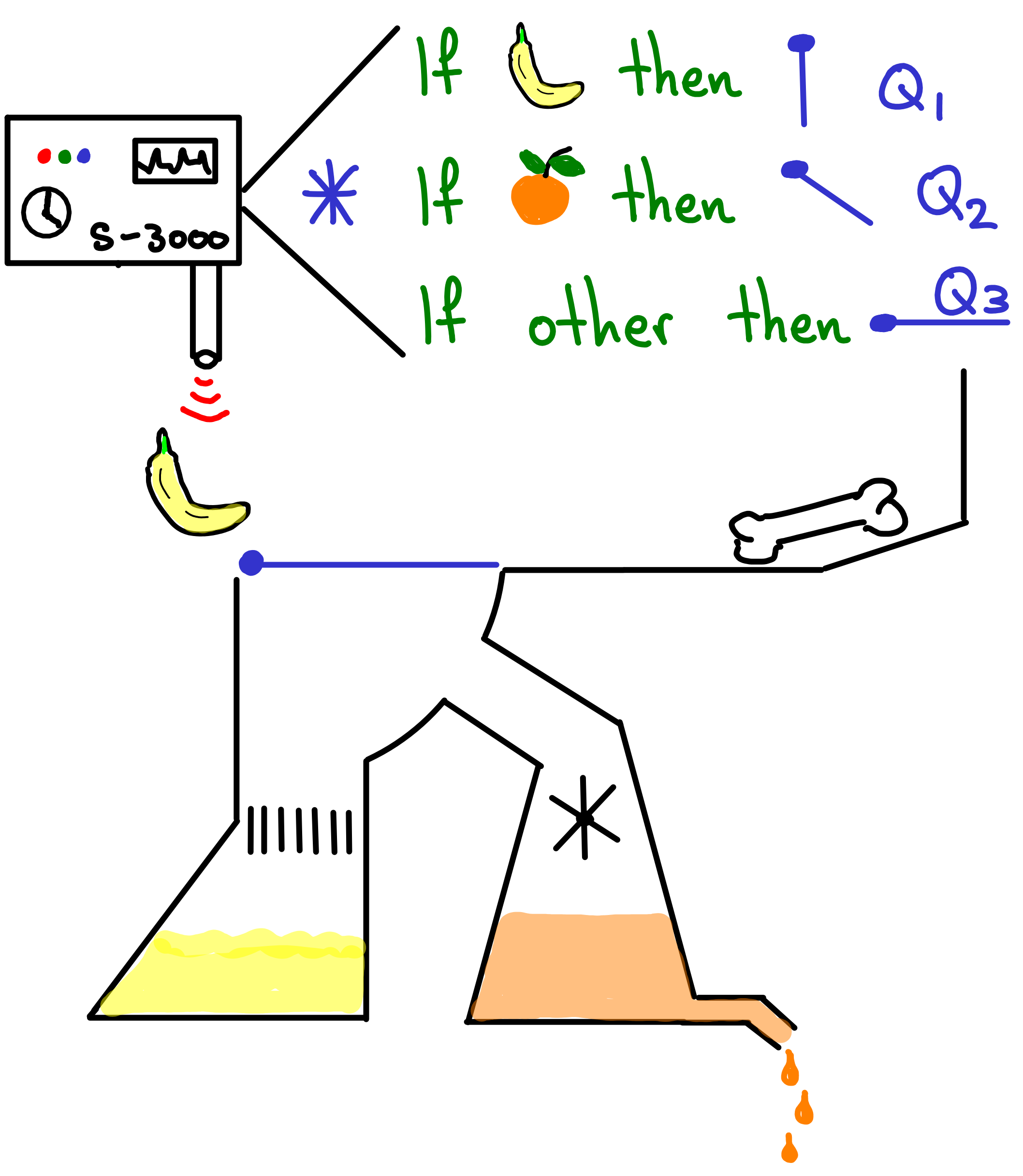
AND

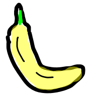
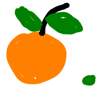
$(P_2 \rightarrow Q_2)$

AND

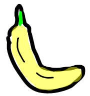
$(\neg(P_1 \text{ OR } P_2) \rightarrow Q_3)$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$ } T → F

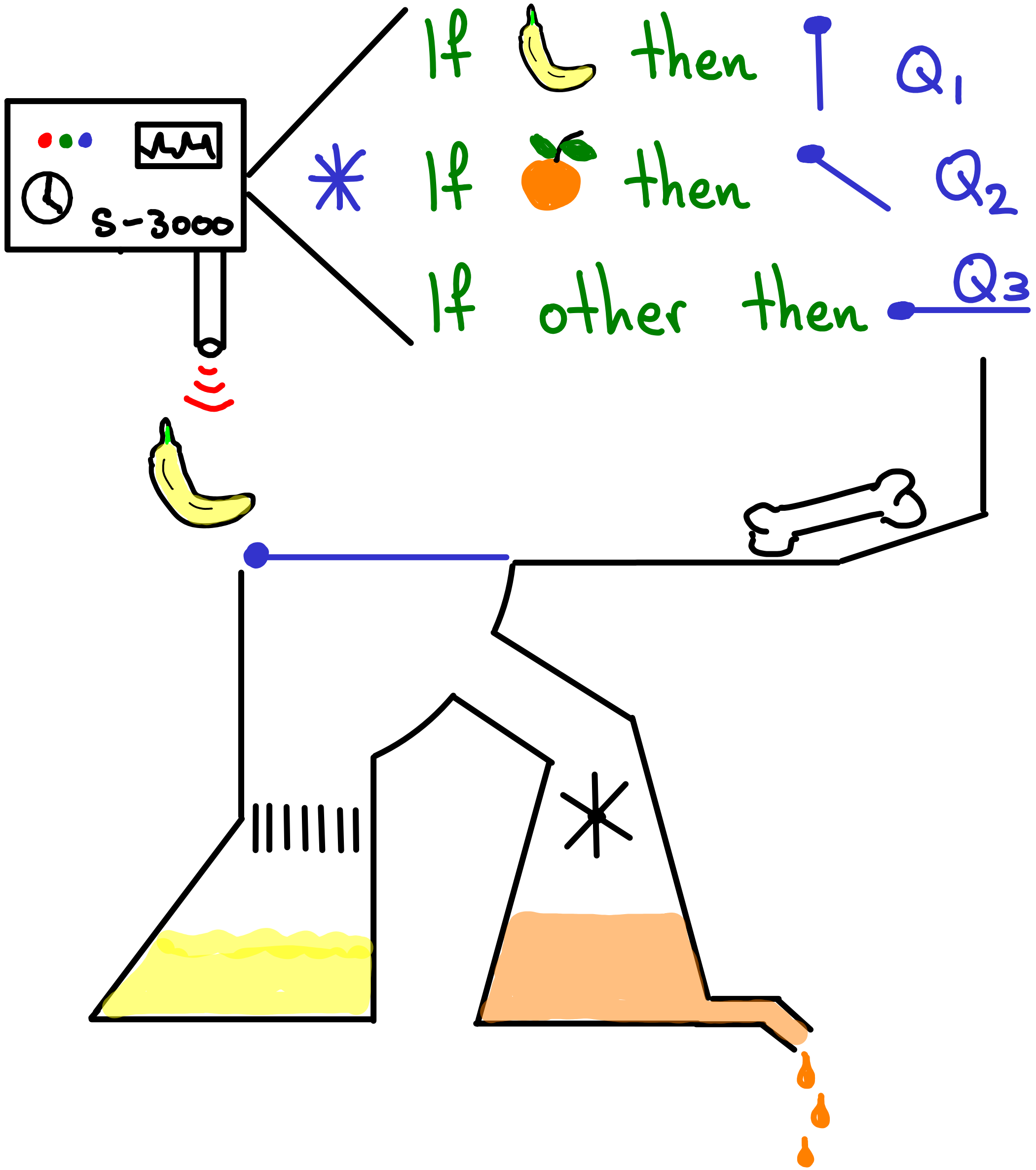
AND

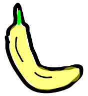
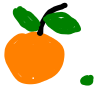
$(P_2 \to Q_2)$

AND

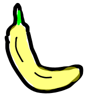
$(\neg(P_1 \text{ OR } P_2) \to Q_3)$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)\ \}$ T→F : F

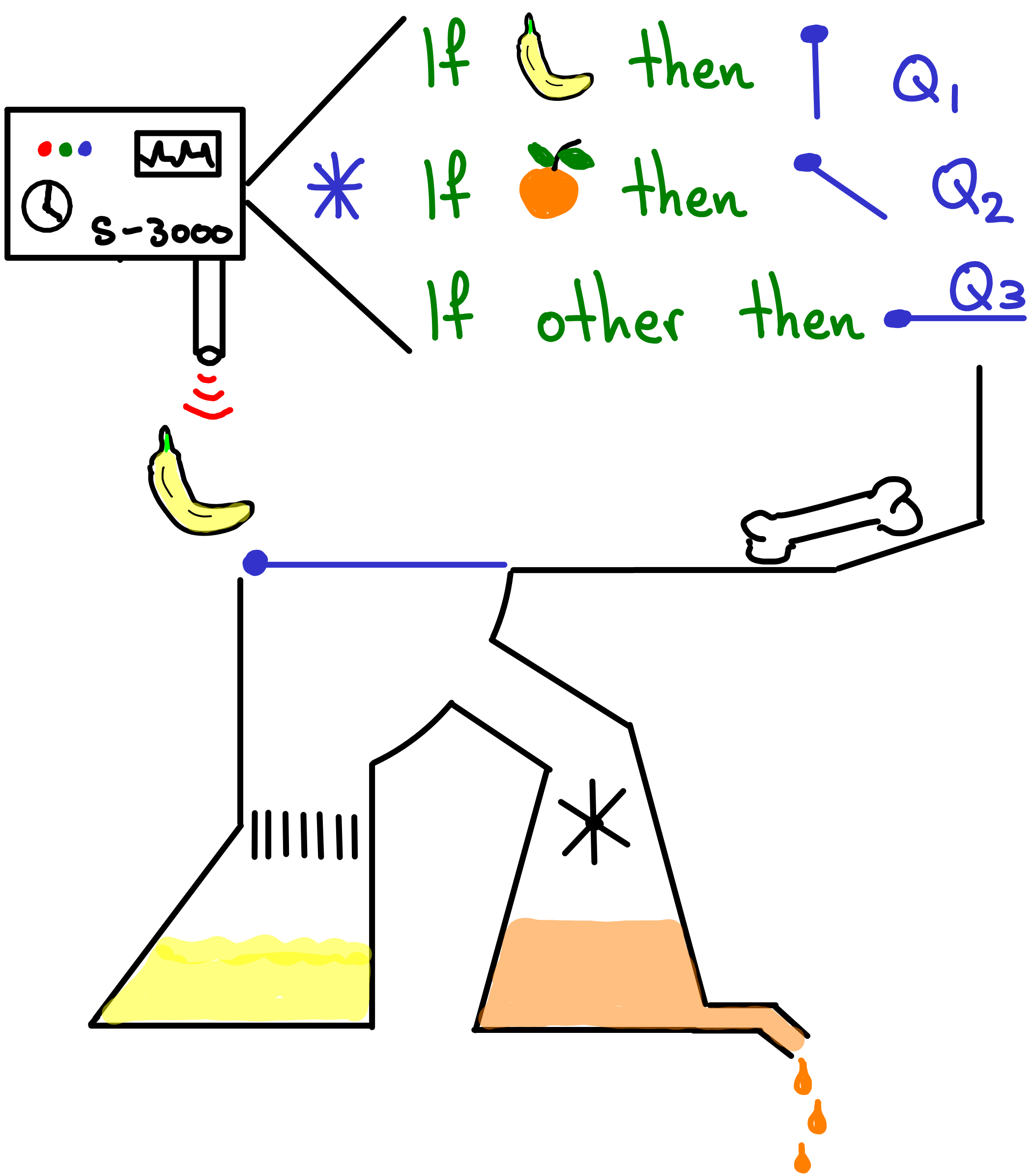
AND

$(P_2 \to Q_2)$

AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$

P	Q	P→Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF 🍌. P_2 IFF 🍊.

Is the machine working? No

e.g., sense 🍌, action = •—

$(P_1 \to Q_1)$ } T → F : F

AND

$(P_2 \to Q_2)$: X

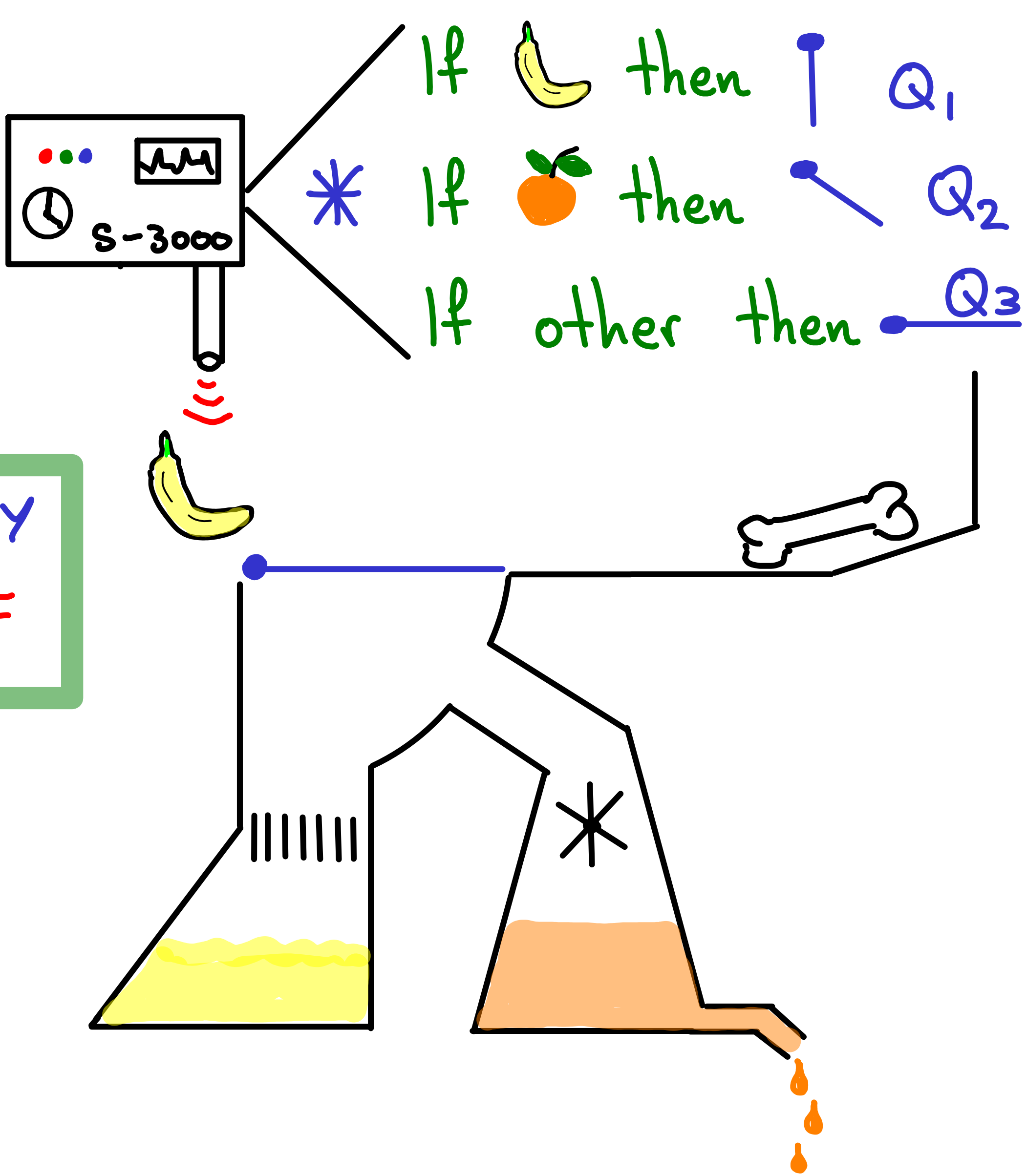
AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$: Y

F AND X AND Y

conclusion: F

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working? **No**

e.g., sense , action =

$(P_1 \to Q_1)$ } T → F : F

AND

$(P_2 \to Q_2)$ } F → F : T

AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$ } $\neg$(T OR F) → T

$\neg$T → T

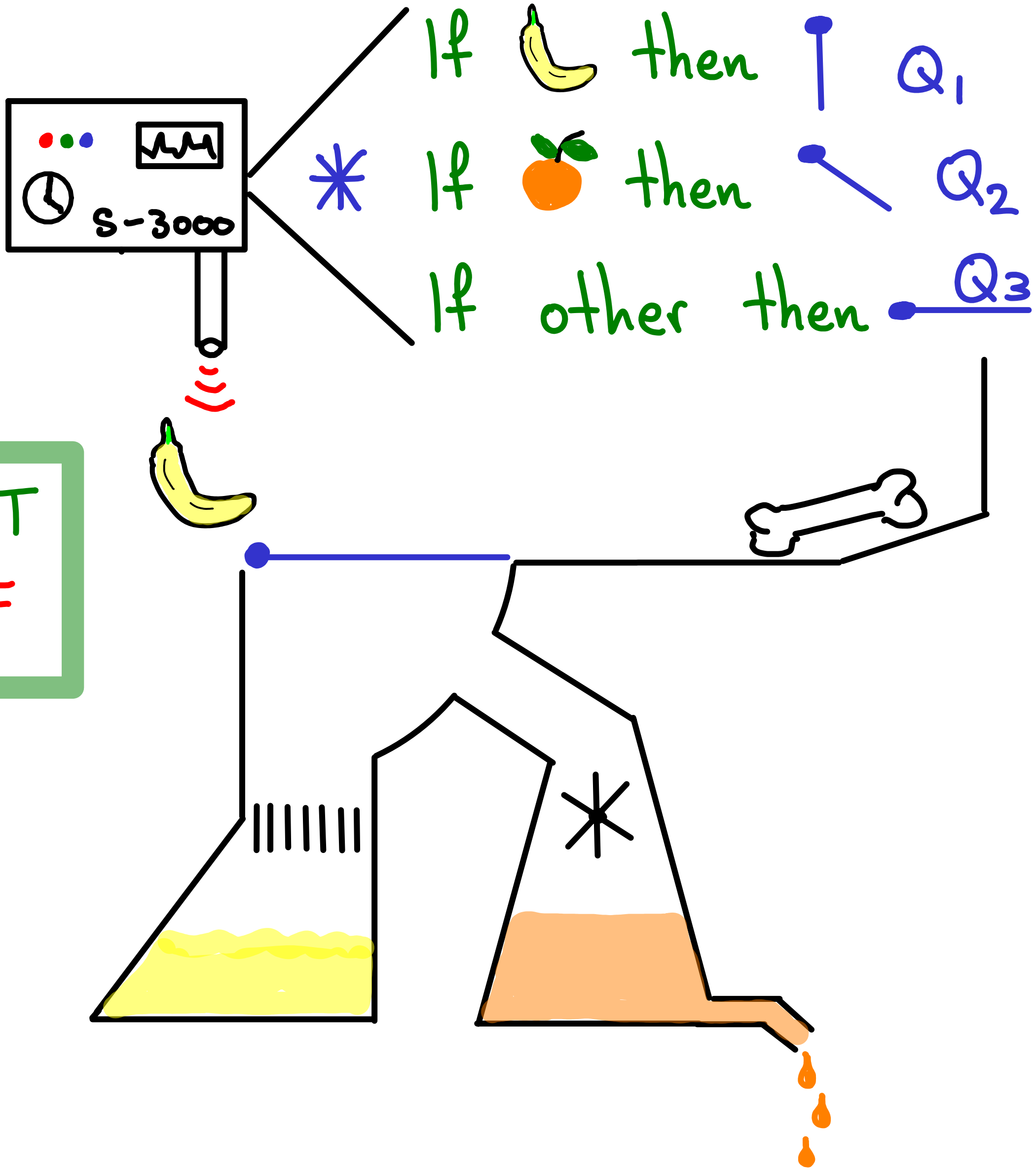
F → T

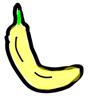
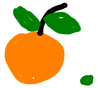
T

F AND T AND T

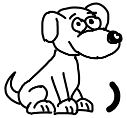
conclusion: F

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action = |

$(P_1 \rightarrow Q_1)$

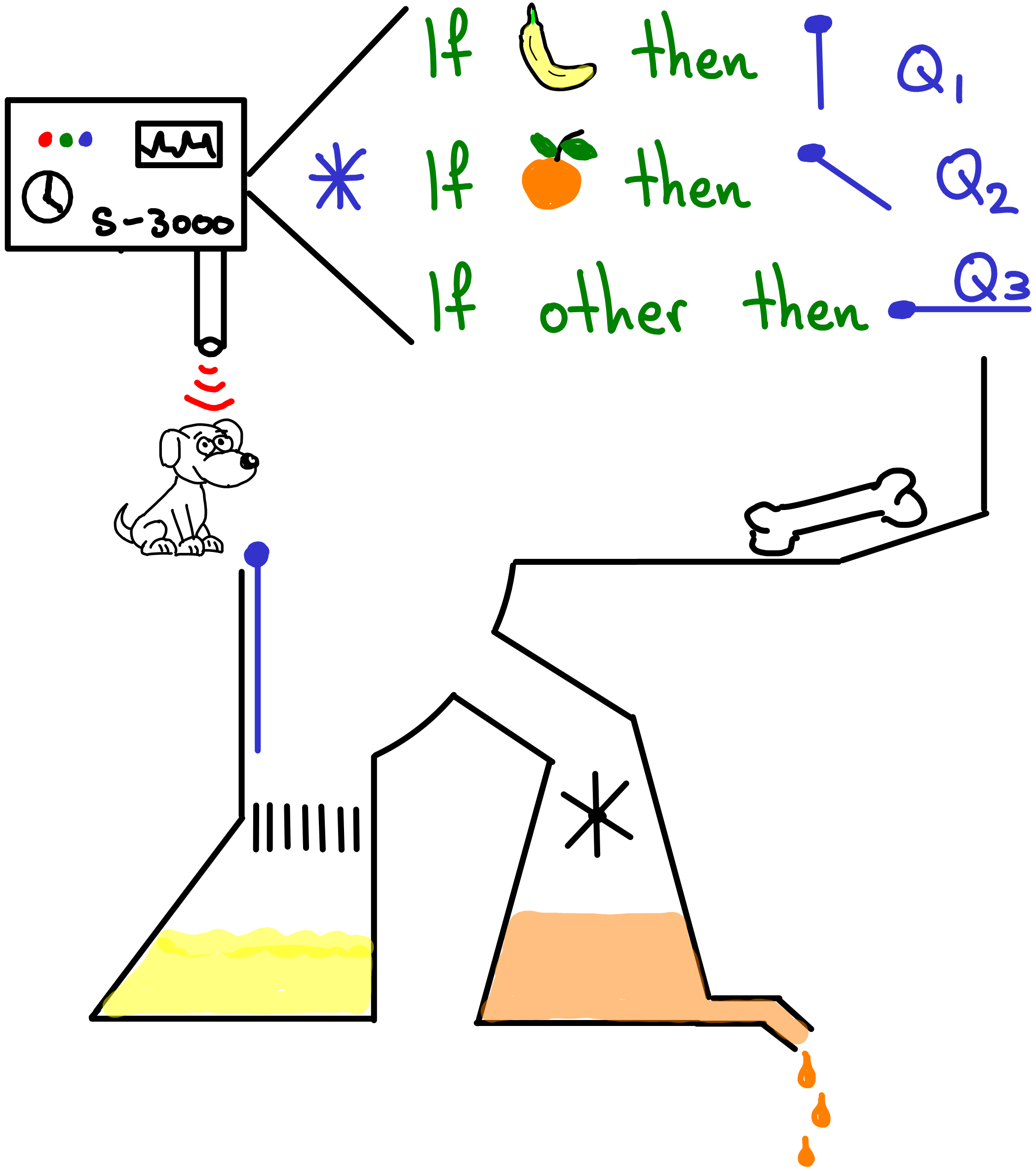
AND

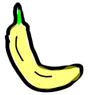
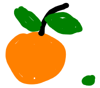
$(P_2 \rightarrow Q_2)$

AND

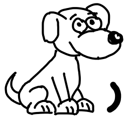
$(\neg(P_1 \text{ OR } P_2) \rightarrow Q_3)$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$ $\}$ F → T

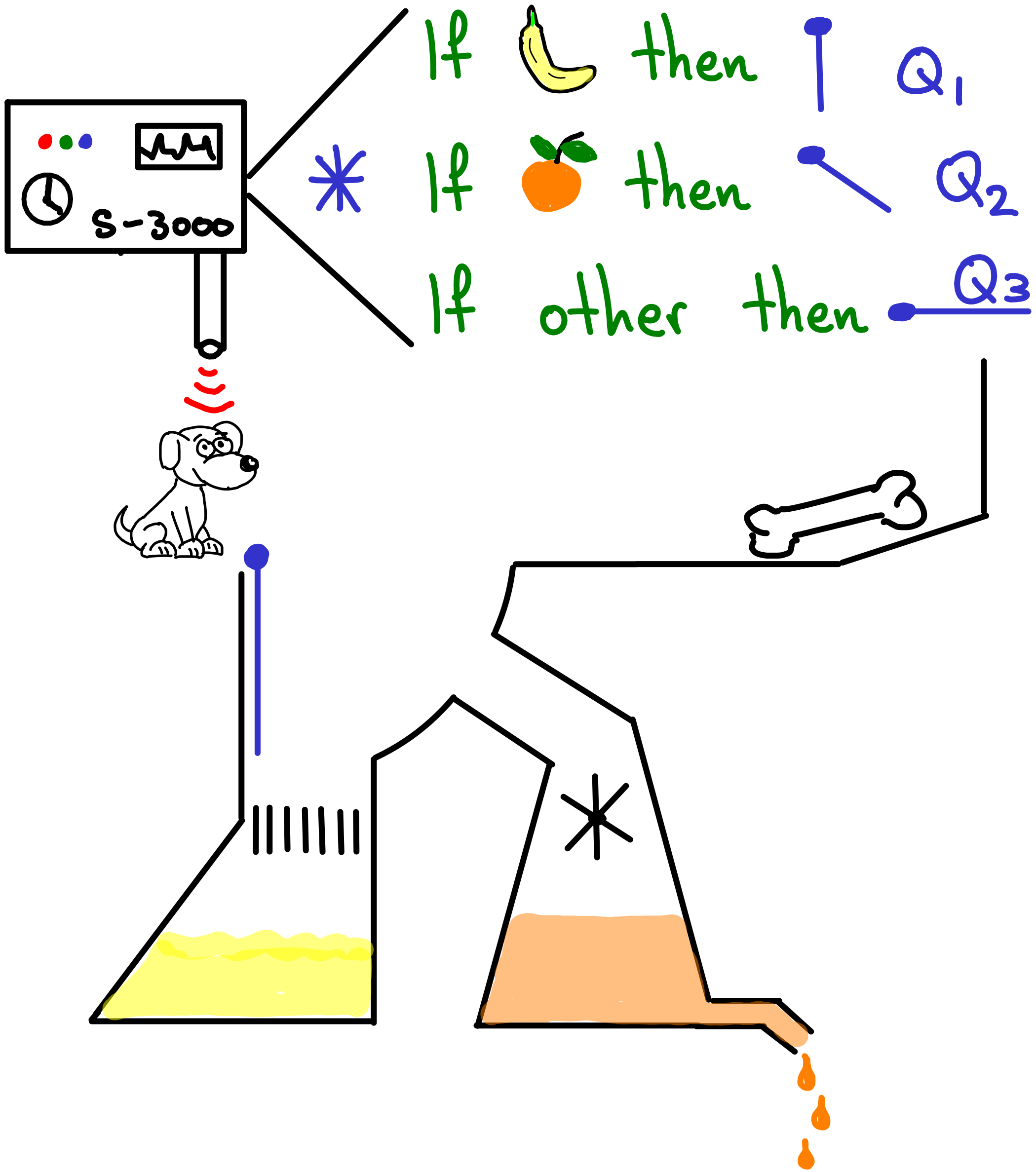
AND

$(P_2 \to Q_2)$

AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \rightarrow Q_1) \}$ $F \rightarrow T$: T

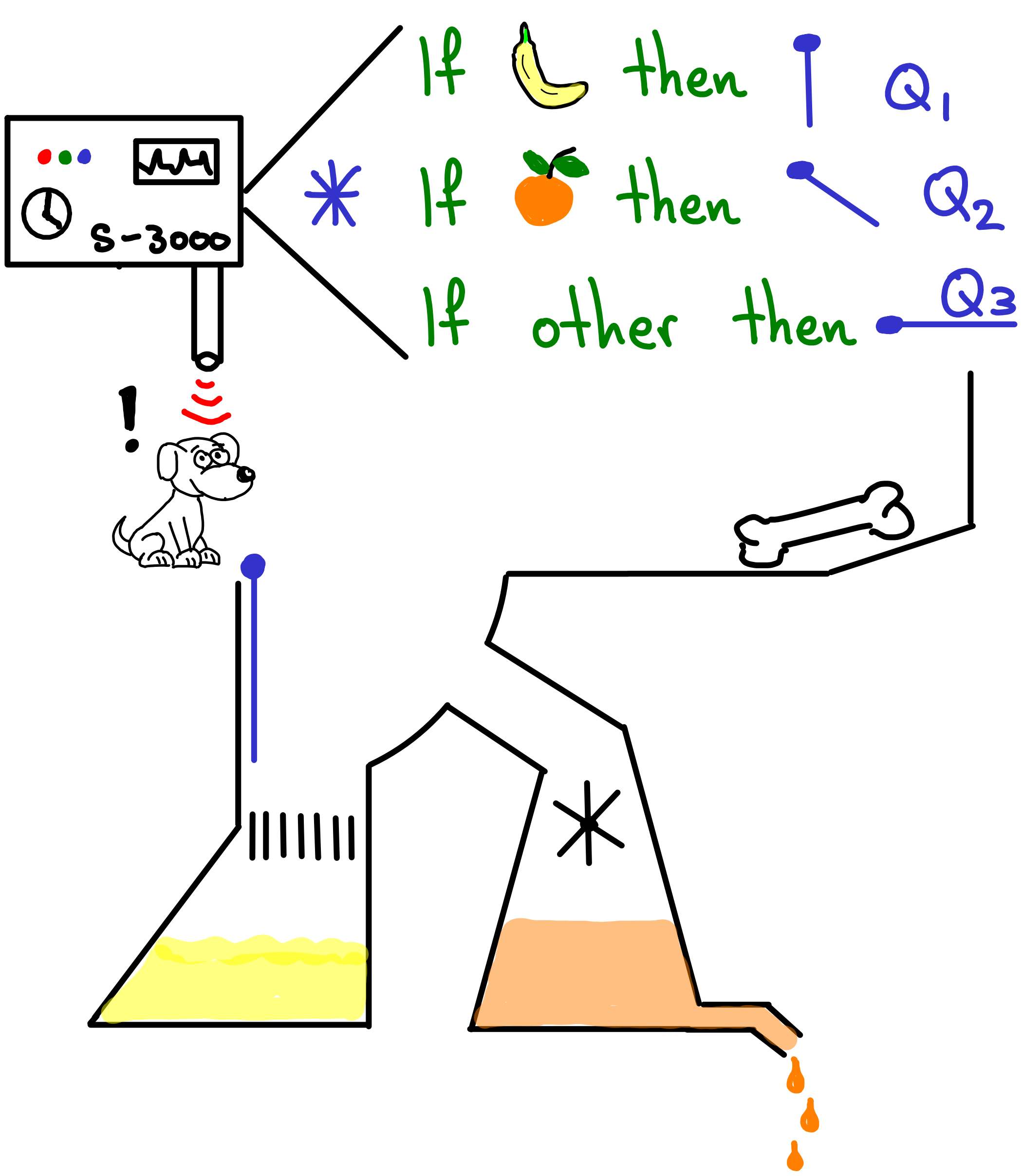
AND

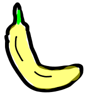
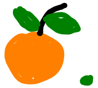
$(P_2 \rightarrow Q_2)$

AND

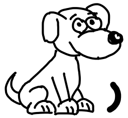
$(\neg(P_1 \text{ OR } P_2) \rightarrow Q_3)$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$ } F → T : T

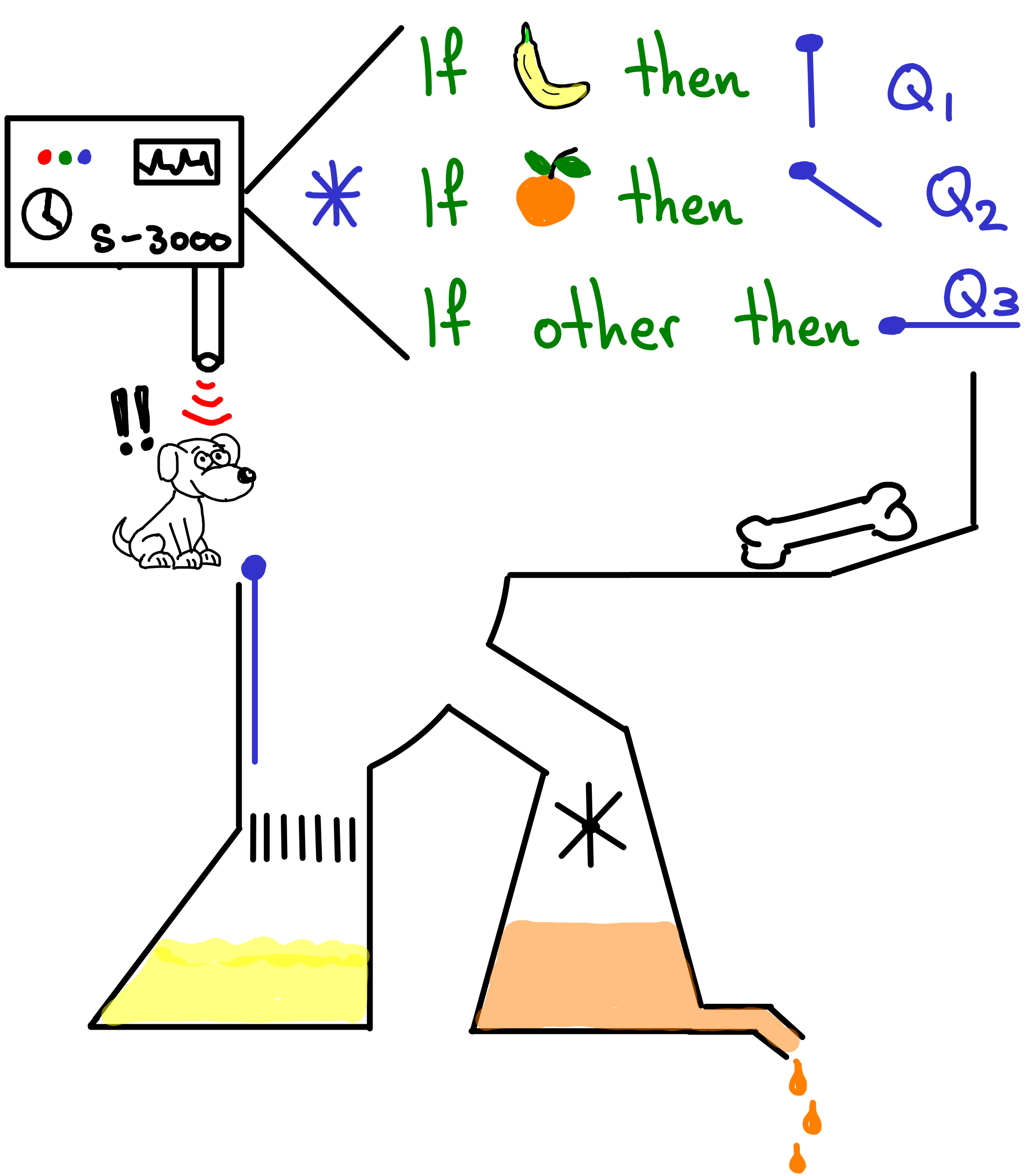
AND

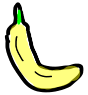
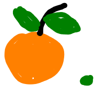
$(P_2 \to Q_2)$ } F → F : T

AND

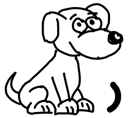
$(\neg(P_1 \text{ OR } P_2) \to Q_3)$

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$ } F → T : T

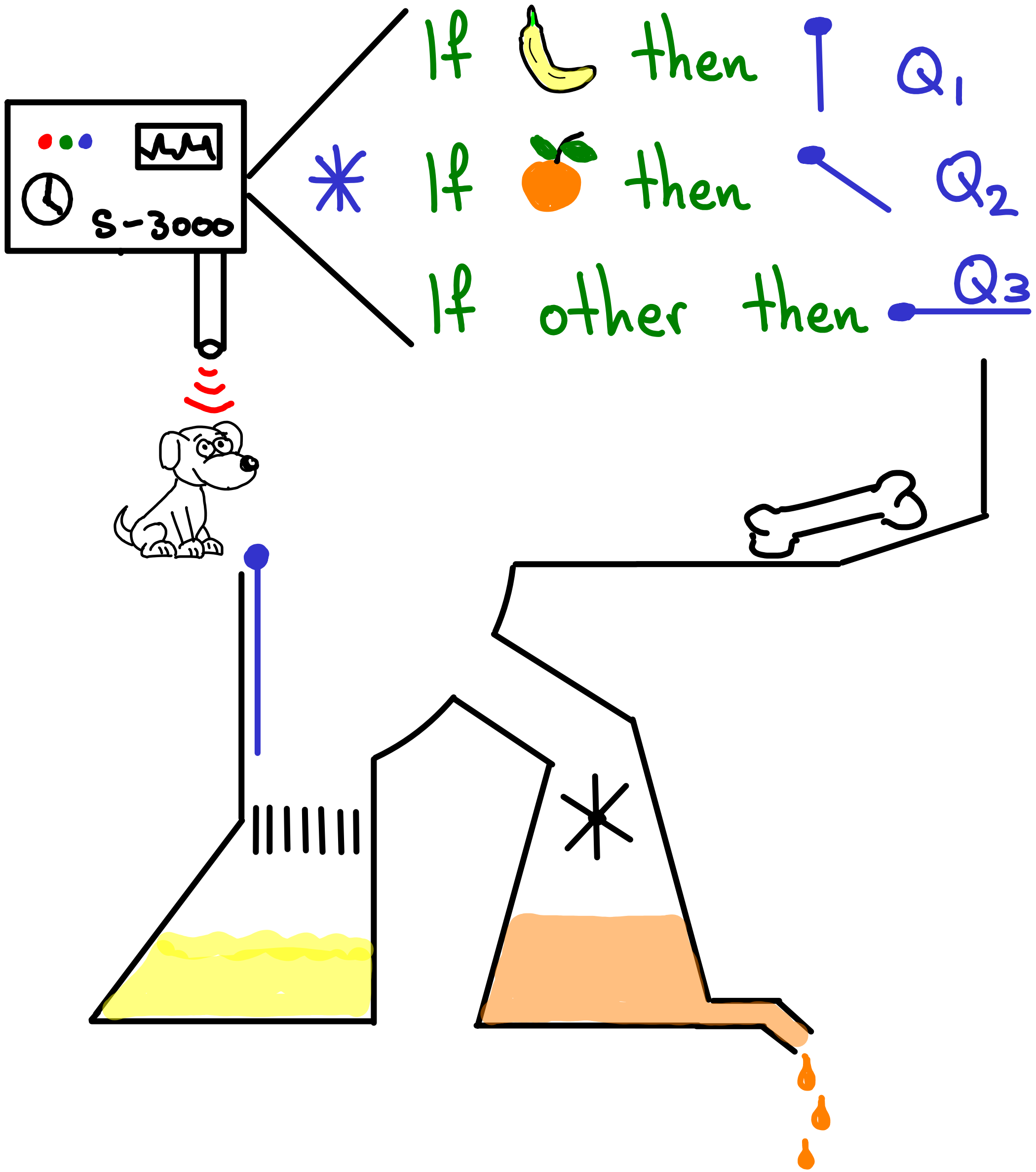
AND

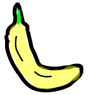
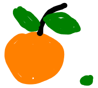
$(P_2 \to Q_2)$ } F → F : T

AND

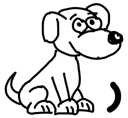
$(\neg(P_1 \text{ OR } P_2) \to Q_3)$ } $\neg$(F OR F) → F

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$ } F → T : T

AND

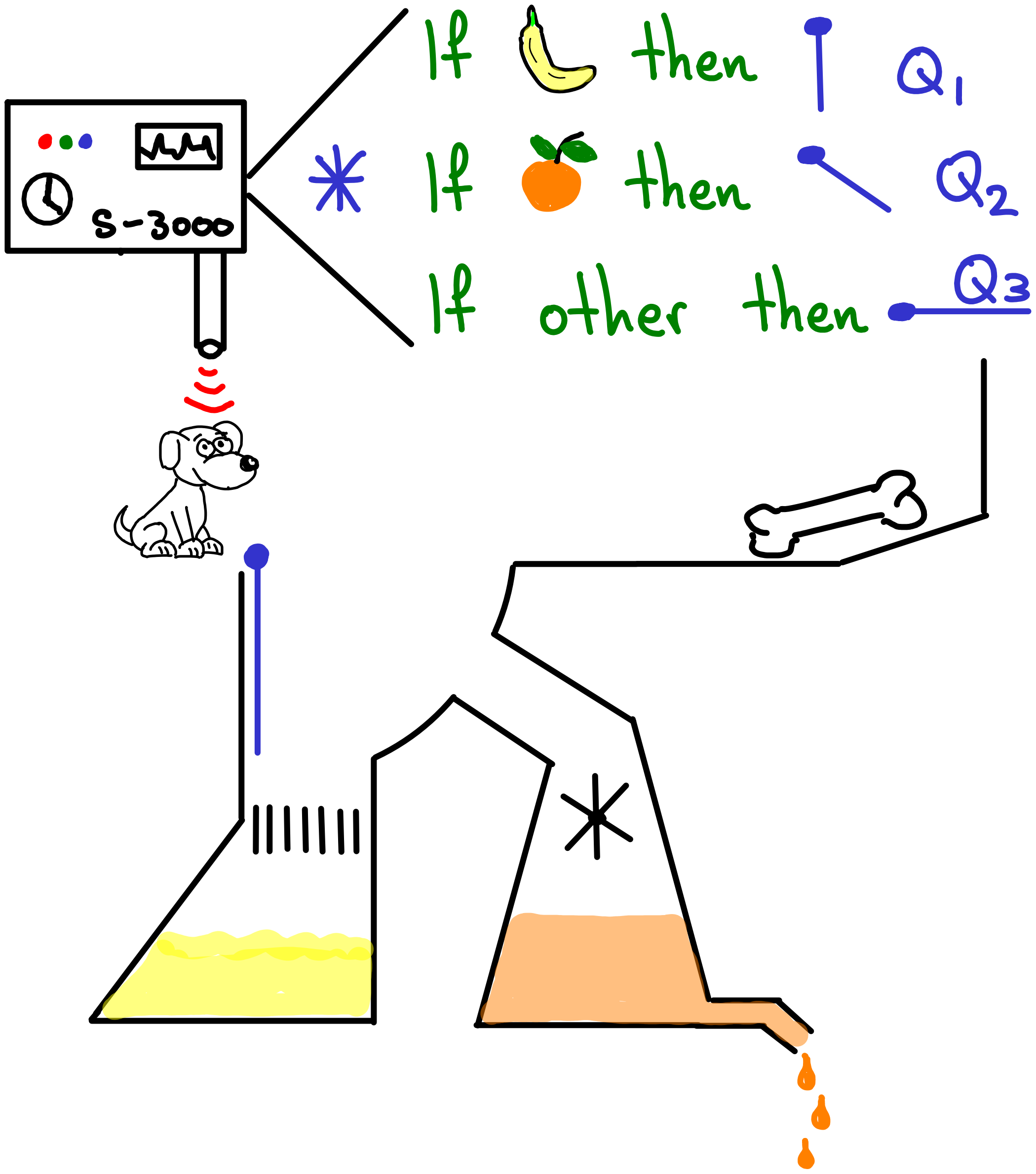
$(P_2 \to Q_2)$ } F → F : T

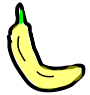
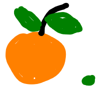
AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$ } $\neg$(F OR F) → F

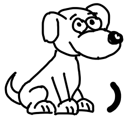
$\neg$F → F

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$ } F → T : T

AND

$(P_2 \to Q_2)$ } F → F : T

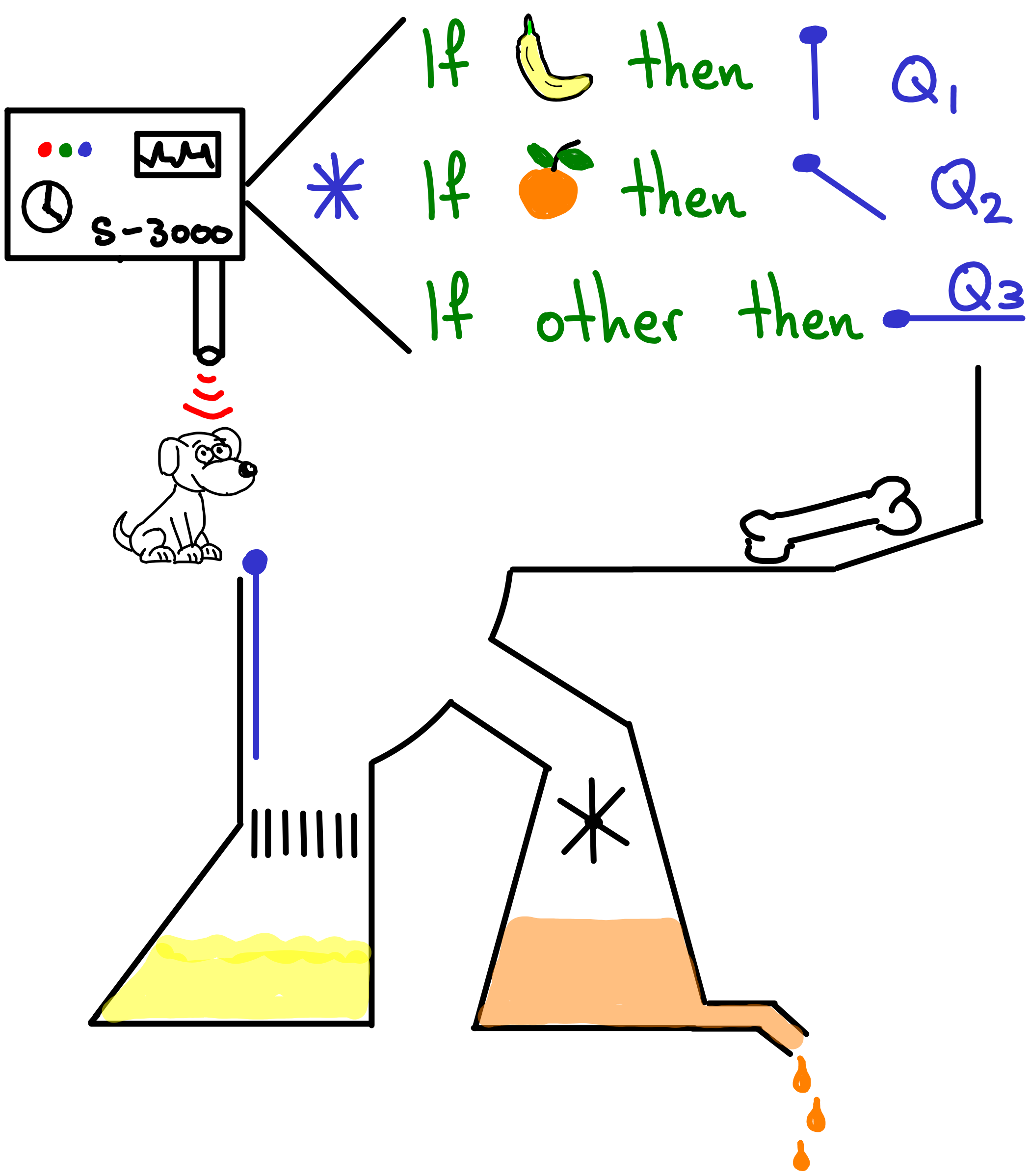
AND

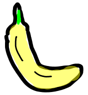
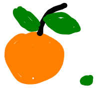
$(\neg(P_1 \text{ OR } P_2) \to Q_3)$ } ¬(F OR F) → F

¬F → F

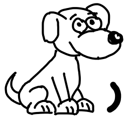
T → F

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working?

e.g., sense , action =

$(P_1 \to Q_1)$ F → T : T

AND

$(P_2 \to Q_2)$ F → F : T

AND

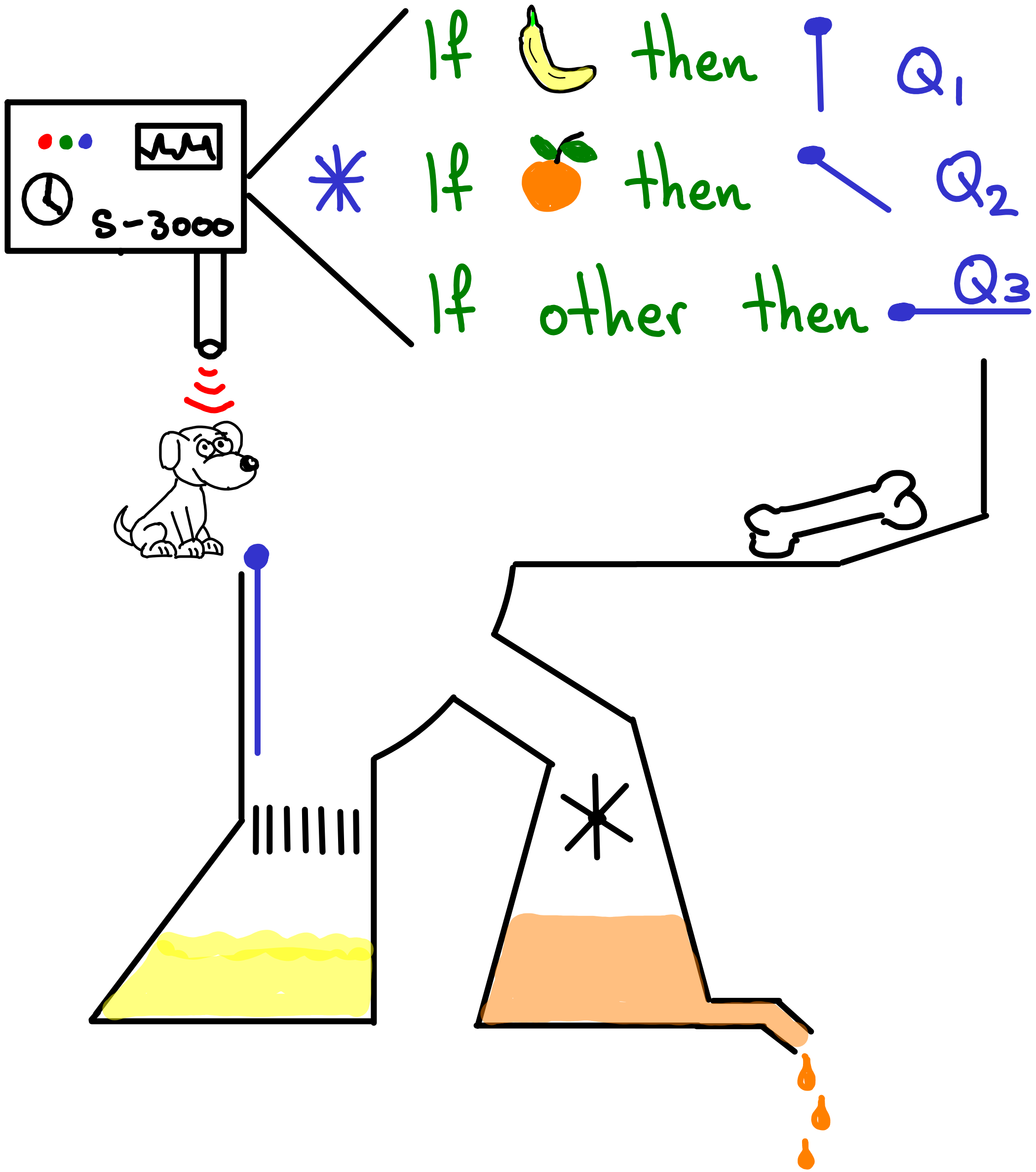
$(\neg(P_1 \text{ OR } P_2) \to Q_3)$ $\neg$(F OR F) → F

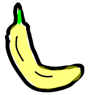
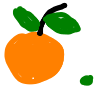
$\neg$F → F

T → F

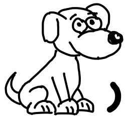
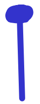
F

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



P_1 IFF . P_2 IFF .

Is the machine working? No

e.g., sense , action = 

$(P_1 \to Q_1)$ } F → T : T

AND

$(P_2 \to Q_2)$ } F → F : T

AND

$(\neg(P_1 \text{ OR } P_2) \to Q_3)$ } $\neg$(F OR F) → F

$\neg$F → F

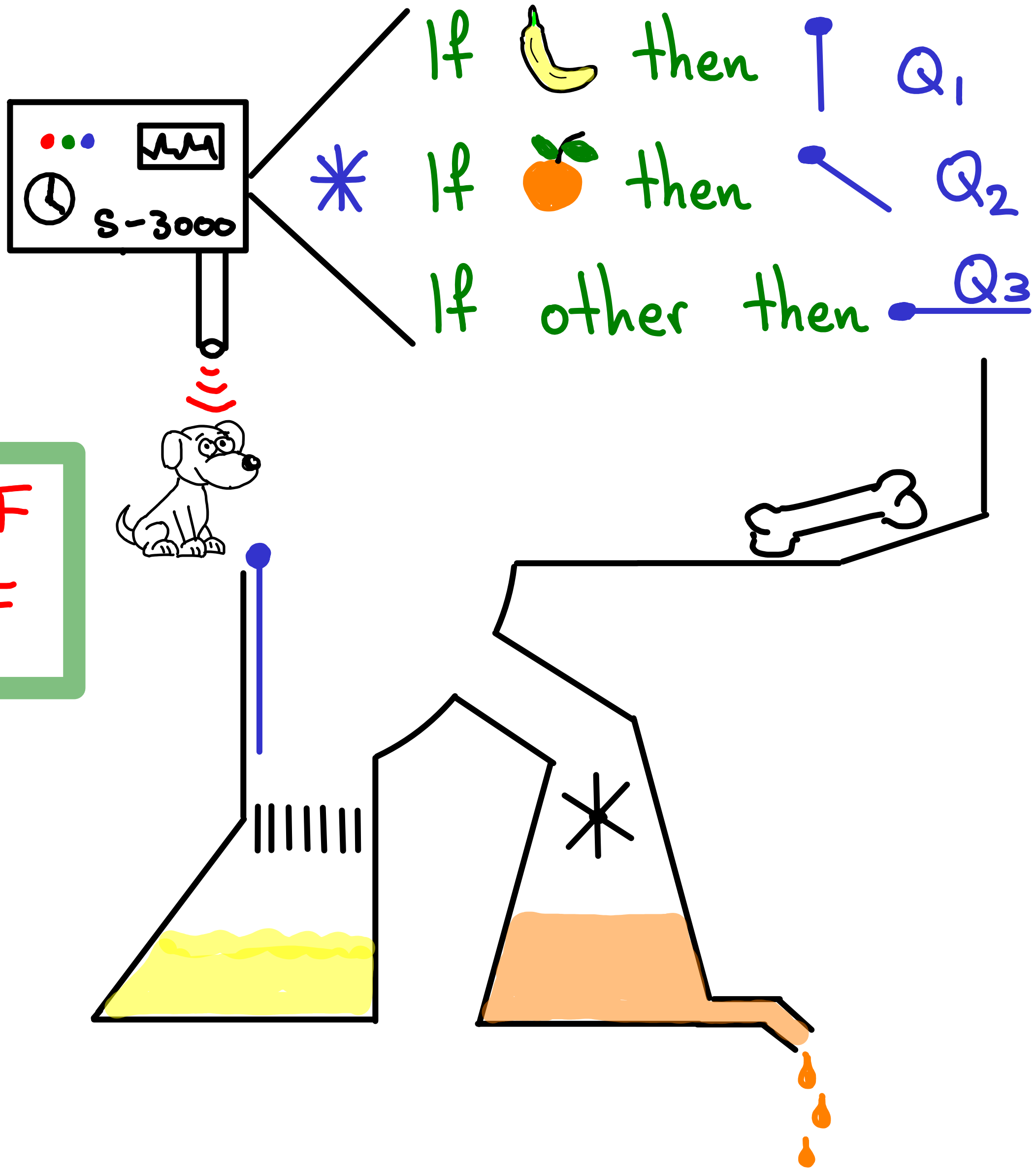
T → F

F

T AND T AND F

conclusion: F

P	Q	P → Q	P OR Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F



(more) PROPOSITIONAL LOGIC

(more) PROPOSITIONAL LOGIC

P: A OR $(\neg A$ AND $B)$

(more) PROPOSITIONAL LOGIC

P: A OR $(\neg A$ AND $B)$

- if A is true then P is true regardless of what $(\neg A$ AND $B)$ is.

(more) PROPOSITIONAL LOGIC

P: A OR $(\neg A$ AND $B)$

2 options

- if A then P regardless of what $(\neg A$ AND $B)$ is.
- if $\neg A$, then we need $(\neg A$ AND $B)$, to get P.

is true

(more) PROPOSITIONAL LOGIC

P: A OR $(\neg A$ AND $B)$

2 options

- if A then P regardless of what $(\neg A$ AND $B)$ is.
- if $\neg A$, then we need $(\underbrace{\neg A}_{\text{But this is true in this case}}$ AND $B)$, to get P.

is true

(more) PROPOSITIONAL LOGIC

P: A OR $(\neg A$ AND $B)$

2 options:

- if A then P regardless of what $(\neg A$ AND $B)$ is.

is true

- if $\neg A$, then we need $(\neg A$ AND $B)$, to get P.

But this is true in this case

So we need $(T$ AND $B)$

(more) PROPOSITIONAL LOGIC

P: A OR $(\neg A$ AND $B)$

2 options:

- if A then P regardless of what $(\neg A$ AND $B)$ is.

is true

- if $\neg A$, then we need $(\underbrace{\neg A}$ AND $B)$, to get P.

But this is true in this case

So we need $(T$ AND $B)$, i.e., we need B

(more) PROPOSITIONAL LOGIC

P: A OR $(\neg A$ AND $B) \iff A$ OR B

2 options:
- if A then P regardless of what $(\neg A$ AND $B)$ is.

OR

- if $\neg A$, then we need $(\underbrace{\neg A}_{\text{But this is true in this case}}$ AND $B)$, to get P.

So we need (T AND B), i.e., we need B

(more) PROPOSITIONAL LOGIC

$$A \text{ OR } (\neg A \text{ AND } B) \quad \longleftrightarrow \quad A \text{ OR } B$$

if $x<0$ do [action C]

else if $(x \geq 0$ and $y > 10)$ do [action C]

(more) PROPOSITIONAL LOGIC

A OR $(\neg A$ AND $B) \longleftrightarrow A$ OR B

if $x<0$ do [action C]
else if $(x \geq 0$ and $y > 10)$ do [action C]

more efficient → if $x<0$ do [action C]
else if $y>10$ do [action C]

(more) PROPOSITIONAL LOGIC

A OR $(\neg A$ AND $B)$ $\longleftrightarrow$ A OR B

A	B	A OR $(\neg A$ AND $B)$	A OR B
T	T		
T	F		
F	T		
F	F		

(more) PROPOSITIONAL LOGIC

A OR $(\neg A$ AND $B)$ $\longleftrightarrow$ A OR B

A	B	A OR $(\neg A$ AND $B)$	A OR B
T	T		T
T	F		T
F	T		T
F	F		F

(more) PROPOSITIONAL LOGIC

$$A \text{ OR } (\neg A \text{ AND } B) \longleftrightarrow A \text{ OR } B$$

A	B	$\overline{A}$ OR $(\overline{\neg A}$ AND $\overline{B})$	A OR B
T	T	T F T	T
T	F		T
F	T		T
F	F		F

(more) PROPOSITIONAL LOGIC

$$A \text{ OR } (\neg A \text{ AND } B) \iff A \text{ OR } B$$

A	B	A OR $(\neg A$ AND $B)$	A OR B
T	T	T F	T
T	F		T
F	T		T
F	F		F

(more) PROPOSITIONAL LOGIC

$$A \text{ OR } (\neg A \text{ AND } B) \longleftrightarrow A \text{ OR } B$$

A	B	A OR $(\neg A$ AND $B)$	A OR B
T	T	T	T
T	F		T
F	T		T
F	F		F

(more) PROPOSITIONAL LOGIC

$$A \text{ OR } (\neg A \text{ AND } B) \iff A \text{ OR } B$$

A	B	A OR (¬A AND B)	A OR B
T	T	T	T
T	F	T OR (F AND F)	T
F	T		T
F	F		F

(more) PROPOSITIONAL LOGIC

A OR $(\neg A$ AND $B)$ $\longleftrightarrow$ A OR B

A	B	A OR $(\neg A$ AND $B)$	A OR B
T	T	T	T
T	F	T	T
F	T		T
F	F		F

(more) PROPOSITIONAL LOGIC

$$A \text{ OR } (\neg A \text{ AND } B) \quad \longleftrightarrow \quad A \text{ OR } B$$

A	B	A OR ($\neg$A AND B)	A OR B
T	T	T	T
T	F	T	T
F	T	F OR (T AND T)	T
F	F		F

(more) PROPOSITIONAL LOGIC

A OR $(\neg A$ AND $B) \longleftrightarrow A$ OR B

A	B	A OR $(\neg A$ AND $B)$	A OR B
T	T	T	T
T	F	T	T
F	T	T	T
F	F		F

(more) PROPOSITIONAL LOGIC

$$A \text{ OR } (\neg A \text{ AND } B) \longleftrightarrow A \text{ OR } B$$

A	B	A OR $(\neg A$ AND $B)$	A OR B
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F OR (T AND F)	F

(more) PROPOSITIONAL LOGIC

$$A \text{ OR } (\neg A \text{ AND } B) \iff A \text{ OR } B$$

A	B	$A \text{ OR } (\neg A \text{ AND } B)$	$A \text{ OR } B$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

PROPOSITIONAL LOGIC NOTATION

	NOT P	$\neg P$	or $\overline{P}$
	P AND Q	$P \wedge Q$	
	P OR Q	$P \vee Q$	
if P then Q,	P implies Q	$P \rightarrow Q$	
	P IFF Q	$P \leftrightarrow Q$	
	P XOR Q	$P \oplus Q$	

MCS: "cryptic ... we mostly stick to words"

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

If I am hungry then I eat

$$P \rightarrow Q$$

P	Q	$P \rightarrow Q$	$Q \rightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

If I am hungry then I eat

$P \rightarrow Q$

If I eat then I am hungry

$Q \rightarrow P$

converse

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$\neg Q \rightarrow \neg P$
T	T	T	T	T
T	F	F	T	
F	T	T	F	
F	F	T	T	T

If I am hungry then I eat

$P \rightarrow Q$

If I eat then I am hungry

$Q \rightarrow P$

converse

contrapositive

If I don't eat then I am not hungry

$\neg Q \rightarrow \neg P$

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$\neg Q \rightarrow \neg P$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

If I am hungry then I eat

$P \rightarrow Q$

contrapositive

If I don't eat then I am not hungry

$\neg Q \rightarrow \neg P$

If I eat then I am hungry

$Q \rightarrow P$

converse

P	Q	$P \to Q$	$Q \to P$	$\neg Q \to \neg P$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

If I am hungry then I eat

$P \to Q$

contrapositive

If I don't eat then I am not hungry

$\neg Q \to \neg P$

If I eat then I am hungry

$Q \to P$

converse

B OR $\neg B$ (Shakespeare?)

B OR $\neg B$ (Shakespeare?)

B	$\neg B$	B OR $\neg B$
T	F	T
F	T	T

If a logic formula is always T then it is valid.

$\underbrace{B \text{ OR } \neg B}_{T}$

(Shakespeare?)

B	$\neg B$	B OR $\neg B$
T	F	T
F	T	T

If a logic formula is always T then it is valid.

$\underbrace{B \text{ OR } \neg B}_{T}$

(Shakespeare?)

B	$\neg B$	B OR $\neg B$
T	F	T
F	T	T

B XOR $\neg B$ .... valid?

If a logic formula is always T then it is valid.

$\underbrace{B \text{ OR } \neg B}_{T}$

(Shakespeare?)

B	¬B	B OR ¬B
T	F	T
F	T	T

B XOR ¬B is valid:

B	¬B	B XOR ¬B
T	F	T
F	T	T

If a logic formula is always T then it is valid.

$\underbrace{B \text{ OR } \neg B}_{T}$

(Shakespeare?)

B	$\neg B$	$B \text{ OR } \neg B$
T	F	T
F	T	T

$B \text{ XOR } \neg B$ is valid:

B	$\neg B$	$B \text{ XOR } \neg B$
T	F	T
F	T	T

If you're asked: "do you want cake now or later?" ...

If a logic formula is always T then it is valid.

$\underbrace{B \text{ OR } \neg B}_{T}$

(Shakespeare ?)

B	¬B	B OR ¬B
T	F	T
F	T	T

B XOR ¬B is valid:

B	¬B	B XOR ¬B
T	F	T
F	T	T

If you're asked: "do you want cake now or later?" ... just say YES

Also works with: "do you want cake or ice cream?"

If a logic formula can be T then it is satisfiable

If a logic formula can be T then it is satisfiable

P is satisfiable IFF $\neg$P is not valid

DISJUNCTIVE FORM "an OR of ANDs"

e.g., (A AND B) OR (A AND C AND D) OR ($\bar{B}$ AND D)

DISJUNCTIVE FORM "an OR of ANDs"

e.g., (A AND B) OR (A AND C AND D) OR ($\bar{B}$ AND D)

CONJUNCTIVE FORM "an AND of ORs"

e.g., (A OR C) AND (A OR $\bar{D}$) AND ($\bar{B}$ OR C OR D)

DISJUNCTIVE FORM "an OR of ANDs"

e.g., (A AND B) OR (A AND C AND D) OR ($\overline{B}$ AND D)

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We can write any propositional formula like this. e.g., A AND (B OR C)

...

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We can write any propositional formula like this. e.g., A AND (B OR C)

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table ☹

DISJUNCTIVE FORM "an OR of ANDs"

e.g., $(A \text{ AND } B) \text{ OR } (A \text{ AND } C \text{ AND } D) \text{ OR } (\overline{B} \text{ AND } D)$

We can write any propositional formula like this. e.g., A AND (B OR C)

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table :(

Find all rows in table where formula is T.

DISJUNCTIVE FORM "an OR of ANDs"

e.g., (A AND B) OR (A AND C AND D) OR ($\bar{B}$ AND D)

We can write any propositional formula like this. e.g., A AND (B OR C)

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table :(

Find all rows in table where formula is T.

Want ≥1 of these rows to be satisfied:

DISJUNCTIVE FORM "an OR of ANDs"

e.g., (A AND B) OR (A AND C AND D) OR ($\bar{B}$ AND D)

We can write any propositional formula like this. e.g., A AND (B OR C)

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table :(

Find all rows in table where formula is T.

Want ≥1 of these rows to be satisfied:

(A AND B AND C)

DISJUNCTIVE FORM "an OR of ANDs"

e.g., $(A \text{ AND } B) \text{ OR } (A \text{ AND } C \text{ AND } D) \text{ OR } (\overline{B} \text{ AND } D)$

We can write any propositional formula like this. e.g., $A \text{ AND } (B \text{ OR } C)$

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table :(

Find all rows in table where formula is T.

Want ≥1 of these rows to be satisfied:

$(A \text{ AND } B \text{ AND } C)$

$\text{OR } (A \text{ AND } B \text{ AND } \overline{C})$

DISJUNCTIVE FORM "an OR of ANDs"

e.g., (A AND B) OR (A AND C AND D) OR ($\overline{B}$ AND D)

We can write any propositional formula like this. e.g., A AND (B OR C)

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table :(

Find all rows in table where formula is T.

Want ≥1 of these rows to be satisfied:

(A AND B AND C)

OR (A AND B AND $\overline{C}$)

OR (A AND $\overline{B}$ AND C)

CONJUNCTIVE FORM "an AND of ORs"

e.g., $(A \text{ OR } C) \text{ AND } (A \text{ OR } \overline{D}) \text{ AND } (\overline{B} \text{ OR } C \text{ OR } D)$

CONJUNCTIVE FORM "an AND of ORs"

e.g., (A OR C) AND (A OR $\bar{D}$) AND ($\bar{B}$ OR C OR D)

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We can write any propositional formula like this. e.g., A AND (B OR C)

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table

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A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table

Find all rows in table where formula is F.

CONJUNCTIVE FORM "an AND of ORs"

e.g., $(A \text{ OR } C) \text{ AND } (A \text{ OR } \bar{D}) \text{ AND } (\bar{B} \text{ OR } C \text{ OR } D)$

We can write any propositional formula like this. e.g., A AND (B OR C)

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table

Find all rows in table where formula is F.

Want ALL of these rows to NOT be satisfied:

CONJUNCTIVE FORM "an AND of ORs"

e.g., (A OR C) AND (A OR $\bar{D}$) AND ($\bar{B}$ OR C OR D)

We can write any propositional formula like this. e.g., A AND (B OR C)

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table

Find all rows in table where formula is F.

Want ALL of these rows to NOT be satisfied:

($\bar{A}$ OR B OR C)

CONJUNCTIVE FORM "an AND of ORs"

e.g., $(A \text{ OR } C) \text{ AND } (A \text{ OR } \overline{D}) \text{ AND } (\overline{B} \text{ OR } C \text{ OR } D)$

We can write any propositional formula like this. e.g., A AND (B OR C)

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table

Find all rows in table where formula is F.

Want ALL of these rows to NOT be satisfied:

$(\overline{A} \text{ OR } B \text{ OR } C)$

$\text{AND } (A \text{ OR } \overline{B} \text{ OR } \overline{C})$

CONJUNCTIVE FORM "an AND of ORs"

e.g., $(A \text{ OR } C) \text{ AND } (A \text{ OR } \bar{D}) \text{ AND } (\bar{B} \text{ OR } C \text{ OR } D)$

We can write any propositional formula like this. e.g., A AND (B OR C)

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table

Find all rows in table where formula is F.

Want ALL of these rows to NOT be satisfied:

$$(\bar{A} \text{ OR } B \text{ OR } C)$$
$$\text{AND } (A \text{ OR } \bar{B} \text{ OR } \bar{C})$$
$$\text{AND } (A \text{ OR } \bar{B} \text{ OR } C)$$

CONJUNCTIVE FORM "an AND of ORs"

e.g., $(A \text{ OR } C) \text{ AND } (A \text{ OR } \overline{D}) \text{ AND } (\overline{B} \text{ OR } C \text{ OR } D)$

We can write any propositional formula like this. e.g., $A \text{ AND } (B \text{ OR } C)$

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table

Find all rows in table where formula is F.

Want ALL of these rows to NOT be satisfied:

$$(\overline{A} \text{ OR } B \text{ OR } C)$$
$$\text{AND } (A \text{ OR } \overline{B} \text{ OR } \overline{C})$$
$$\text{AND } (A \text{ OR } \overline{B} \text{ OR } C)$$
$$\text{AND } (A \text{ OR } B \text{ OR } \overline{C})$$

CONJUNCTIVE FORM "an AND of ORs"

e.g., (A OR C) AND (A OR $\bar{D}$) AND ($\bar{B}$ OR C OR D)

We can write any propositional formula like this. e.g., A AND (B OR C)

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Fill in entire truth table

Find all rows in table where formula is F.

Want ALL of these rows to NOT be satisfied:

($\bar{A}$ OR B OR C)

AND (A OR $\bar{B}$ OR $\bar{C}$)

AND (A OR $\bar{B}$ OR C)

AND (A OR B OR $\bar{C}$)

AND (A OR B OR C)

What we got was actually DISJUNCTIVE NORMAL FORM

& CONJUNCTIVE NORMAL FORM

Every variable is present in each term within parentheses.

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Every variable is present in each term within parentheses.

We can often simplify DNF, CNF

e.g.,

$$\underbrace{\begin{array}{l} (A \text{ AND } B \text{ AND } C) \\ \text{OR } (A \text{ AND } B \text{ AND } \overline{C}) \\ \text{OR } (A \text{ AND } \overline{B} \text{ AND } C) \end{array}}$$

$$\begin{array}{l} (A \text{ AND } B) \\ \text{OR } (A \text{ AND } C) \end{array}$$

What we got was actually DISJUNCTIVE NORMAL FORM

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Every variable is present in each term within parentheses.

We can often simplify DNF, CNF

e.g.,

A	B	C	A AND (B OR C)
T	T	T	T
T	T	F	T
T	F	T	T

$$\underbrace{\begin{array}{l} (A \text{ AND } B \text{ AND } C) \\ \text{OR } (A \text{ AND } B \text{ AND } \overline{C}) \\ \text{OR } (A \text{ AND } \overline{B} \text{ AND } C) \end{array}}$$

(A AND B)

OR (A AND C)

SIMPLIFYING PROPOSITIONAL FORMULAS

or just trying to show equivalence of two formulas

SIMPLIFYING PROPOSITIONAL FORMULAS

Can replace $A \to B$ with $\neg A$ OR B

...

SIMPLIFYING PROPOSITIONAL FORMULAS

Can replace $A \to B$ with $\neg A$ OR B

$$\underbrace{\neg A}_{\text{case 1}} \text{ OR } \underbrace{A}_{\text{case 2}}$$

SIMPLIFYING PROPOSITIONAL FORMULAS

Can replace $A \rightarrow B$ with $\neg A$ OR B

$$\underbrace{\neg A}_{\text{case 1}} \text{ OR } (\underbrace{A}_{\text{case 2}} \text{ AND } B)$$

SIMPLIFYING PROPOSITIONAL FORMULAS

Can replace $A \to B$ with $\neg A \text{ OR } B$

Recall, $\overbrace{\underbrace{\neg A}_{\text{case 1}} \text{ OR } (\underbrace{A}_{\text{case 2}} \text{ AND } B)}$

SIMPLIFYING PROPOSITIONAL FORMULAS

Can replace $A \to B$ with $\neg A$ OR B

vacuous $A \to B$

because A

Recall, $\underbrace{\neg A}_{\text{case1}}$ OR $(\underbrace{A}_{\text{case2}}$ AND $B)$

SIMPLIFYING PROPOSITIONAL FORMULAS

Can replace $A \to B$ with $\neg A$ OR B

Can replace $A \leftrightarrow B$ with $(A \to B)$ AND $(B \to A)$

SIMPLIFYING PROPOSITIONAL FORMULAS

Can replace $A \to B$ with $\neg A$ OR B

Can replace $A \leftrightarrow B$ with $(A \to B)$ AND $(B \to A)$, then

$(\neg A$ OR $B)$ AND $(\neg B$ OR $A)$

SIMPLIFYING PROPOSITIONAL FORMULAS

Can replace $A \rightarrow B$ with $\neg A$ OR B

Can replace $A \leftrightarrow B$ with $(A \rightarrow B)$ AND $(B \rightarrow A)$, then

$(\neg A$ OR $B)$ AND $(\neg B$ OR $A)$

- Can replace A XOR B with $(A$ OR $B)$ AND $\neg(A$ AND $B)$

SIMPLIFYING PROPOSITIONAL FORMULAS

Can replace $A \to B$ with $\neg A$ OR B

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$(\neg A$ OR $B)$ AND $(\neg B$ OR $A)$

Can replace A XOR B with $(A$ OR $B)$ AND $\neg(A$ AND $B)$

So we can get everything in terms of AND, OR, NOT.

Next we see several rules that can help to simplify/modify further

F AND A $\longleftrightarrow$ F   T OR A $\longleftrightarrow$ T

F AND A $\longleftrightarrow$ F

T AND A $\longleftrightarrow$?

T OR A $\longleftrightarrow$ T

F OR A $\longleftrightarrow$?

$$F \text{ AND } A \longleftrightarrow F \qquad T \text{ OR } A \longleftrightarrow T$$

$$T \text{ AND } A \longleftrightarrow A \qquad F \text{ OR } A \longleftrightarrow A$$

F AND A $\longleftrightarrow$ F

T AND A $\longleftrightarrow$ A

T OR A $\longleftrightarrow$ T

F OR A $\longleftrightarrow$ A

A AND $\neg$A $\longleftrightarrow$ F

A OR $\neg$A $\longleftrightarrow$ T

F AND A $\longleftrightarrow$ F

T AND A $\longleftrightarrow$ A

T OR A $\longleftrightarrow$ T

F OR A $\longleftrightarrow$ A

A AND $\neg$A $\longleftrightarrow$ F

A OR $\neg$A $\longleftrightarrow$ T

A AND A $\longleftrightarrow$ A

A OR A $\longleftrightarrow$ A

$$F \text{ AND } A \longleftrightarrow F \qquad T \text{ OR } A \longleftrightarrow T$$

$$T \text{ AND } A \longleftrightarrow A \qquad F \text{ OR } A \longleftrightarrow A$$

$$A \text{ AND } \neg A \longleftrightarrow F \qquad A \text{ AND } A \longleftrightarrow A$$

$$A \text{ OR } \neg A \longleftrightarrow T \qquad A \text{ OR } A \longleftrightarrow A$$

$$A \longleftrightarrow \neg(\neg A)$$

A AND B $\longleftrightarrow$ B AND A

A OR B $\longleftrightarrow$ B OR A

commutativity

$$A \text{ AND } B \longleftrightarrow B \text{ AND } A$$

commutativity

$$A \text{ OR } B \longleftrightarrow B \text{ OR } A$$

$$(A \text{ AND } B) \text{ AND } C \longleftrightarrow A \text{ AND } (B \text{ AND } C)$$

$$(A \text{ OR } B) \text{ OR } C \longleftrightarrow A \text{ OR } (B \text{ OR } C)$$

$$A \text{ AND } B \longleftrightarrow B \text{ AND } A$$

$$A \text{ OR } B \longleftrightarrow B \text{ OR } A$$

commutativity

$$(A \text{ AND } B) \text{ AND } C \longleftrightarrow A \text{ AND } (B \text{ AND } C)$$

$$A \text{ AND } B \text{ AND } C$$

associativity

$$(A \text{ OR } B) \text{ OR } C \longleftrightarrow A \text{ OR } (B \text{ OR } C)$$

$$A \text{ OR } B \text{ OR } C$$

$$A \text{ AND } B \longleftrightarrow B \text{ AND } A$$

$$A \text{ OR } B \longleftrightarrow B \text{ OR } A$$

commutativity

$$(A \text{ AND } B) \text{ AND } C \longleftrightarrow A \text{ AND } (B \text{ AND } C)$$

$$A \text{ AND } B \text{ AND } C$$

$$(A \text{ OR } B) \text{ OR } C \longleftrightarrow A \text{ OR } (B \text{ OR } C)$$

$$A \text{ OR } B \text{ OR } C$$

associativity

$$A \text{ AND } (B \text{ OR } C) \longleftrightarrow (A \text{ AND } B) \text{ OR } (A \text{ AND } C)$$

$$A \text{ AND } B \longleftrightarrow B \text{ AND } A$$

$$A \text{ OR } B \longleftrightarrow B \text{ OR } A$$

commutativity

$$(A \text{ AND } B) \text{ AND } C \longleftrightarrow A \text{ AND } (B \text{ AND } C)$$

$$A \text{ AND } B \text{ AND } C$$

associativity

$$(A \text{ OR } B) \text{ OR } C \longleftrightarrow A \text{ OR } (B \text{ OR } C)$$

$$A \text{ OR } B \text{ OR } C$$

$$A \text{ AND } (B \text{ OR } C) \longleftrightarrow (A \text{ AND } B) \text{ OR } (A \text{ AND } C)$$

$$A \text{ OR } (B \text{ AND } C) \longleftrightarrow (A \text{ OR } B) \text{ AND } (A \text{ OR } C) \quad ?$$

A AND B $\longleftrightarrow$ B AND A

A OR B $\longleftrightarrow$ B OR A

commutativity

(A AND B) AND C $\longleftrightarrow$ A AND (B AND C)

A AND B AND C

(A OR B) OR C $\longleftrightarrow$ A OR (B OR C)

A OR B OR C

associativity

A AND (B OR C) $\longleftrightarrow$ (A AND B) OR (A AND C)

A OR (B AND C) $\longleftrightarrow$ (A OR B) AND (A OR C)

distributivity

Two more important rules

$$\neg(A \text{ AND } B) \longleftrightarrow \neg A \text{ OR } \neg B$$

e.g., not (rich and famous) $\longleftrightarrow$ not rich or not famous

$$\neg(A \text{ AND } B) \longleftrightarrow \neg A \text{ OR } \neg B$$

$$\neg(A \text{ OR } B) \longleftrightarrow \neg A \text{ AND } \neg B$$

e.g., not (fast or strong) $\longleftrightarrow$ not fast and not strong

$$\neg(A \text{ AND } B) \longleftrightarrow \neg A \text{ OR } \neg B$$

$$\neg(A \text{ OR } B) \longleftrightarrow \neg A \text{ AND } \neg B$$

De Morgan's Laws

$$(A \rightarrow B) \text{ AND } (B \rightarrow A) \overset{?}{=} (A \text{ AND } B) \text{ OR } (\neg A \text{ AND } \neg B)$$

$(A \to B)$ AND $(B \to A)$ $\stackrel{?}{=}$ $(A$ AND $B)$ OR $(\neg A$ AND $\neg B)$

$=$ $(\neg A$ OR $B)$ AND $(\neg B$ OR $A)$

by earlier result

Can replace $A \to B$ with $\neg A$ OR B

vacuous $A \to B$

because A

Recall, $\underbrace{\neg A}_{\text{case 1}}$ OR $\underbrace{(A \text{ AND } B)}_{\text{case 2}}$

$$(A \to B) \text{ AND } (B \to A) \stackrel{?}{=} (A \text{ AND } B) \text{ OR } (\neg A \text{ AND } \neg B)$$

$$= (\neg A \text{ OR } B) \text{ AND } (\neg B \text{ OR } A)$$ by earlier result

$$= (\neg A \text{ AND } (\neg B \text{ OR } A)) \text{ OR } (B \text{ AND } (\neg B \text{ OR } A))$$ distr.

treat $(\neg B \text{ OR } A)$ as C

$$(A \to B) \text{ AND } (B \to A) \overset{?}{=} (A \text{ AND } B) \text{ OR } (\neg A \text{ AND } \neg B)$$

$$= (\neg A \text{ OR } B) \text{ AND } (\neg B \text{ OR } A) \qquad \text{by earlier result}$$

$$= \underline{(\neg A \text{ AND } (\neg B \text{ OR } A))} \text{ OR } \underline{(B \text{ AND } (\neg B \text{ OR } A))} \qquad \text{distr.}$$

$$= \underline{((\neg A \text{ AND } \neg B) \text{ OR } (\neg A \text{ AND } A))} \text{ OR } \underline{((B \text{ AND } \neg B) \text{ OR } (B \text{ AND } A))} \quad \gg$$

$$(A \to B) \text{ AND } (B \to A) \stackrel{?}{=} (A \text{ AND } B) \text{ OR } (\neg A \text{ AND } \neg B)$$

$$= (\neg A \text{ OR } B) \text{ AND } (\neg B \text{ OR } A) \qquad \text{by earlier result}$$

$$= (\neg A \text{ AND } (\neg B \text{ OR } A)) \text{ OR } (B \text{ AND } (\neg B \text{ OR } A)) \qquad \text{distr.}$$

$$= ((\neg A \text{ AND } \neg B) \text{ OR } (\underline{\neg A \text{ AND } A})) \text{ OR } ((\underline{B \text{ AND } \neg B}) \text{ OR } (B \text{ AND } A)) \quad \gg$$

$$= ((\neg A \text{ AND } \neg B) \text{ OR } F) \text{ OR } (F \text{ OR } (B \text{ AND } A))$$

$$(A \to B) \text{ AND } (B \to A) \stackrel{?}{=} (A \text{ AND } B) \text{ OR } (\neg A \text{ AND } \neg B)$$

$$= (\neg A \text{ OR } B) \text{ AND } (\neg B \text{ OR } A) \qquad \text{by earlier result}$$

$$= (\neg A \text{ AND } (\neg B \text{ OR } A)) \text{ OR } (B \text{ AND } (\neg B \text{ OR } A)) \qquad \text{distr.}$$

$$= ((\neg A \text{ AND } \neg B) \text{ OR } (\neg A \text{ AND } A)) \text{ OR } ((B \text{ AND } \neg B) \text{ OR } (B \text{ AND } A)) \quad \text{"}$$

$$= ((\neg A \text{ AND } \neg B) \text{ OR } F) \text{ OR } (F \text{ OR } (B \text{ AND } A))$$

$$= (\neg A \text{ AND } \neg B) \text{ OR } F \text{ OR } F \text{ OR } (B \text{ AND } A) \qquad \text{assoc.}$$

$$(A \to B) \text{ AND } (B \to A) \stackrel{?}{=} (A \text{ AND } B) \text{ OR } (\neg A \text{ AND } \neg B)$$

$$= (\neg A \text{ OR } B) \text{ AND } (\neg B \text{ OR } A)$$ by earlier result

$$= (\neg A \text{ AND } (\neg B \text{ OR } A)) \text{ OR } (B \text{ AND } (\neg B \text{ OR } A))$$ distr.

$$= ((\neg A \text{ AND } \neg B) \text{ OR } (\neg A \text{ AND } A)) \text{ OR } ((B \text{ AND } \neg B) \text{ OR } (B \text{ AND } A))$$ "

$$= ((\neg A \text{ AND } \neg B) \text{ OR } F) \text{ OR } (F \text{ OR } (B \text{ AND } A))$$

$$= (\neg A \text{ AND } \neg B) \text{ OR } F \text{ OR } F \text{ OR } (B \text{ AND } A)$$ assoc.

$$= (\neg A \text{ AND } \neg B) \text{ OR } (B \text{ AND } A)$$

$$(A \to B) \text{ AND } (B \to A) \stackrel{?}{=} (A \text{ AND } B) \text{ OR } (\neg A \text{ AND } \neg B)$$

$$= (\neg A \text{ OR } B) \text{ AND } (\neg B \text{ OR } A) \qquad \text{by earlier result}$$

$$= (\neg A \text{ AND } (\neg B \text{ OR } A)) \text{ OR } (B \text{ AND } (\neg B \text{ OR } A)) \qquad \text{distr.}$$

$$= ((\neg A \text{ AND } \neg B) \text{ OR } (\neg A \text{ AND } A)) \text{ OR } ((B \text{ AND } \neg B) \text{ OR } (B \text{ AND } A)) \qquad \text{"}$$

$$= ((\neg A \text{ AND } \neg B) \text{ OR } F) \text{ OR } (F \text{ OR } (B \text{ AND } A))$$

$$= (\neg A \text{ AND } \neg B) \text{ OR } F \text{ OR } F \text{ OR } (B \text{ AND } A) \qquad \text{assoc.}$$

$$= (\neg A \text{ AND } \neg B) \text{ OR } (B \text{ AND } A)$$

$$= (A \text{ AND } B) \text{ OR } (\neg A \text{ AND } \neg B) \qquad \text{comm.}$$

"for all" $\forall$ vs $\exists$ "exists"

"for all" $\forall$ vs $\exists$ "exists"

$\forall x \in \mathbb{R}.\ x^2 \geq 0$

"for all" $\forall$ vs $\exists$ "exists"

$\forall x \in \mathbb{R}.\ x^2 \geqslant 0$ $\exists x \in \mathbb{R}.\ x - \pi^2 + \sqrt{2} = 0$

"for all" $\forall$ vs $\exists$ "exists"

$\forall x \in \mathbb{R}.\ x^2 \geqslant 0$ $\qquad$ $\exists x \in \mathbb{R}.\ x - \pi^2 + \sqrt{2} = 0$

For every action there is a reaction. $\forall a \exists r$

"for all" $\forall$ vs $\exists$ "exists"

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There is an answer for every question.

"for all" $\forall$ vs $\exists$ "exists"

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Ambiguous $\longrightarrow$ One answer for all questions? $\exists a \forall q$

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There is an answer for every question.

Ambiguous → One answer for all questions? $\exists a \forall q$

→ For every question there is an answer. $\forall q \exists a$

"for all" $\forall$ vs $\exists$ "exists"

$\forall x \in \mathbb{R}.\ x^2 \geq 0$ $\qquad$ $\exists x \in \mathbb{R}.\ x - \pi^2 + \sqrt{2} = 0$

For every action there is a reaction. $\forall a \exists r \neq \exists r \forall a$ *

There is an answer for every question.

Ambiguous → One answer for all questions? $\exists a \forall q$

$\neq$

For every question there is an answer. $\forall q \exists a$

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* Inconsistency in literature

Every coin has two sides: ?

Every coin has two sides: $\forall c\, \exists s_1(c)\, \exists s_2(c)$

Every coin has two sides: $\forall c\, \exists s_1(c)\, \exists s_2(c)$

P = prime numbers. X = even integers > 2.

$\forall n \in X\, \exists a \in P\, \exists b \in P.\ n = a + b$

?

Every coin has two sides: $\forall c \exists s_1(c) \exists s_2(c)$

P = prime numbers. X = even integers > 2.

$$\forall n \in X \, \exists a \in P \, \exists b \in P. \; n = a + b$$

For every integer n greater than 2,

Every coin has two sides: $\forall c\, \exists s_1(c)\, \exists s_2(c)$

P = prime numbers. X = even integers > 2.

$$\forall n \in X\ \underline{\exists a \in P\, \exists b \in P}.\ n = a + b$$

For every integer n greater than 2,

<u>there exist prime numbers a and b</u>

Every coin has two sides: $\forall c \, \exists s_1(c) \, \exists s_2(c)$

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For every integer n greater than 2,

there exist prime numbers a and b <u>such that</u>

Every coin has two sides: $\forall c \, \exists s_1(c) \, \exists s_2(c)$

P = prime numbers. X = even integers > 2.

$$\forall n \in X \; \exists a \in P \; \exists b \in P. \; n = a + b$$

For every integer n greater than 2,
there exist prime numbers a and b such that $n = a + b$.

Every integer greater than 2 is the sum of two primes.
(Goldbach's conjecture)

$\forall n \in X \, \exists a \in P \, \exists b \in P. \; n = a + b$

Every integer greater than 2 is the sum of two primes.

$$\forall n \in X\ \exists a \in P\ \exists b \in P.\ n = a + b$$

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$\forall n \in X\ \exists a \in P\ \exists b \in P.\ n = a + b$

Every integer greater than 2 is the sum of two primes.

$\neq$

$\forall a \in P\ \forall b \in P\ \exists n \in X.\ n = a + b$

For every pair of primes there is an integer >2 that is their sum.

$\forall n \in X\ \exists a \in P\ \exists b \in P.\ n = a + b$

Every integer greater than 2 is the sum of two primes.

$\forall a \in P\ \forall b \in P\ \exists n \in X.\ n = a + b$

For every pair of primes there is an integer >2 that is their sum.

$\exists a \in P\ \exists b \in P.\ \forall n \in X\ n = a + b$

?

$\forall n \in X\, \exists a \in P\, \exists b \in P.\ n = a + b$

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For every pair of primes there is an integer >2 that is their sum.

$\exists a \in P\, \exists b \in P.\, \forall n \in X\ n = a + b$

There exist 2 primes such that their sum is equal to every integer >2

$\forall n \in X\, \exists a \in P\, \exists b \in P.\; n = a + b$

Every integer greater than 2 is the sum of two primes.

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For every pair of primes there is an integer >2 that is their sum.

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There exist 2 primes such that their sum is equal to every integer >2

$\exists a \in P\, \exists b \in P\, \forall n \in X.\; n = a + b$?

$\forall n \in X\, \exists a \in P\, \exists b \in P.\ n = a + b$

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For every pair of primes there is an integer >2 that is their sum.

$\exists a \in P\, \exists b \in P.\, \forall n \in X\ n = a + b$

There exist 2 primes such that their sum is equal to every integer >2

$\exists a \in P\, \exists b \in P\, \forall n \in X.\ n = a + b$ (poor form)

Every integer greater than 2 is the sum of two primes.

$\forall x.\ P(x)$ For all x, proposition P (with x as a variable) is true.

that makes sense for P

e.g., P(Alex) : Alex likes logic.

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Not everybody likes logic. ?

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e.g., $P(\text{Alex})$: Alex likes logic.

Not (everybody likes logic).

$\neg(\forall x. P(x))$

$\forall x. P(x)$ For all x, proposition P (with x as a $\underbrace{\text{variable}}_{\text{that makes sense for } P}$) is true.

e.g., $P(\text{Alex})$: Alex likes logic.

Not everybody likes logic. $\neg(\forall x. P(x))$

There is someone who doesn't like logic. ?

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Not everybody likes logic. $\neg(\forall x. P(x))$

There is someone who doesn't like logic. $\exists x. \neg(P(x))$

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Nobody can lick their own elbow. actually not true. ?

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Every person can't lick their own elbow. ?

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e.g., $P(\text{Alex})$: Alex likes logic.

Not everybody likes logic. $\neg(\forall x. P(x))$

There is someone who doesn't like logic. $\exists x. \neg(P(x))$

actually not true.

Nobody can lick their own elbow. $\neg(\exists x. P(x))$

Every person can't lick their own elbow. $\forall x. \neg(P(x))$

$\forall x.\, P(x)$ For all x, proposition P (with x as a variable) is true.

that makes sense for P

e.g., P(Alex) : Alex likes logic.

Not everybody likes logic. $\neg(\forall x.\, P(x))$

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actually not true.

Nobody can lick their own elbow. $\neg(\exists x.\, P(x))$

Every person can't lick their own elbow. $\forall x.\, \neg(P(x))$