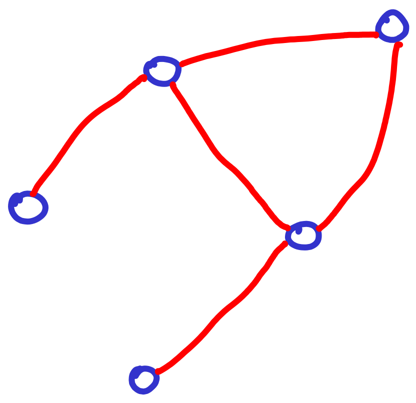
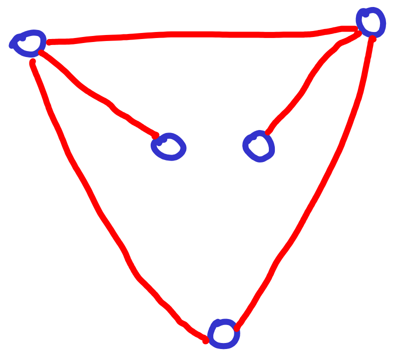
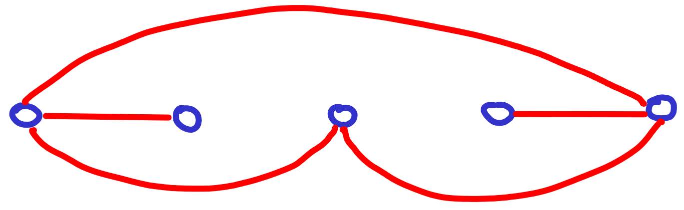
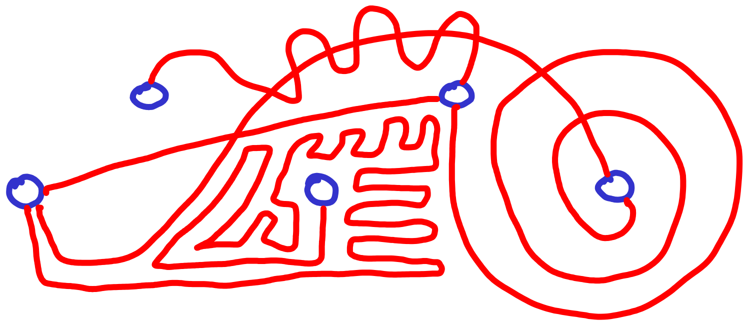
GRAPHS

vertices and edges

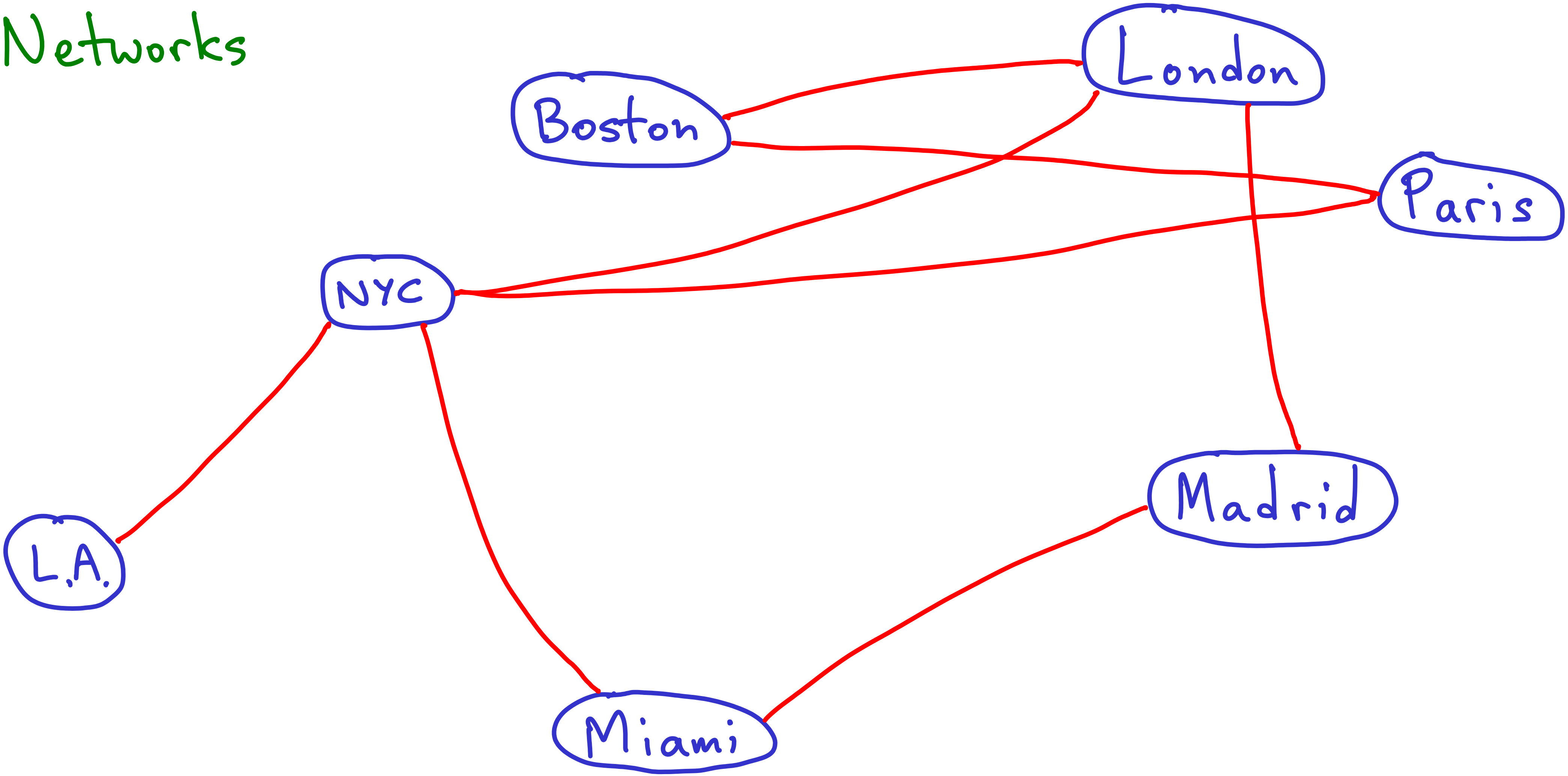




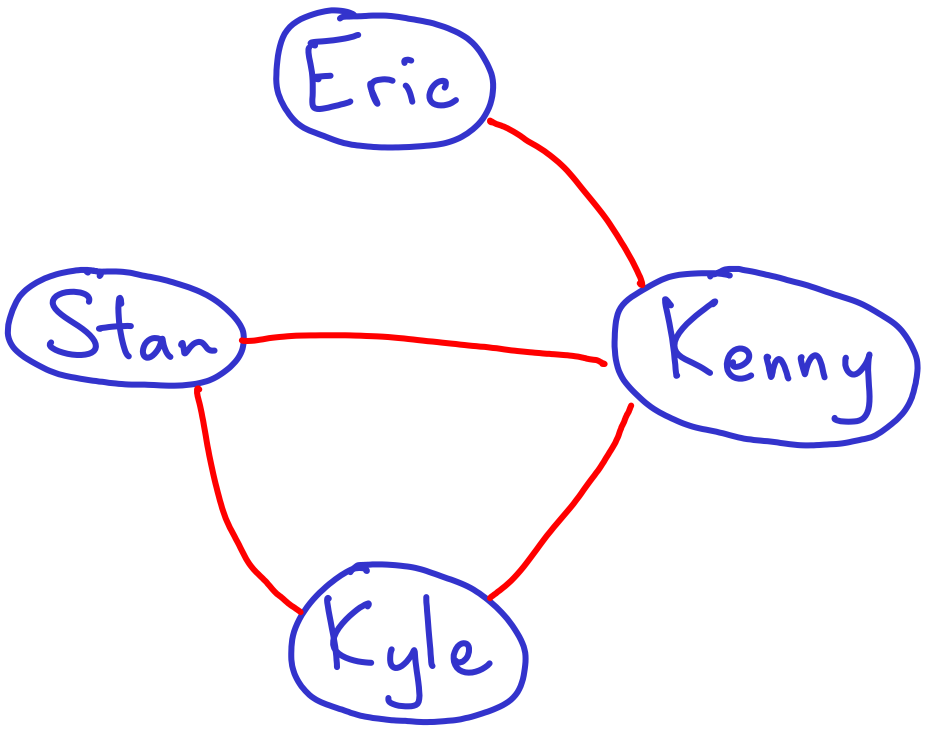




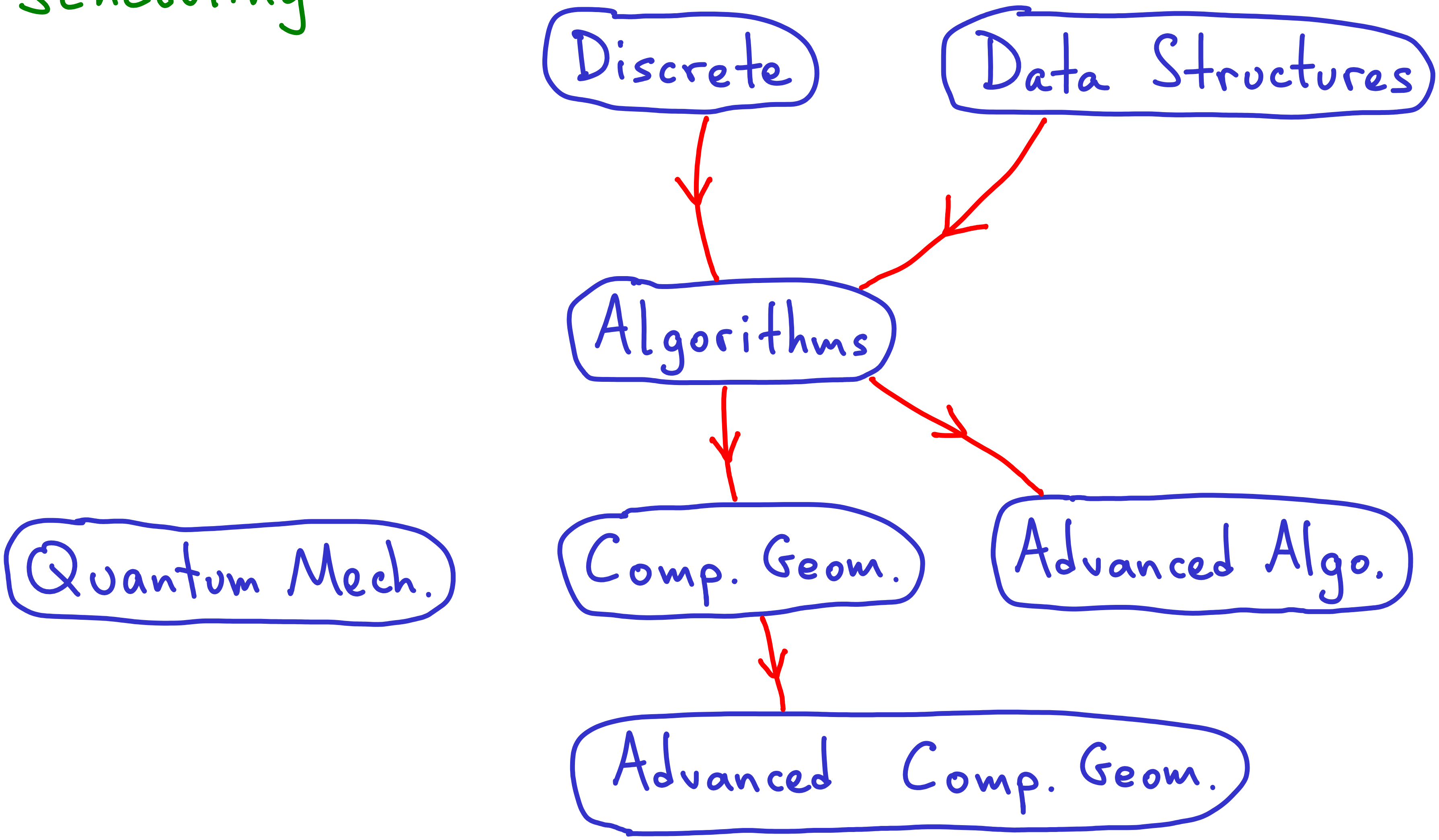
Networks



Social networks



Scheduling



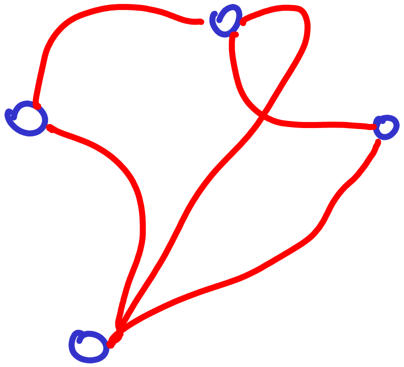
(Directed Graph)

Graphs can be "abstract" or "geometric" / "embedded"

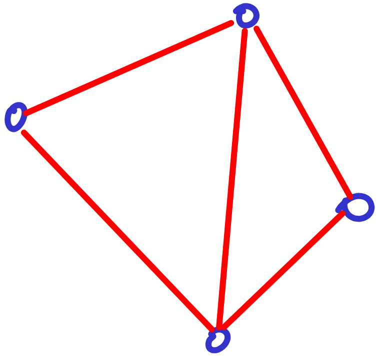
co-ordinates & drawings don't matter

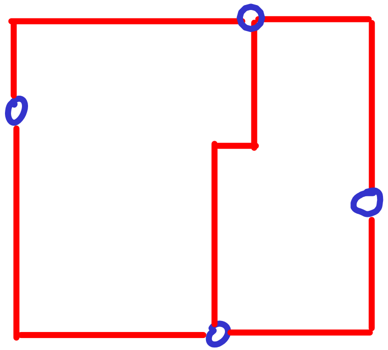
representing physical restrictions

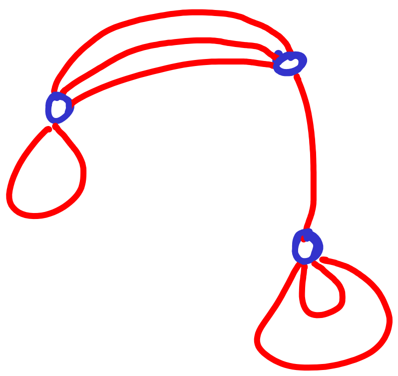
although sometimes it helps to draw & visualize



VS





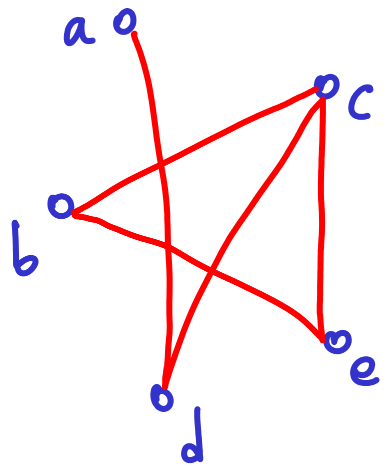


Often it is assumed that there are no loops or multiple edges

Assume this unless specified

$$G = (V, E)$$

$$V(G) \qquad E(G)$$

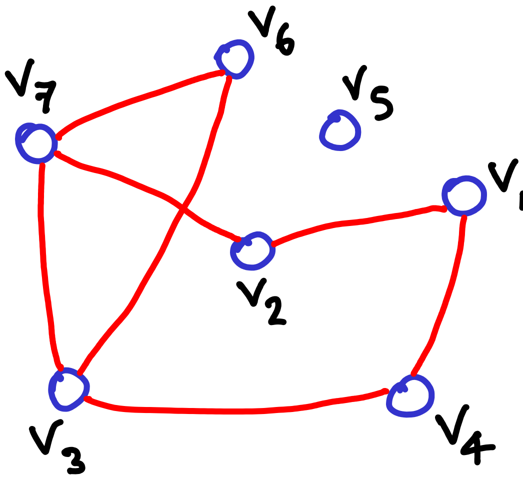


$$(\{a, b, c, d, e\}, \{ad, \underbrace{bc}, be, ce, cd\})$$

b & c are adjacent

(share an edge)

Representation



$$G = \{V, E\}$$

"is y a neighbor of x"? → matrix

"report all neighbors of x" → list

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \left.\begin{matrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{matrix}\right\} \end{array}$$

Adjacency matrix

size: $|V|^2$

(symmetric for undirected)

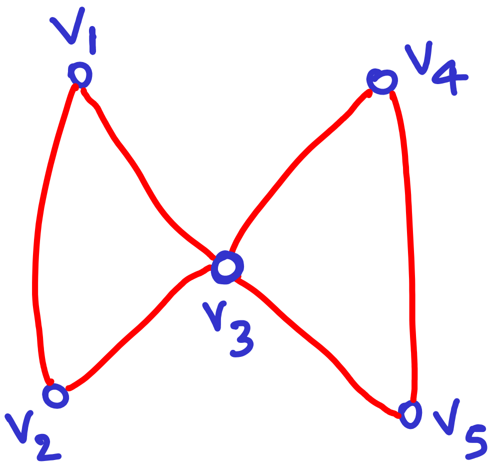
Adjacency list

size: $|V| + 2|E|$

(undirected)

$$\left\{\begin{array}{l} 1 \to 2 \to 4 \\ 2 \to 1 \to 7 \\ 3 \to 4 \to 6 \to 7 \\ 4 \to 1 \to 3 \\ 5 \\ 6 \to 3 \to 7 \\ 7 \to 2 \to 3 \to 6 \end{array}\right.$$

Vertex degree



v_3 has 4 adjacent vertices = 4 neighbors

$d(v_3) = 4$

$\sum d(v) = 2+2+4+2+2 = 12$

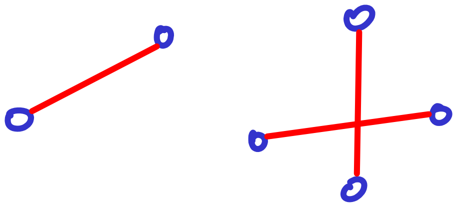
$|E| = 6$

$$\sum_{v \in G} d(v) = 2 \cdot |E|$$

(just doublecounting)

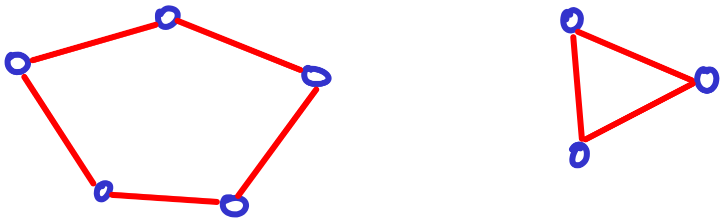
Regular graphs : all vertices have the same degree

d = 1 ?



1-regular

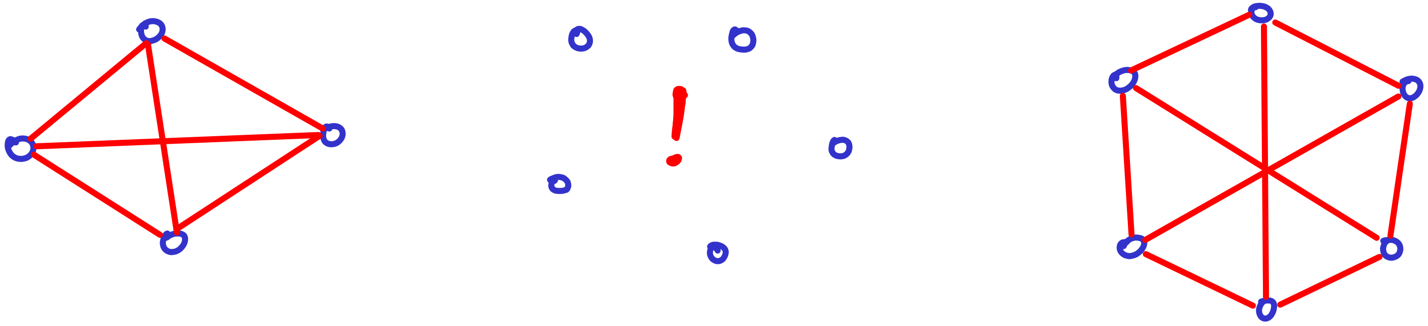
d = 2 ?



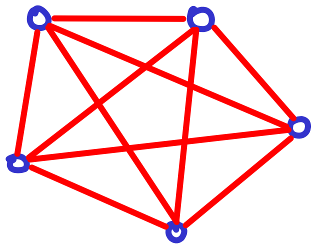
2-regular

d = 3 ?

3-regular



$n=5$



4-regular

($n-1$ regular) $\rightarrow$ complete graph

edges? $n-1 + n-2 + n-3 + \cdots + 3 + 2 + 1 = \binom{n}{2}$

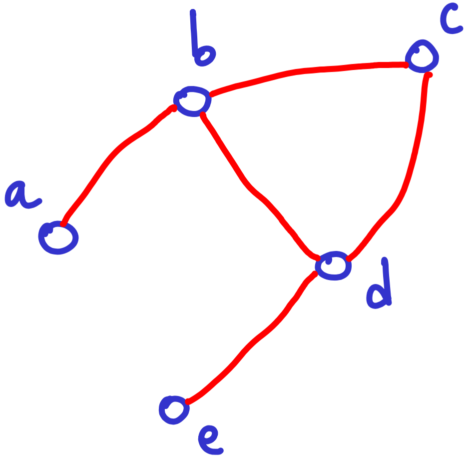
$$= \sum_{i=1}^{n-1} i = \frac{n \cdot (n-1)}{2}$$

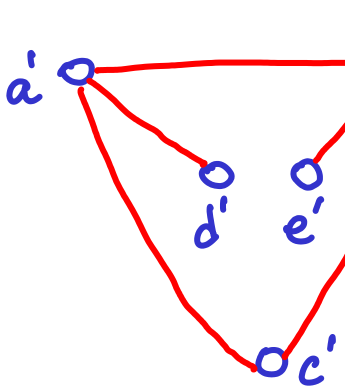
possible graphs on n vertices? $2^{\binom{n}{2}}$

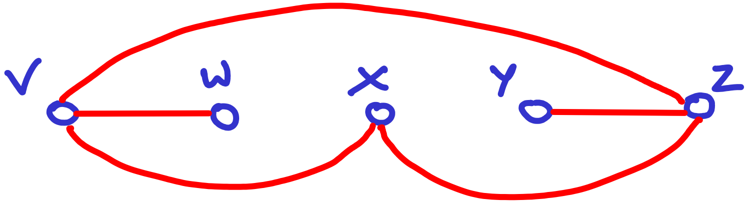
... but some of them are similar in shape isomorphism → map vertices of graph G to vertices of graph H

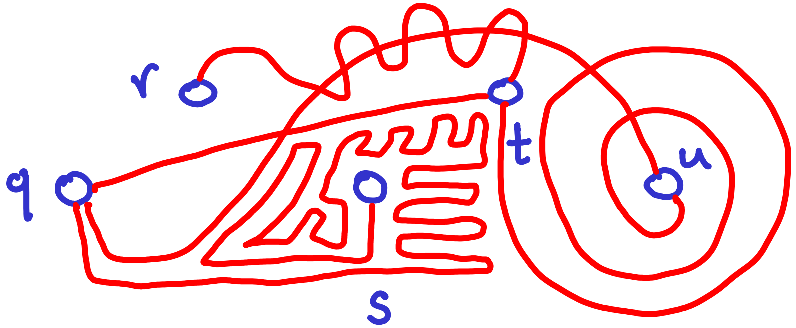
if vertices $\alpha, \beta \in G$ share an edge in G then

vertices $m(\alpha), m(\beta) \in H$ must share an edge in H.









a :	d' :	w :	r
b :	a' :	v :	t
c :	c' :	x :	u
d :	b' :	z :	q
e :	e' :	y :	s

Determining if two graphs are isomorphic (without a mapping)

is difficult (time complexity as function of n)

(not just because drawings look complicated)

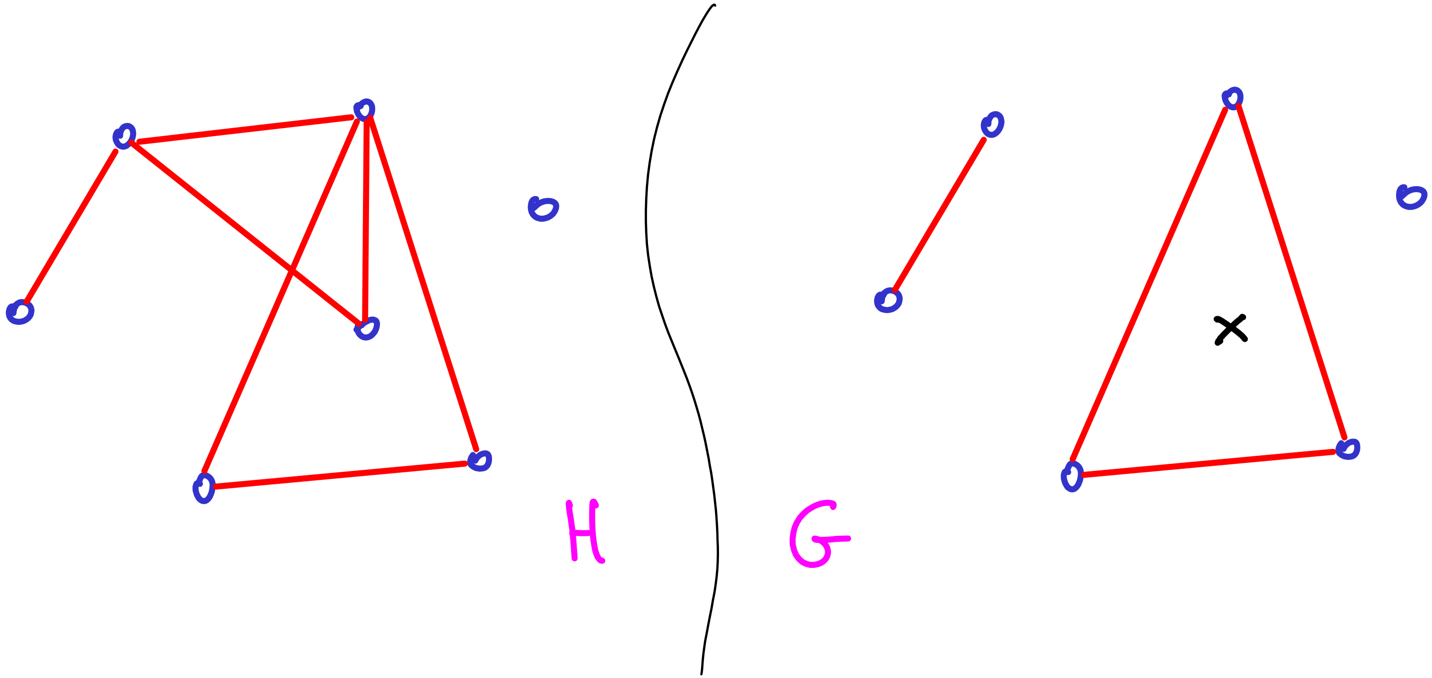
Counting # possible graphs without "doublecounting" isomorphs

is complicated

Subgraphs

G is a subgraph of H if $\begin{cases} V(G) \subseteq V(H) \\ E(G) \subseteq E(H) \end{cases}$ $\Big\}$ if equal then it's a "spanning" subgraph

unlike



Subgraphs

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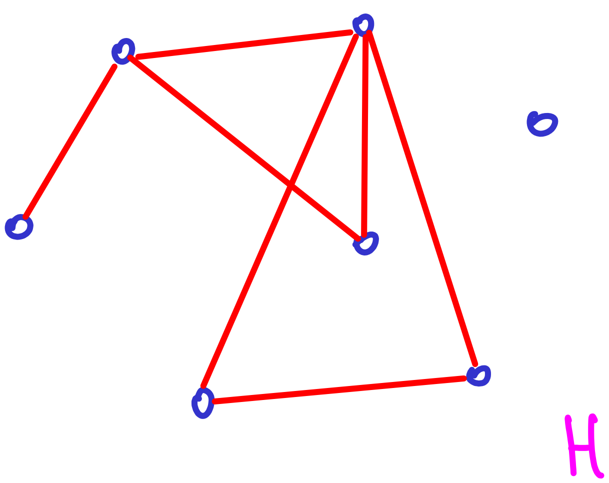

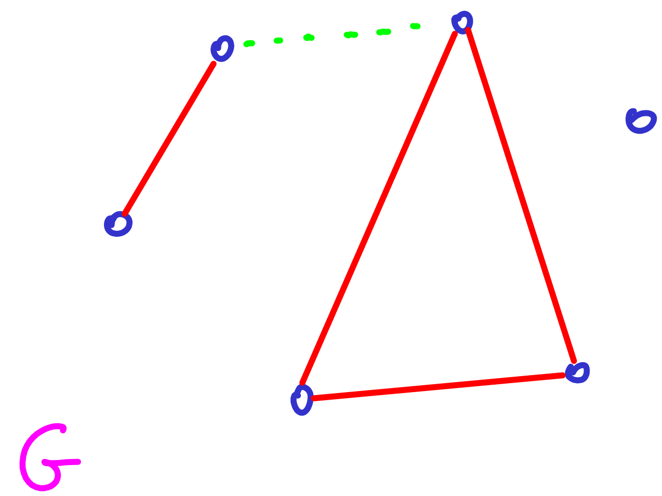

If you only remove edges as a result of removing vertices then G is an "induced" subgraph

unlike