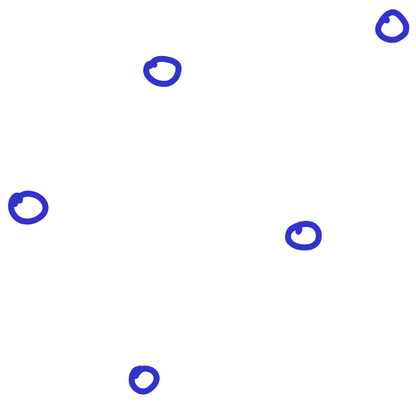
GRAPHS

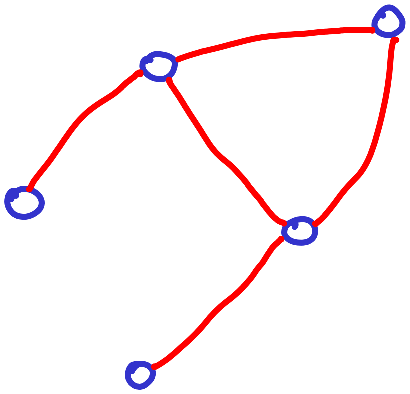
GRAPHS

vertices



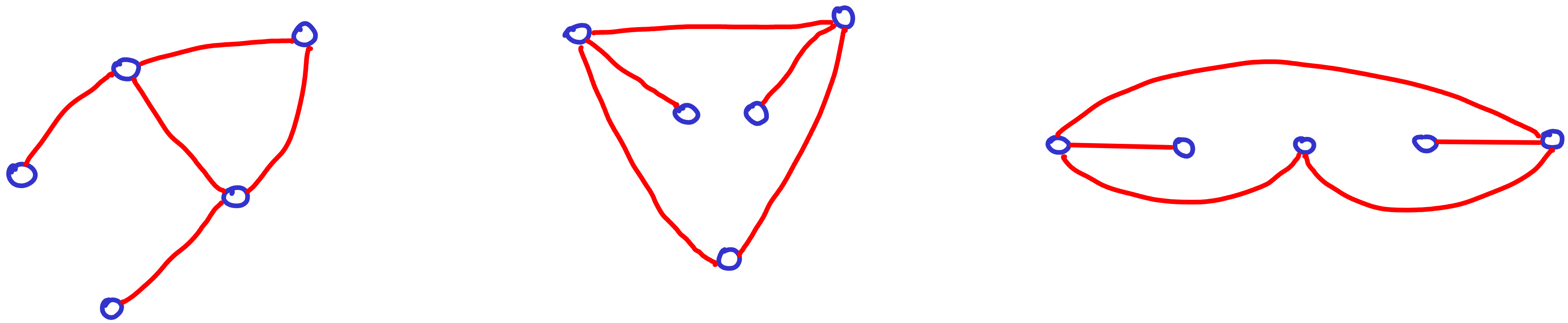
GRAPHS

vertices and edges



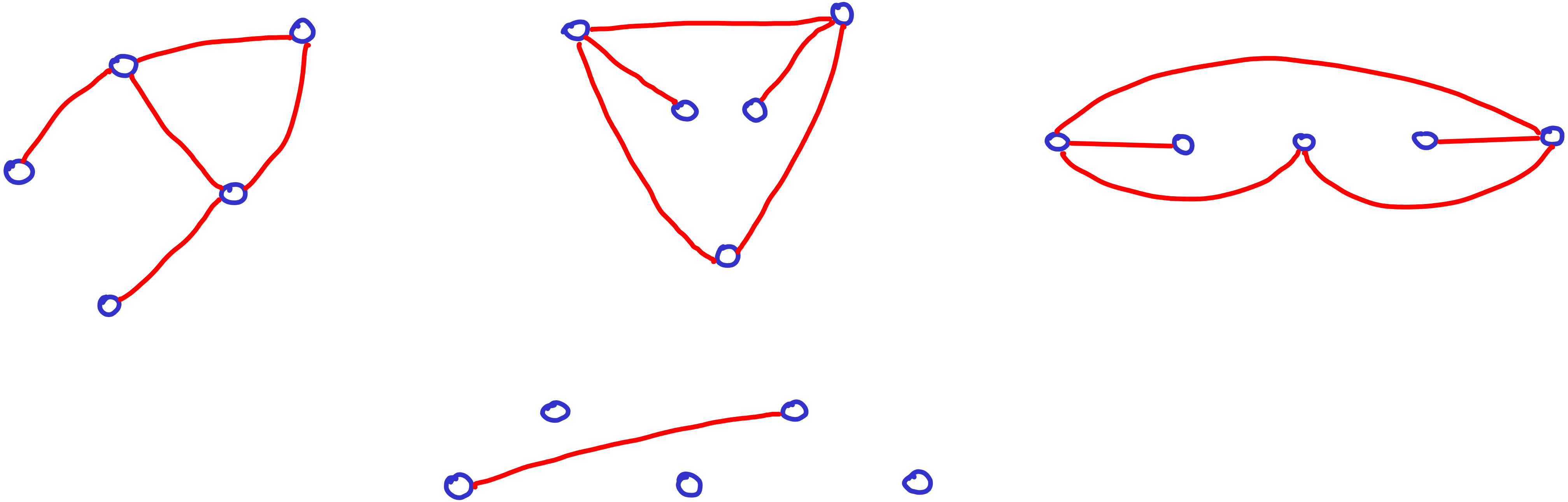
GRAPHS

vertices and edges



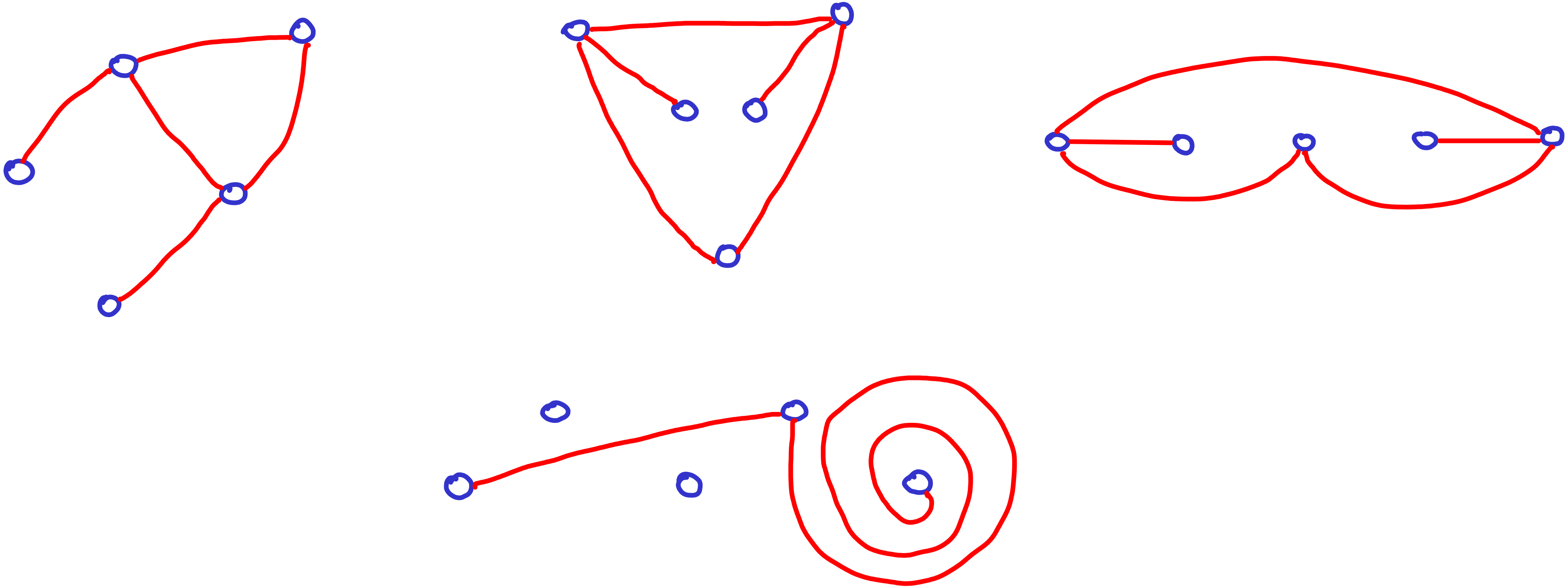
GRAPHS

vertices and edges



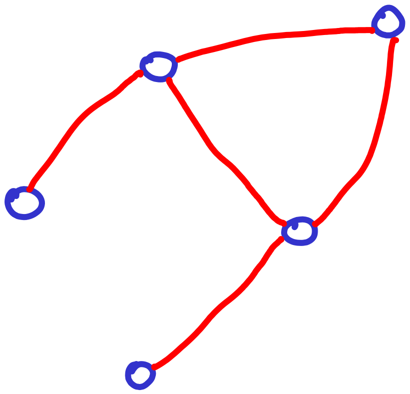
GRAPHS

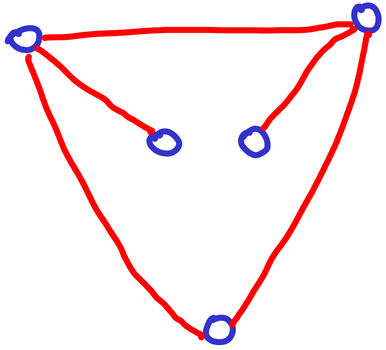
vertices and edges

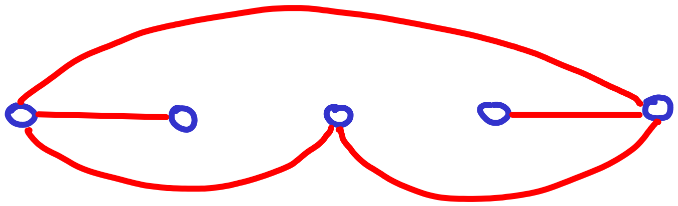


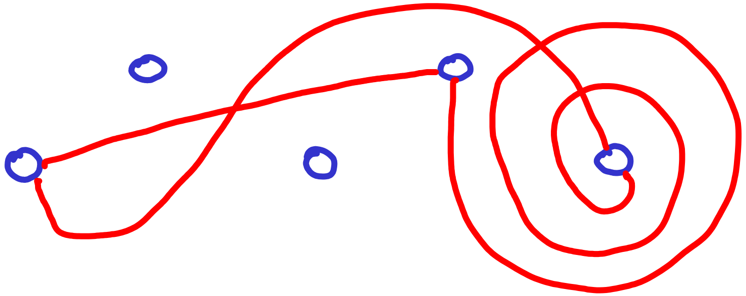
GRAPHS

vertices and edges



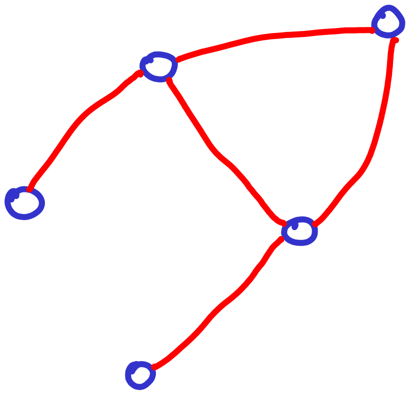


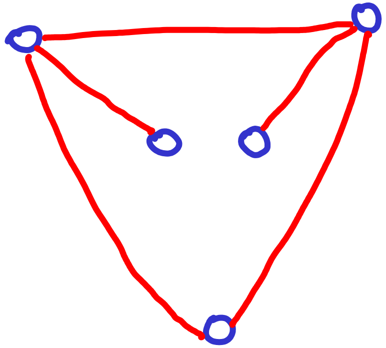


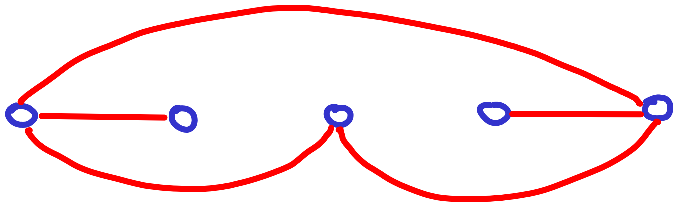


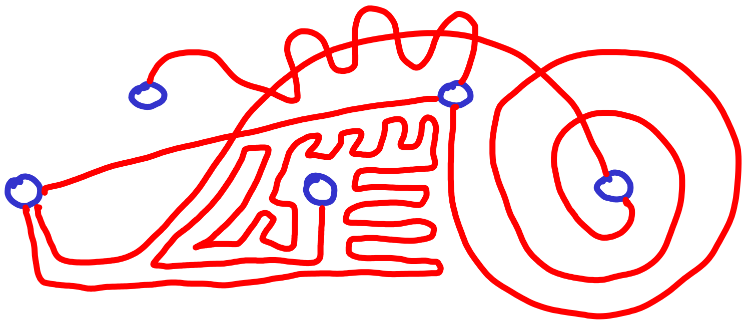
GRAPHS

vertices and edges

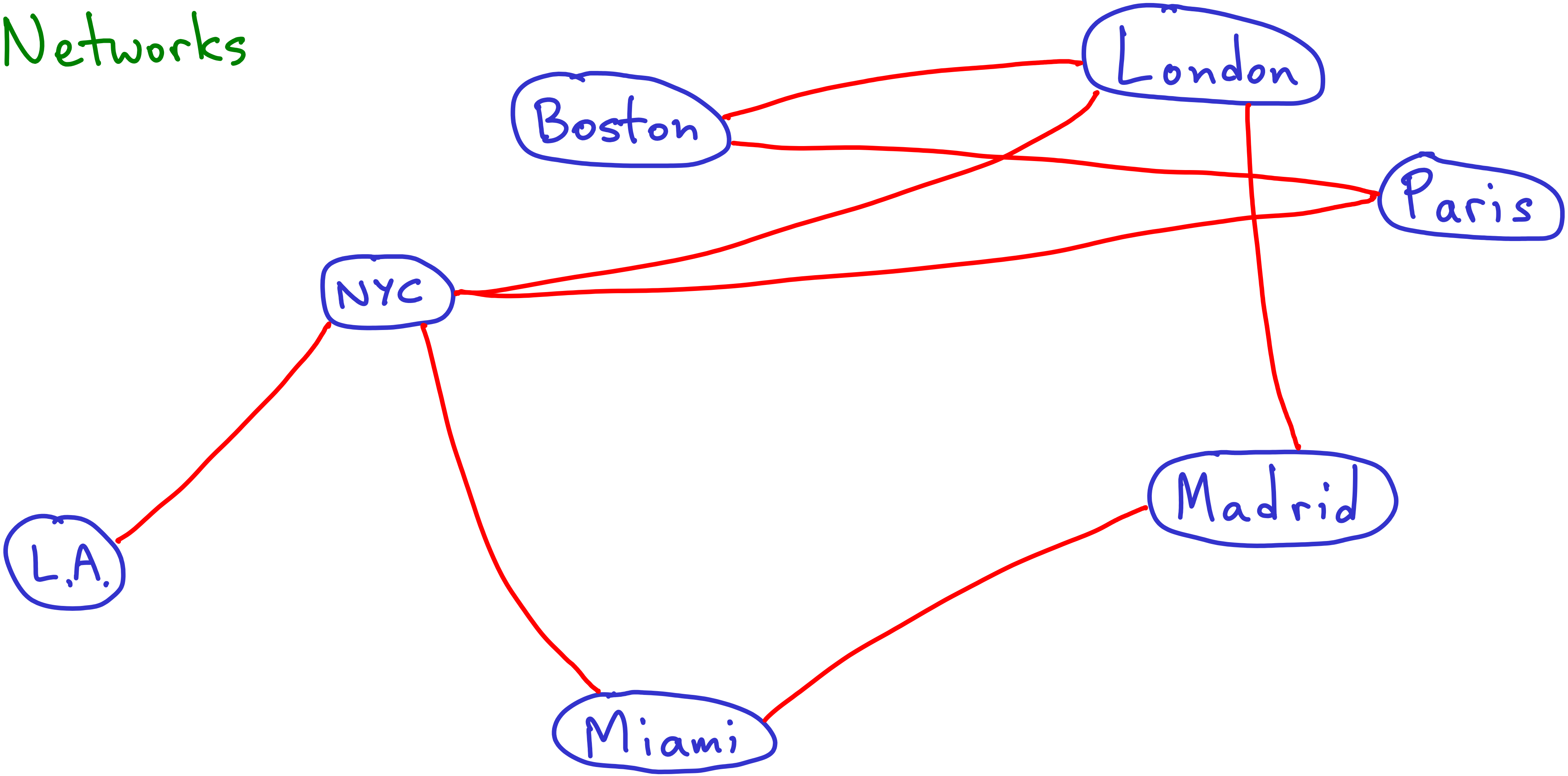




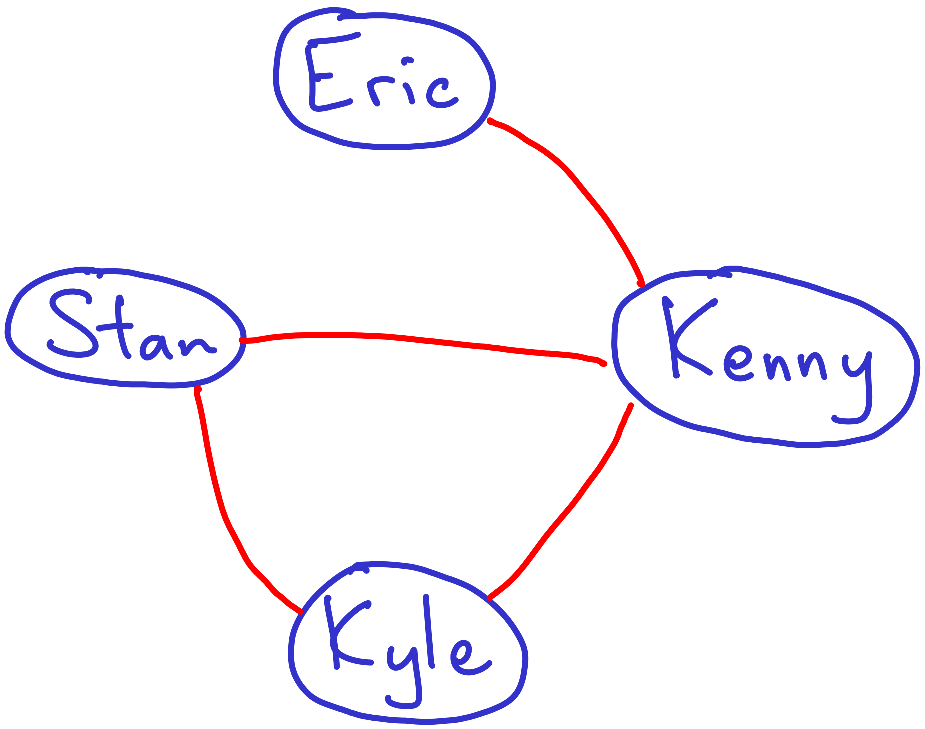




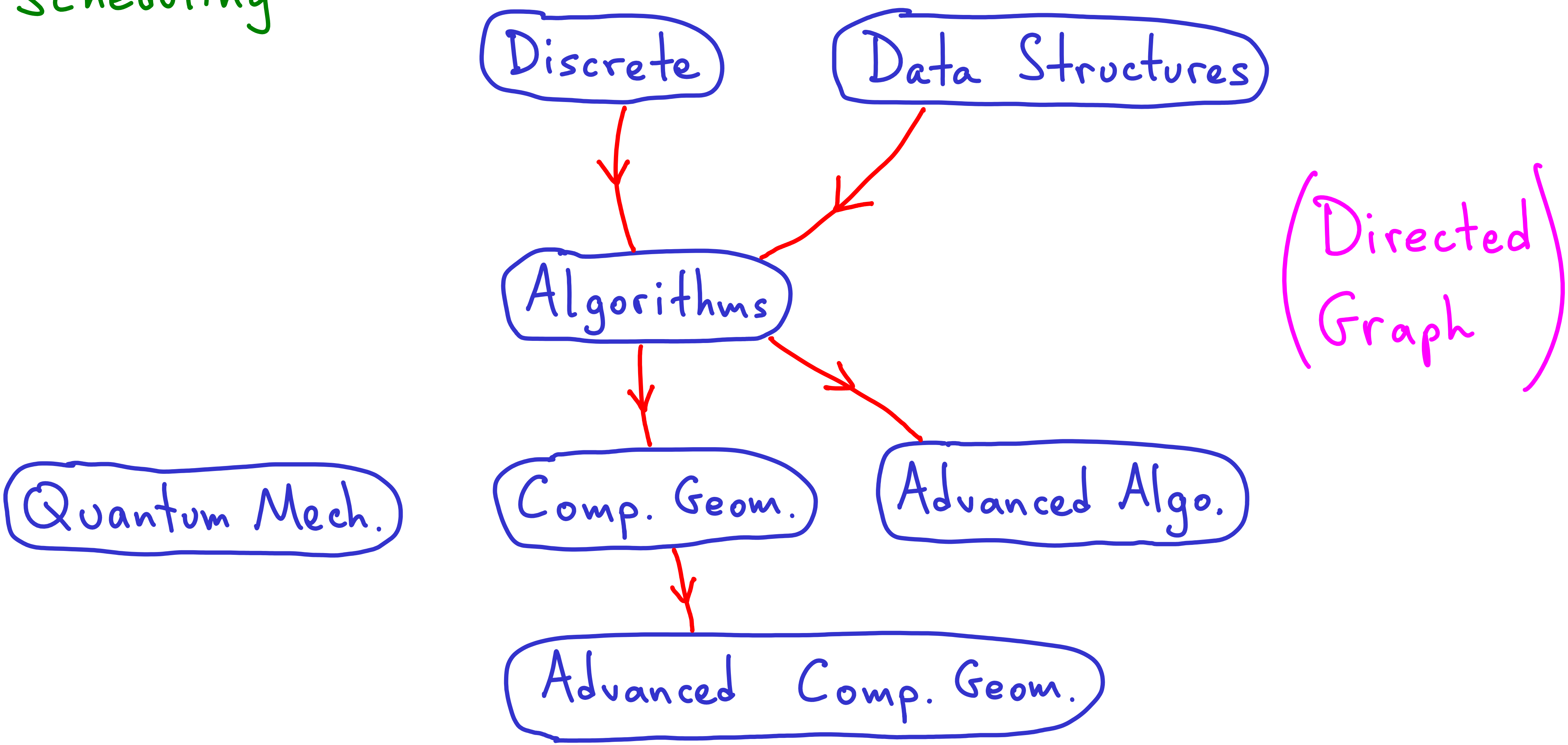
Networks



Social networks



Scheduling

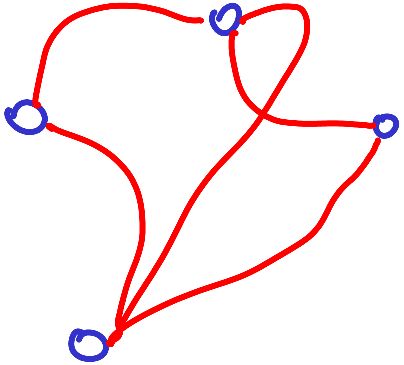

Discrete
Data Structures
Algorithms
Directed Graph
Quantum Mech.
Comp. Geom.
Advanced Algo.
Advanced Comp. Geom.

Graphs can be "abstract"

$\underbrace{}_{}$

co-ordinates
& drawings
don't matter

although sometimes
it helps to draw & visualize

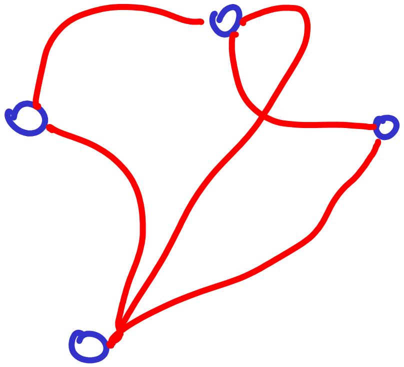


Graphs can be "abstract" or "geometric" / "embedded"

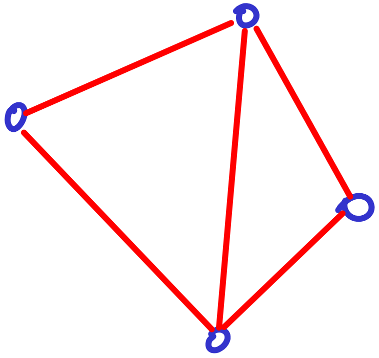
co-ordinates
& drawings
don't matter

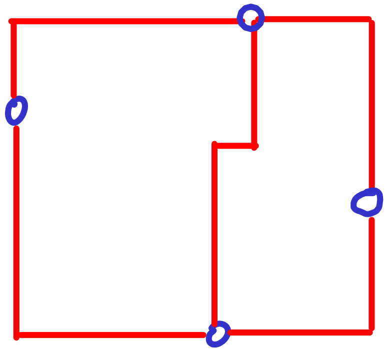
representing physical
restrictions

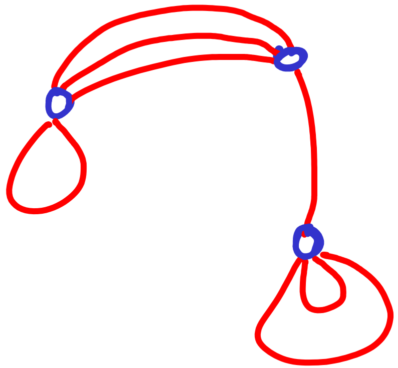
although sometimes
it helps to draw & visualize



vs

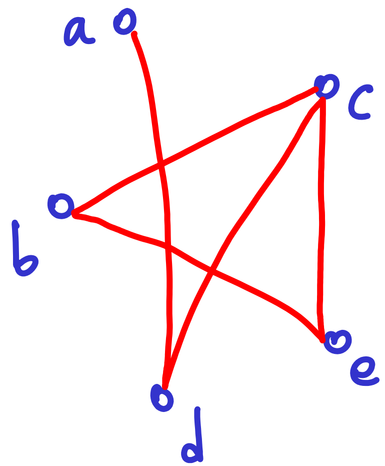






Often it is assumed that there are no loops or multiple edges

Assume this unless specified

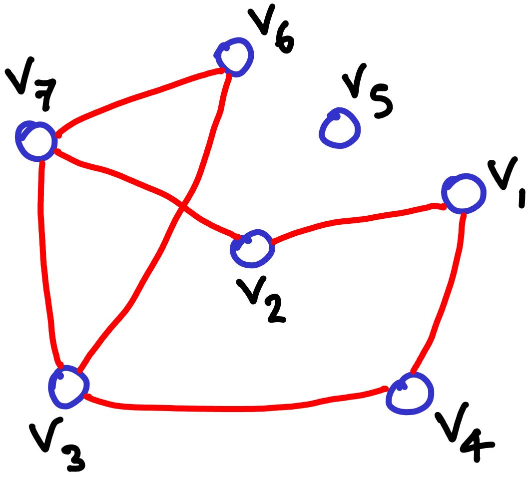


$$G = (V, E)$$

$$\begin{array}{cc} V & E \\ \downarrow & \downarrow \\ V(G) & E(G) \end{array}$$

$$(\{a, b, c, d, e\}, \{ad, \underbrace{bc}_{\text{b \& c are adjacent (share an edge)}}, be, ce, cd\})$$

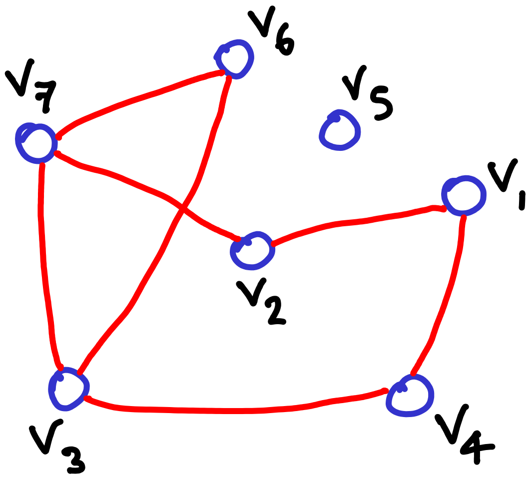
Representation



$$G = \{V, E\}$$

?

Representation



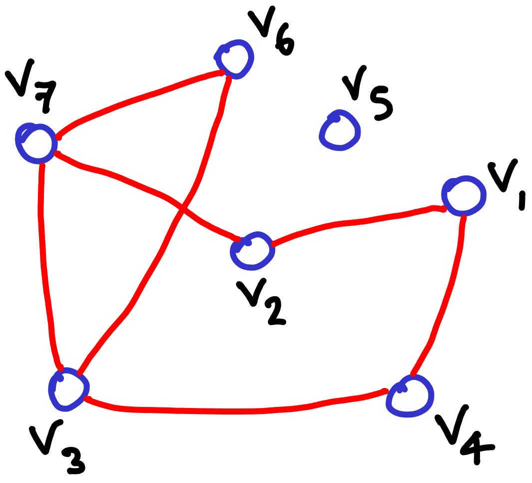
$$G = \{V, E\}$$

$$\left.\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
7 & 0 & 1 & 1 & 0 & 0 & 1 & 0
\end{array}\right\}$$

Adjacency matrix

size: ?

Representation



$$
\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
7 & 0 & 1 & 1 & 0 & 0 & 1 & 0
\end{array}
\Bigg\}
$$

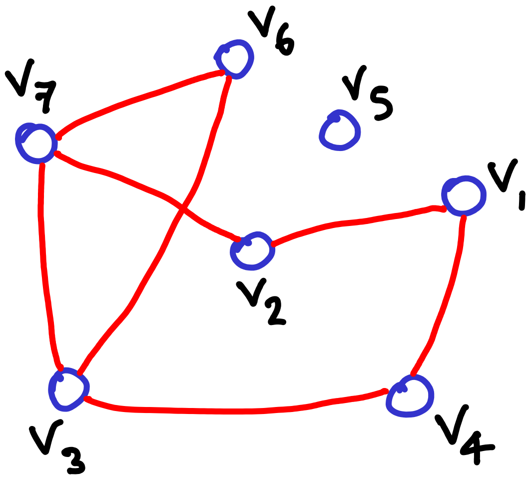
Adjacency matrix

size: $|V|^2$

(symmetric for undirected)

$$G = \{V, E\}$$

Representation



$G = \{V, E\}$

$$
\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
7 & 0 & 1 & 1 & 0 & 0 & 1 & 0
\end{array}
$$

Adjacency matrix

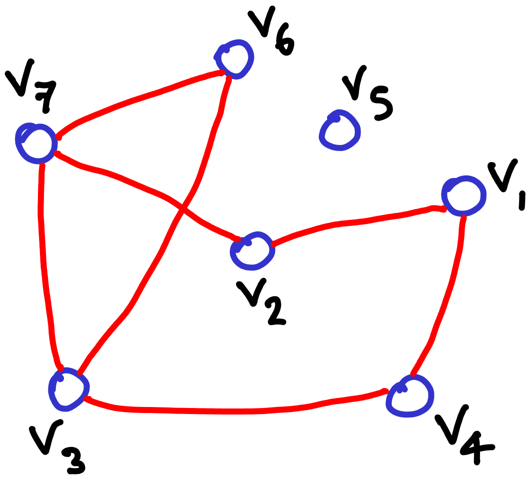
size: $|V|^2$

(symmetric for undirected)

Adjacency list

1 → 2 → 4
2 → 1 → 7
3 → 4 → 6 → 7
4 → 1 → 3
5
6 → 3 → 7
7 → 2 → 3 → 6

Representation



$G = \{V, E\}$

	1	2	3	4	5	6	7
1	0	1	0	1	0	0	0
2	1	0	0	0	0	0	1
3	0	0	0	1	0	1	1
4	1	0	1	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	1	0	0	0	1
7	0	1	1	0	0	1	0

Adjacency matrix

size: $|V|^2$

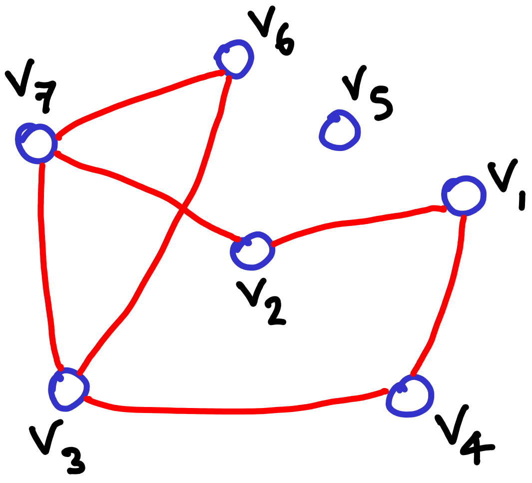
(symmetric for undirected)

Adjacency list

size: ?

1 → 2 → 4
2 → 1 → 7
3 → 4 → 6 → 7
4 → 1 → 3
5
6 → 3 → 7
7 → 2 → 3 → 6

Representation



$$G = \{V, E\}$$

$$\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
7 & 0 & 1 & 1 & 0 & 0 & 1 & 0
\end{array}$$

Adjacency matrix

size: $|V|^2$

(symmetric for undirected)

Adjacency list

size: $|V| + 2|E|$

(undirected)

$1 \to 2 \to 4$

$2 \to 1 \to 7$

$3 \to 4 \to 6 \to 7$

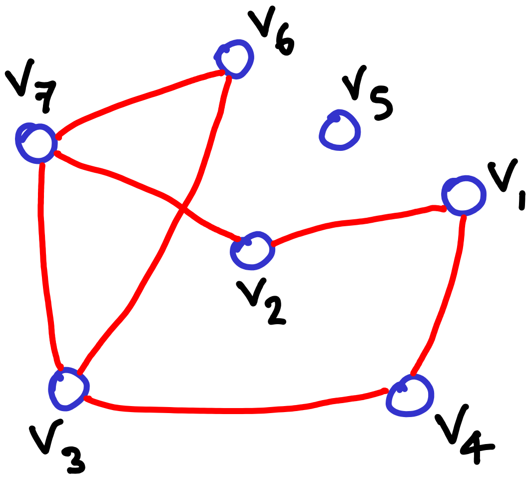
$4 \to 1 \to 3$

5

$6 \to 3 \to 7$

$7 \to 2 \to 3 \to 6$

Representation



$G = \{V, E\}$

	1	2	3	4	5	6	7
1	0	1	0	1	0	0	0
2	1	0	0	0	0	0	1
3	0	0	0	1	0	1	1
4	1	0	1	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	1	0	0	0	1
7	0	1	1	0	0	1	0

Adjacency matrix

size: $|V|^2$

(symmetric for undirected)

Adjacency list

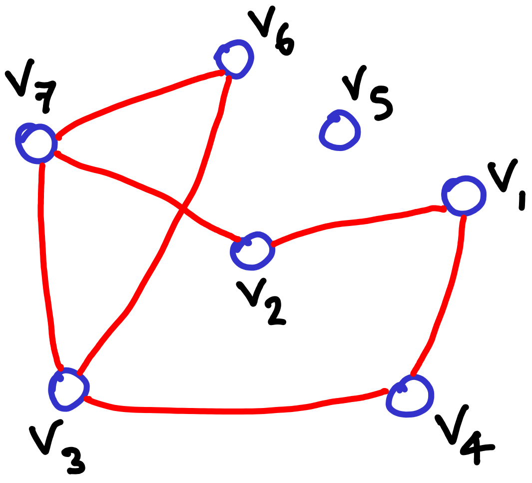
size: $|V| + 2|E|$

(undirected)

1 → 2 → 4
2 → 1 → 7
3 → 4 → 6 → 7
4 → 1 → 3
5
6 → 3 → 7
7 → 2 → 3 → 6

give one reason for using matrix vs list

Representation



$G = \{V, E\}$

"is y a neighbor of x"? → matrix

"report all neighbors of x" → list

	1	2	3	4	5	6	7
1	0	1	0	1	0	0	0
2	1	0	0	0	0	0	1
3	0	0	0	1	0	1	1
4	1	0	1	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	1	0	0	0	1
7	0	1	1	0	0	1	0

Adjacency matrix

size: $|V|^2$

(symmetric for undirected)

Adjacency list

size: $|V| + 2|E|$

(undirected)

$1 \to 2 \to 4$

$2 \to 1 \to 7$

$3 \to 4 \to 6 \to 7$

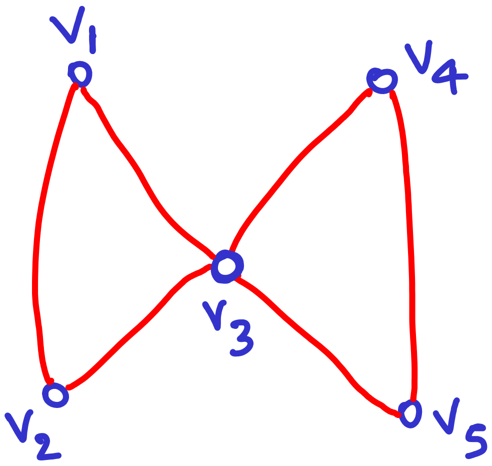
$4 \to 1 \to 3$

5

$6 \to 3 \to 7$

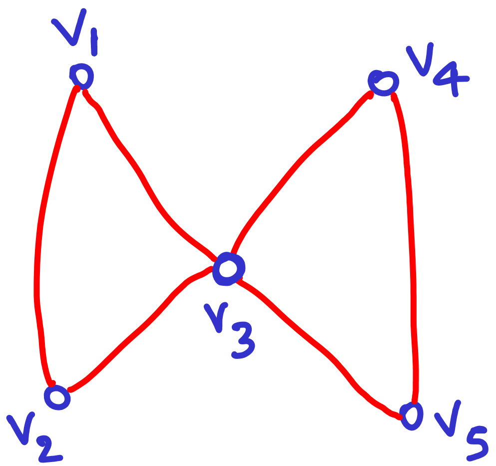
$7 \to 2 \to 3 \to 6$

Vertex degree



v_3 has 4 adjacent vertices = 4 neighbors

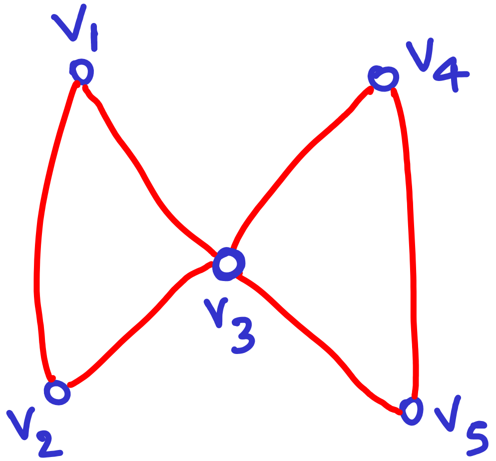
Vertex degree



v_3 has 4 adjacent vertices = 4 neighbors

$d(v_3) = 4$

Vertex degree

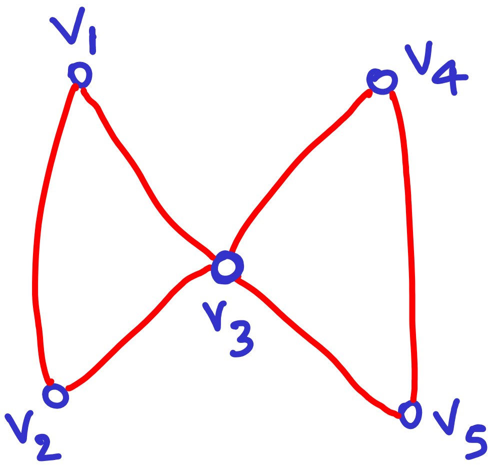


v_3 has 4 adjacent vertices = 4 neighbors

$$d(v_3) = 4$$

$$\sum_{v \in G} d(v) = ?$$

Vertex degree



v_3 has 4 adjacent vertices = 4 neighbors

$d(v_3) = 4$

$\sum d(v) = 2+2+4+2+2 = 12$

$|E| = 6$

$$\sum_{v \in G} d(v) = 2 \cdot |E|$$

(just doublecounting)

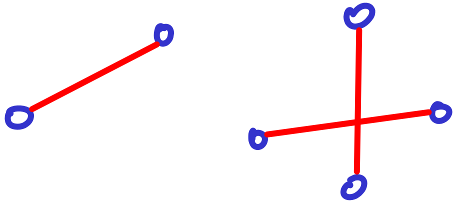
Regular graphs : all vertices have the same degree

Regular graphs: all vertices have the same degree

$d=1$?

Regular graphs : all vertices have the same degree

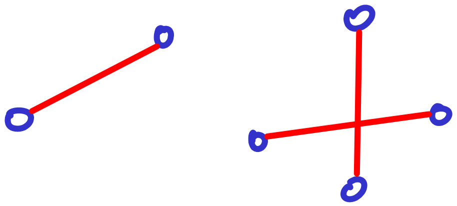
$d=1$?



1-regular

Regular graphs : all vertices have the same degree

$d=1$?

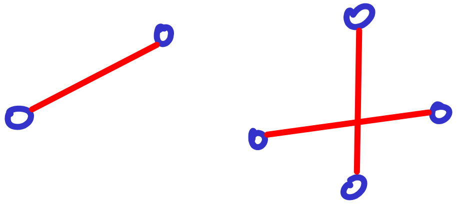


1-regular

$d=2$?

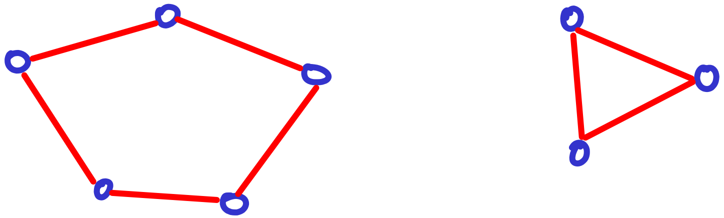
Regular graphs: all vertices have the same degree

$d = 1$?



1-regular

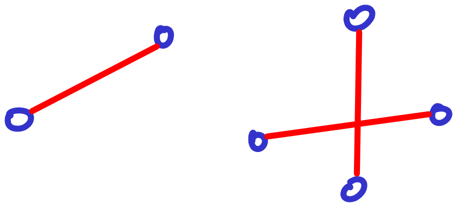
$d = 2$?



2-regular

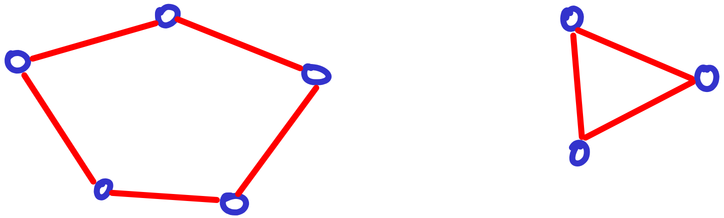
Regular graphs: all vertices have the same degree

$d = 1$?



1-regular

$d = 2$?

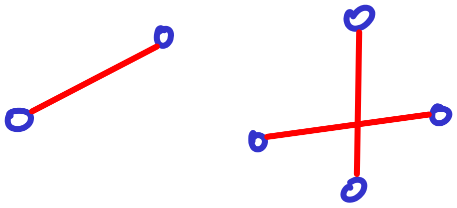


2-regular

$d = 3$?

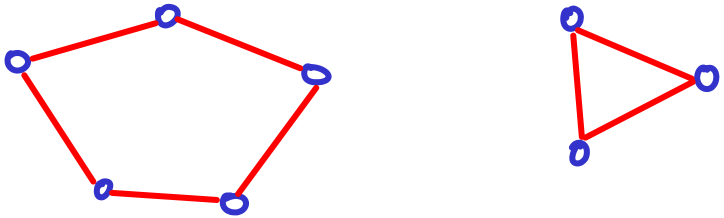
Regular graphs: all vertices have the same degree

$d=1$?



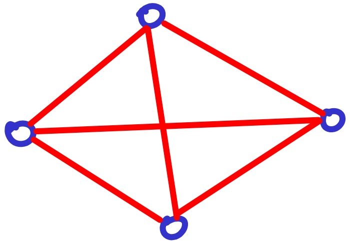
1-regular

$d=2$?



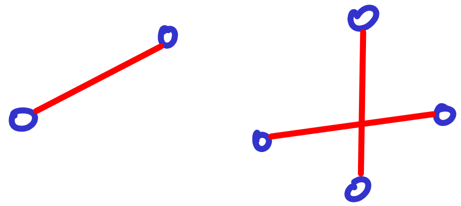
2-regular

$d=3$?



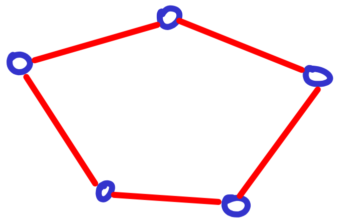
Regular graphs: all vertices have the same degree

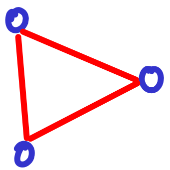
$d = 1$?



1-regular

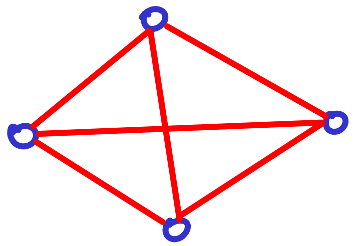
$d = 2$?

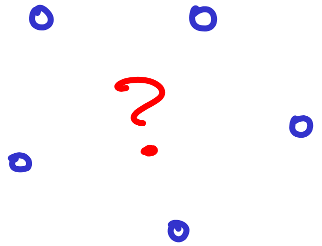




2-regular

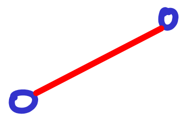
$d = 3$?

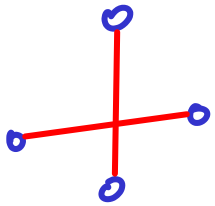




Regular graphs : all vertices have the same degree

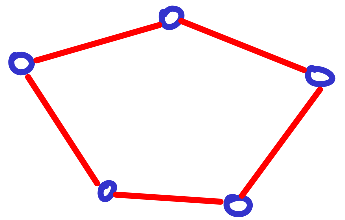
$d = 1$?

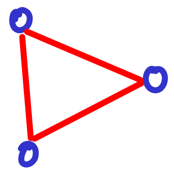




1-regular

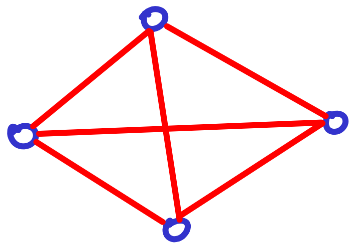
$d = 2$?

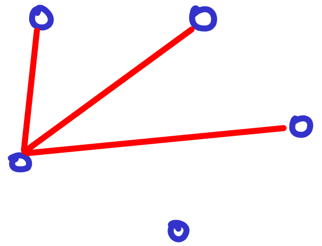




2-regular

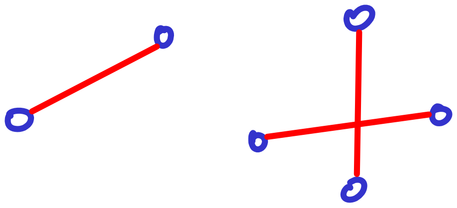
$d = 3$?





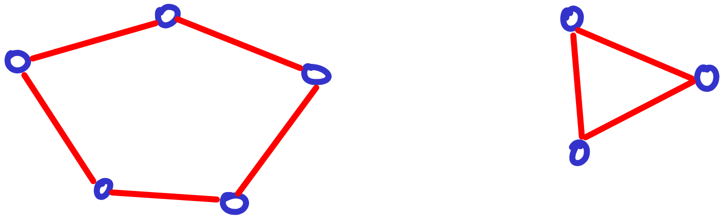
Regular graphs : all vertices have the same degree

d = 1 ?



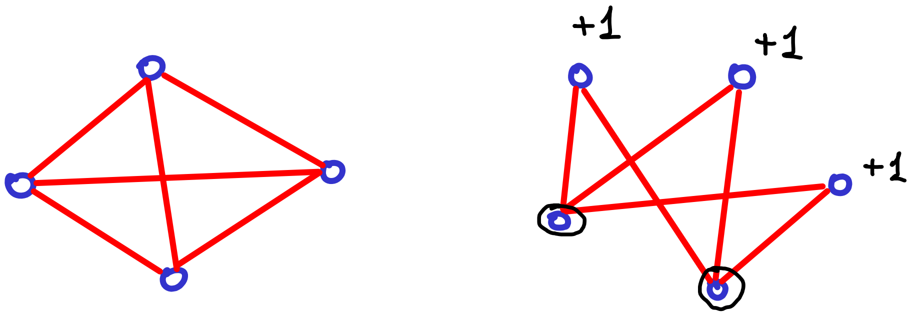
1- regular

d = 2 ?



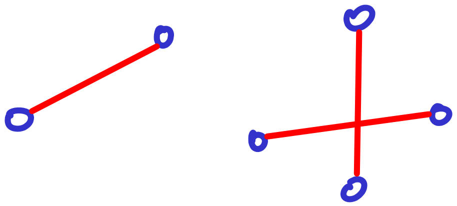
2- regular

d = 3 ?



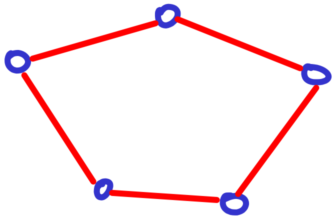
Regular graphs: all vertices have the same degree

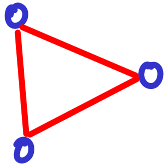
$d=1$?



1-regular

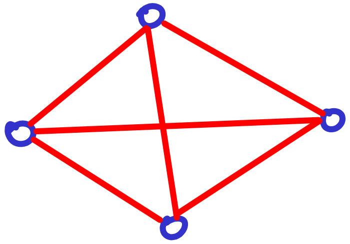
$d=2$?

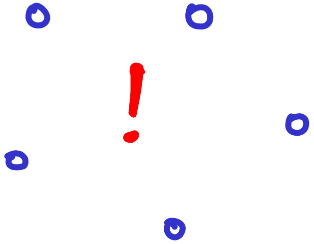




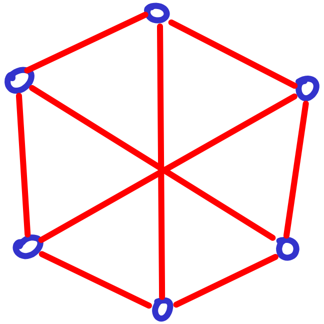
2-regular

$d=3$?

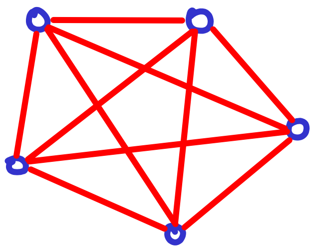




3-regular

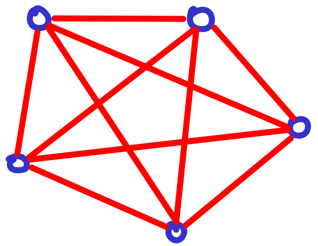


$n=5$



4-regular

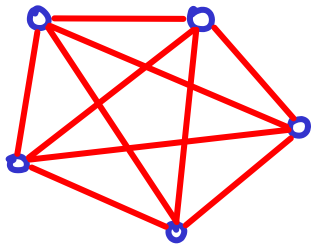
$n=5$



4-regular

($n-1$ regular) $\rightarrow$ complete graph

$n=5$

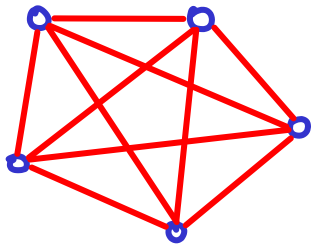


4-regular

($n-1$ regular) → complete graph

edges?

$n=5$



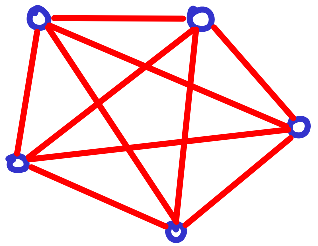
4-regular

($n-1$ regular) → complete graph

edges?

$n-1 + n-2 + n-3 + \cdots + 3 + 2 + 1$

$n=5$



4-regular

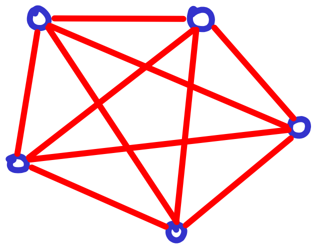
($n-1$ regular) $\rightarrow$ complete graph

\# edges?

$$n-1 + n-2 + n-3 + \cdots + 3 + 2 + 1 = \binom{n}{2}$$

$$= \sum_{i=1}^{n-1} i =$$

$n=5$



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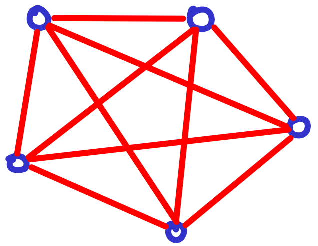
($n-1$ regular) $\rightarrow$ complete graph

edges?

$n-1 + n-2 + n-3 + \cdots + 3 + 2 + 1 = \binom{n}{2}$

$$= \sum_{i=1}^{n-1} i = \frac{n\cdot(n-1)}{2}$$

$n=5$



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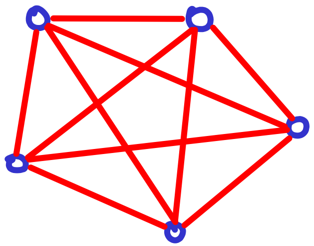
($n-1$ regular) → complete graph

\# edges? $n-1 + n-2 + n-3 + \cdots + 3 + 2 + 1 = \binom{n}{2}$

$$= \sum_{i=1}^{n-1} i = \frac{n\cdot(n-1)}{2}$$

\# possible graphs on n vertices?

$n=5$



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possible graphs on n vertices? $\quad 2^{\binom{n}{2}}$

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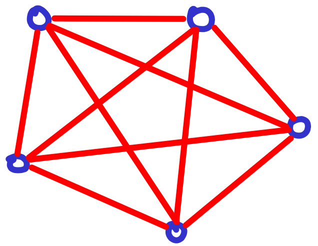

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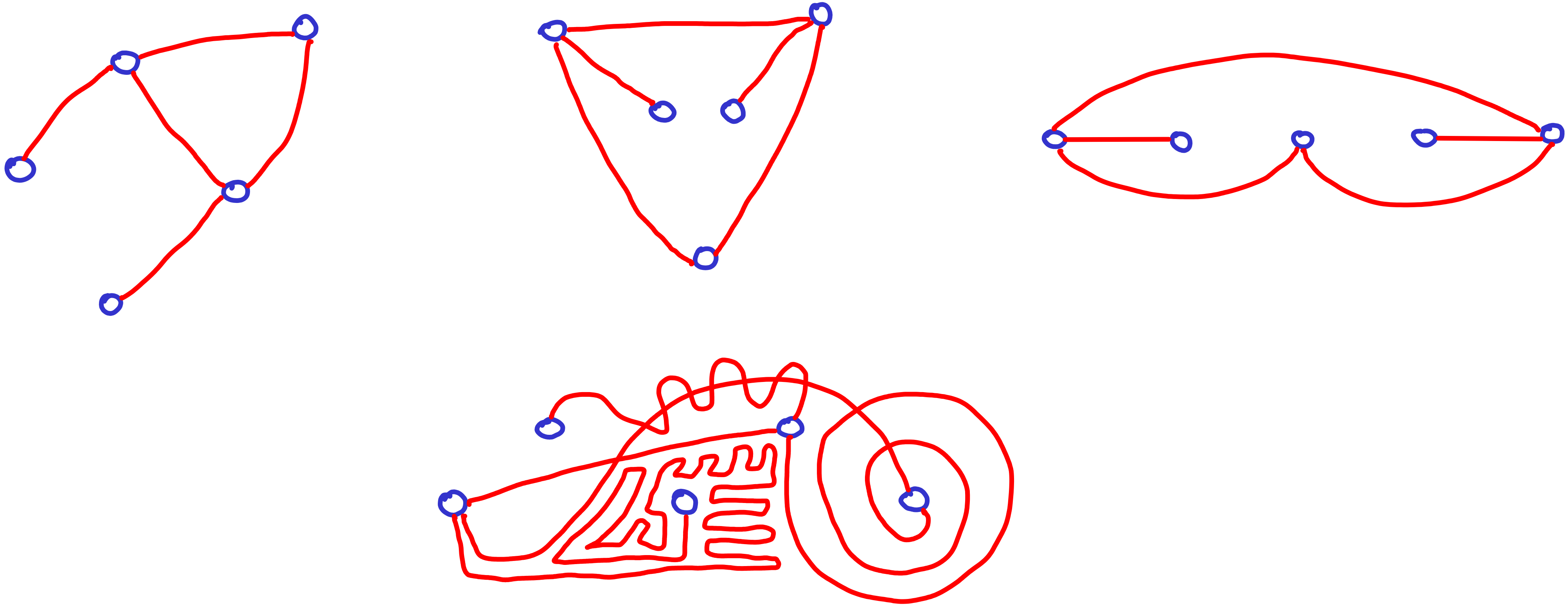
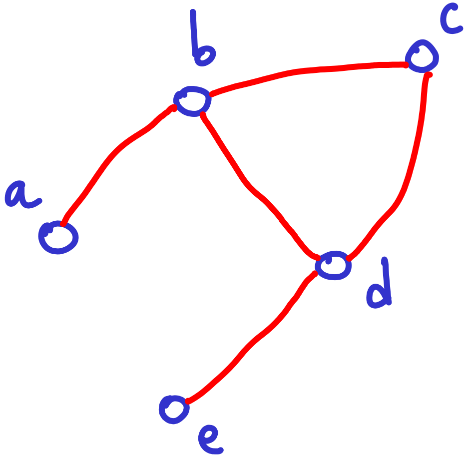
possible graphs on n vertices? $\quad 2^{\binom{n}{2}}$

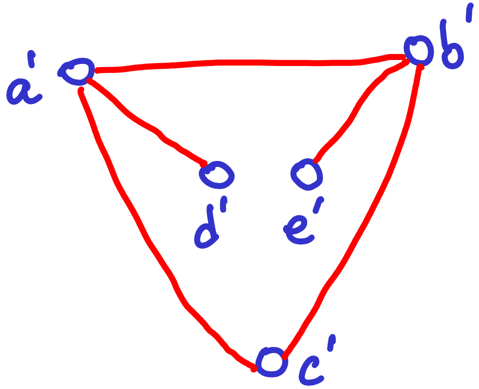
... but some of them are similar in shape

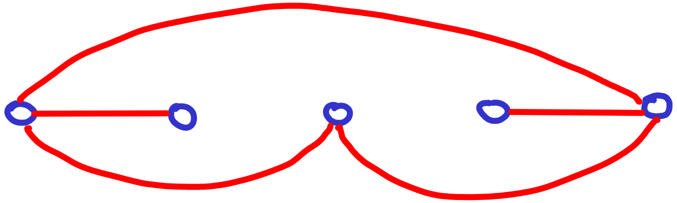
isomorphism

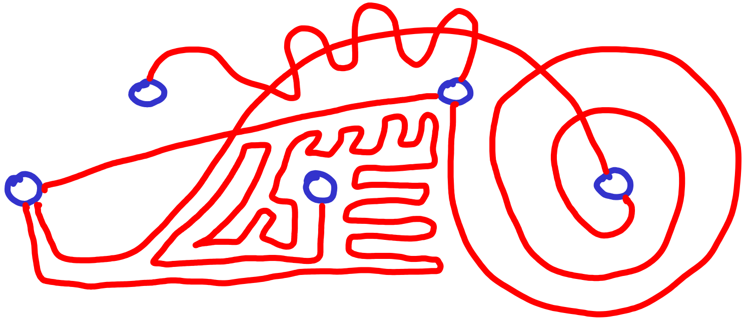


isomorphism









$a : d'$

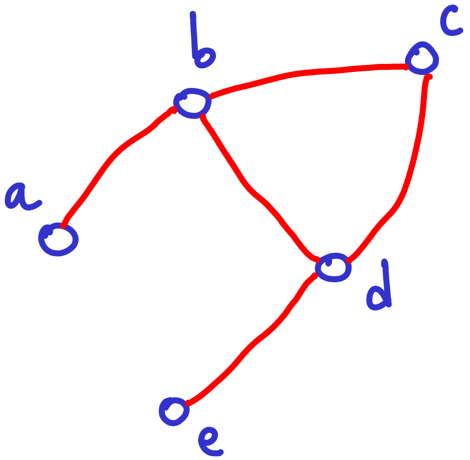
$b : a'$

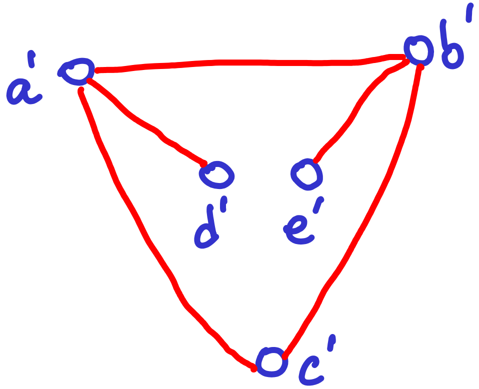
$c : c'$

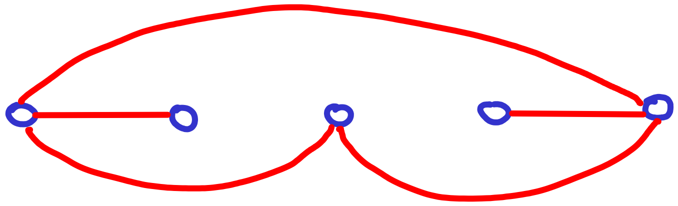
$d : b'$

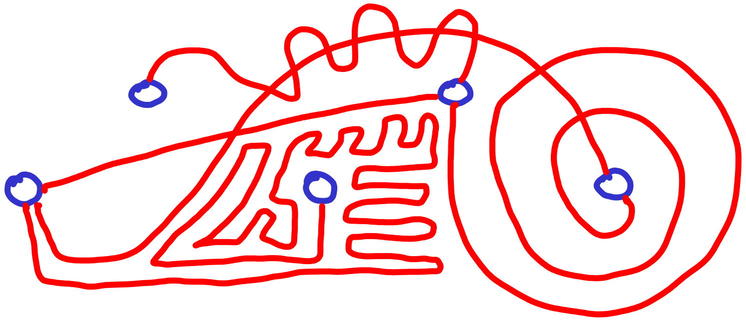
$e : e'$

isomorphism → map vertices of graph G to vertices of graph H









a : d'

b : a'

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d : b'

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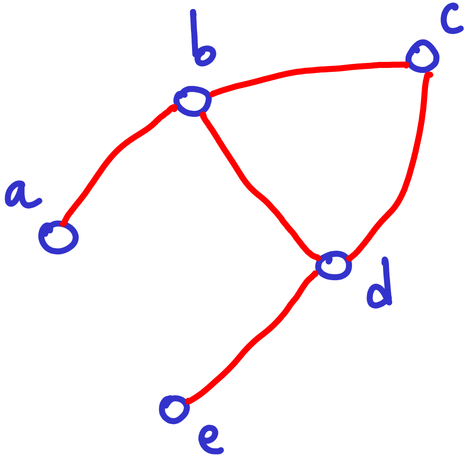
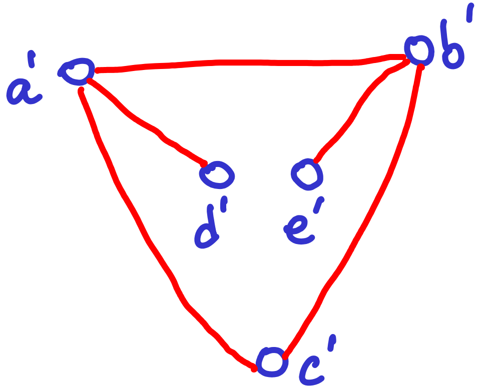
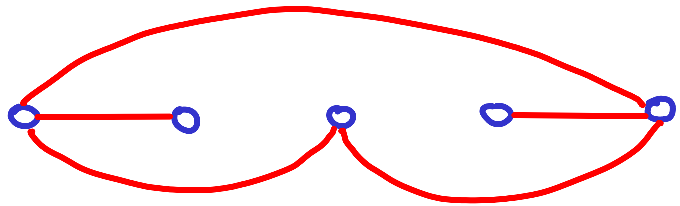
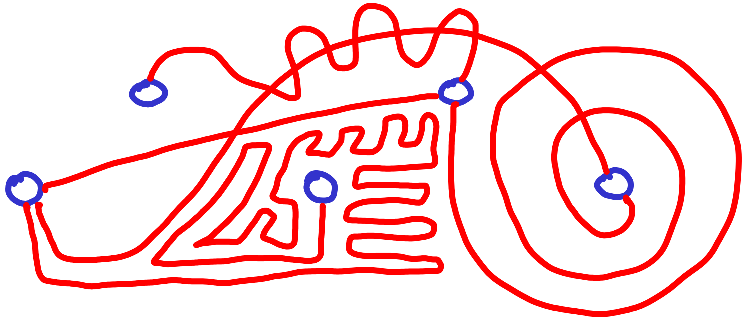
if vertices $\alpha, \beta \in G$ share an edge in G then vertices $m(\alpha), m(\beta) \in H$ must share an edge in H.









a : d'
b : a'
c : c'
d : b'
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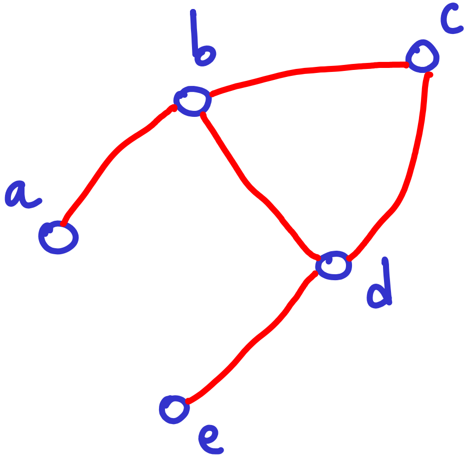

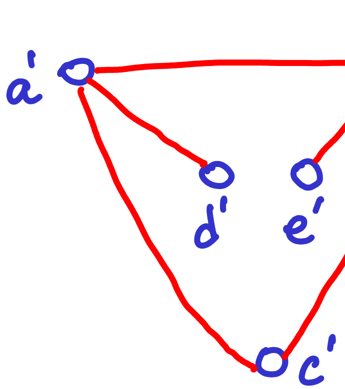

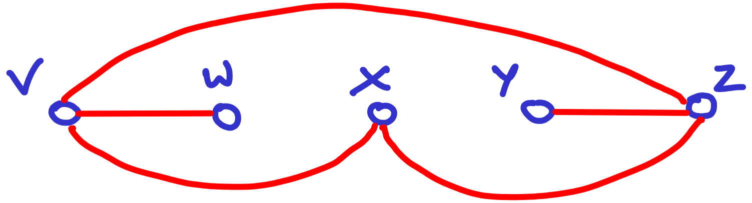

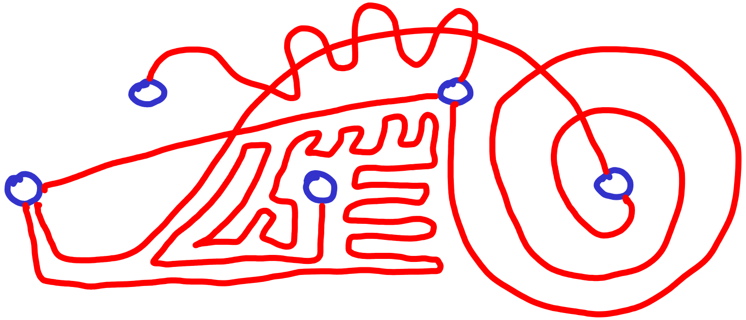

a : d' : ?

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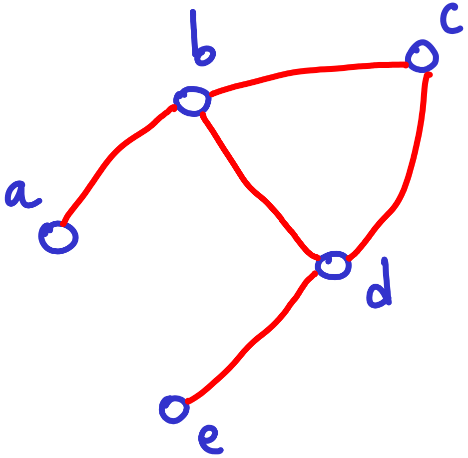

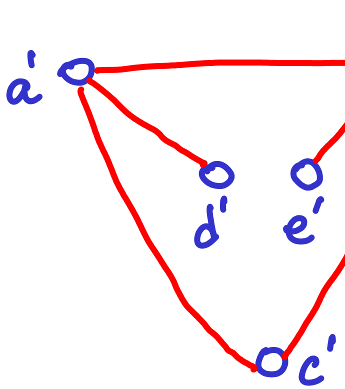

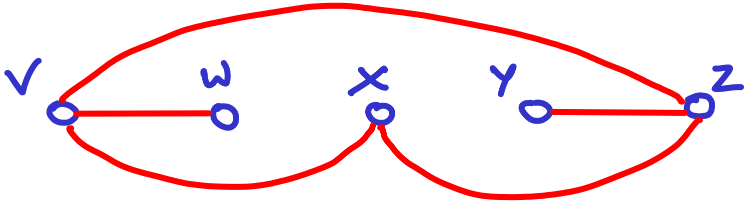

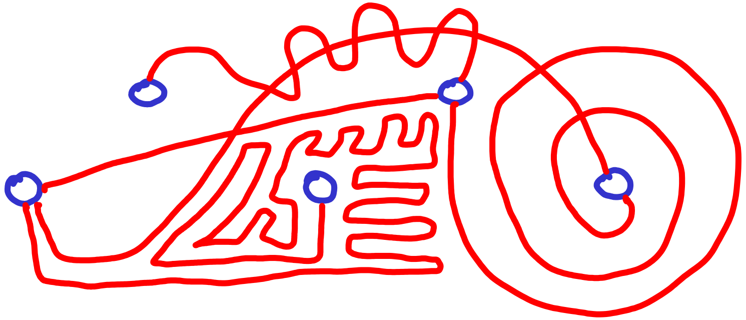

a	d'	w
b	a'	v
c	c'	x
d	b'	z
e	e'	y

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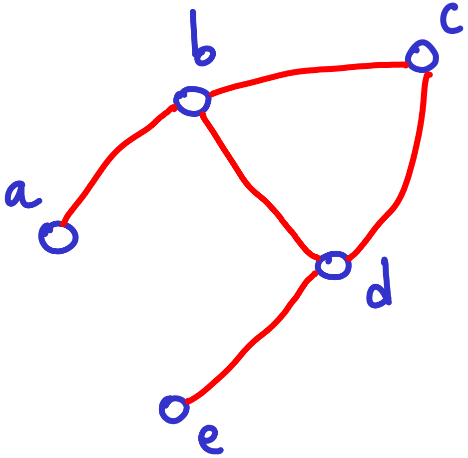

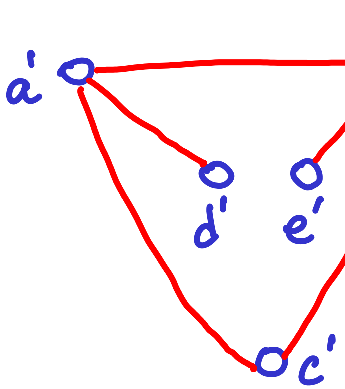

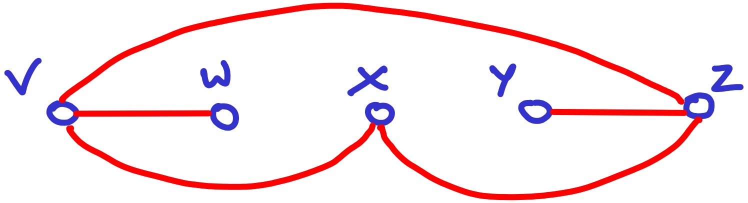

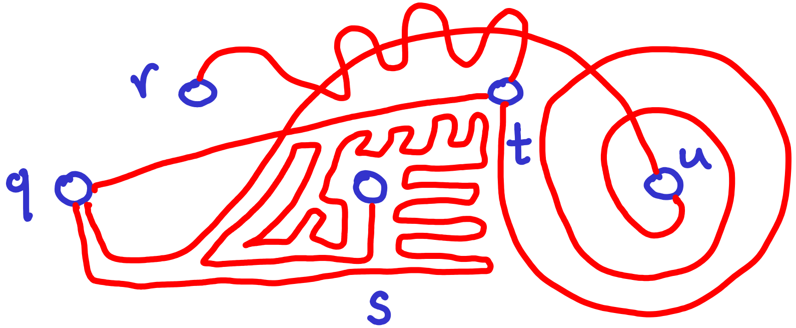

a	d'	w	?
b	a'	v	?
c	c'	x	?
d	b'	z	?
e	e'	y	?

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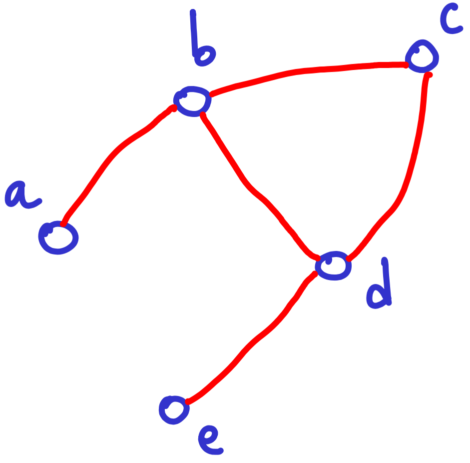

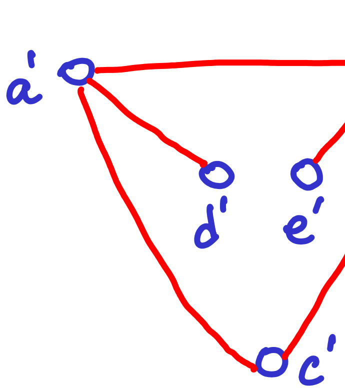

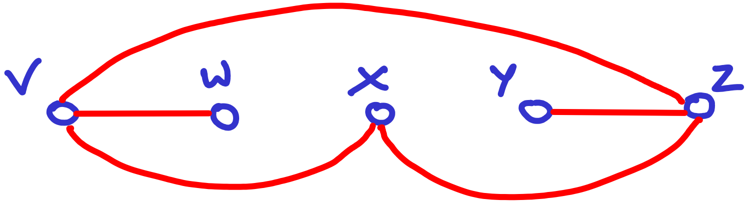

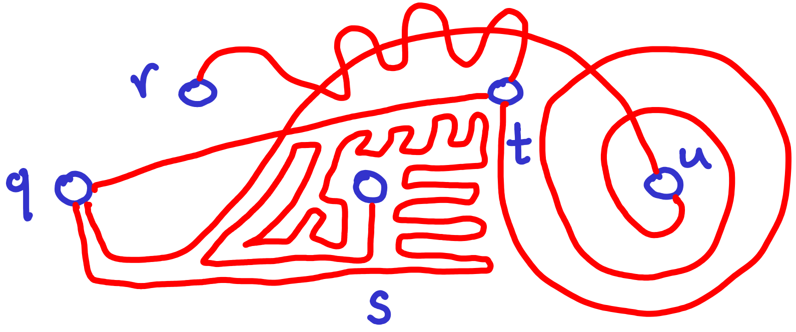

a	d'	w	r
b	a'	v	t
c	c'	x	u
d	b'	z	q
e	e'	y	s

Determining if two graphs are isomorphic (without a mapping)

is difficult (time complexity as function of n)

(not just because drawings look complicated)

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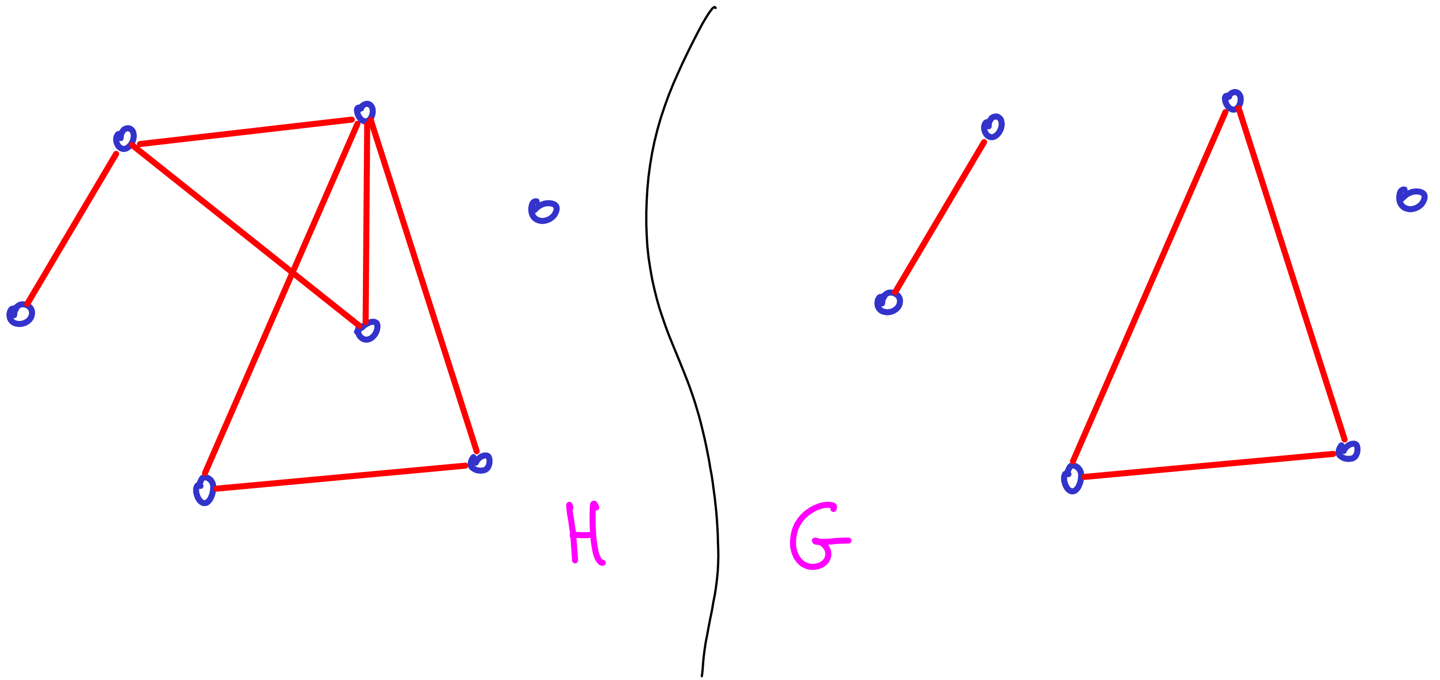
Counting # possible graphs without "doublecounting" isomorphs

is complicated

Subgraphs

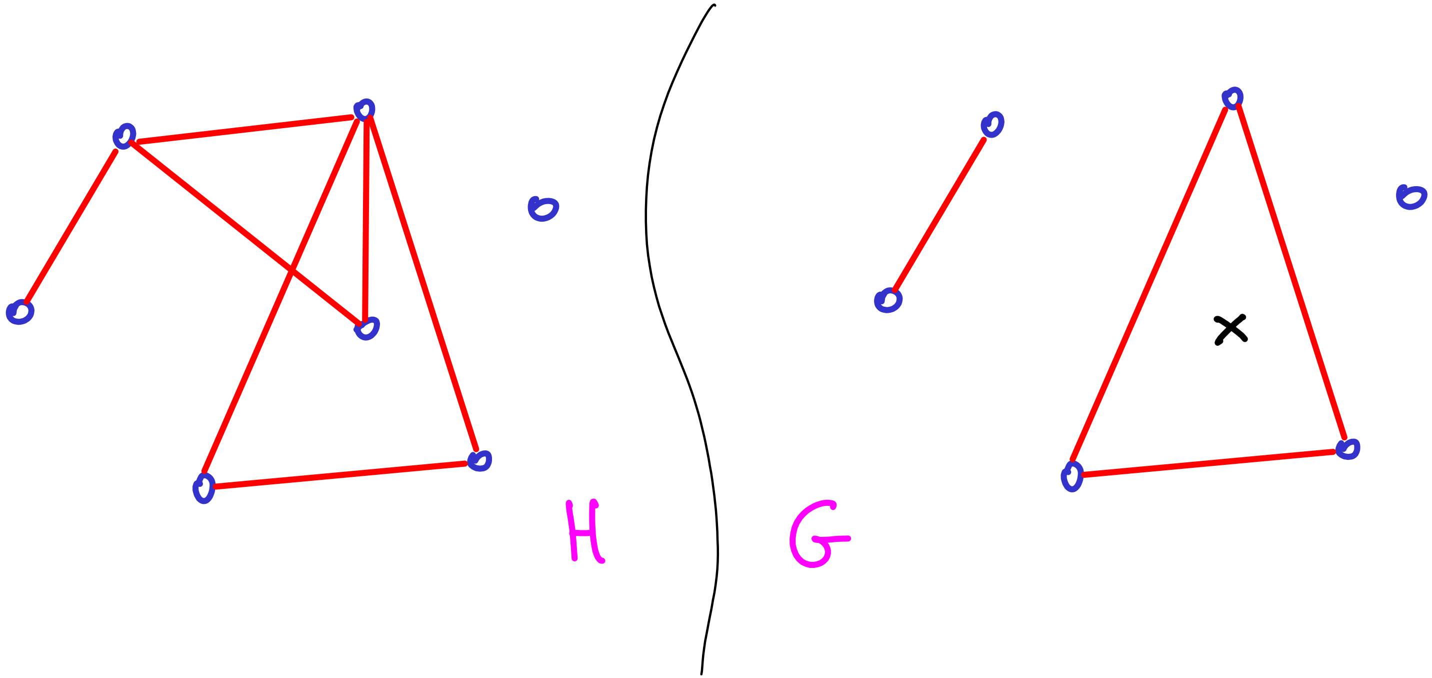
G is a subgraph of H if $\begin{cases} V(G) \subseteq V(H) \\ E(G) \subseteq E(H) \end{cases}$



Subgraphs

G is a subgraph of H if $\begin{cases} V(G) \subseteq V(H) \\ E(G) \subseteq E(H) \end{cases}$ $\Big\}$ if equal then it's a "spanning" subgraph

unlike



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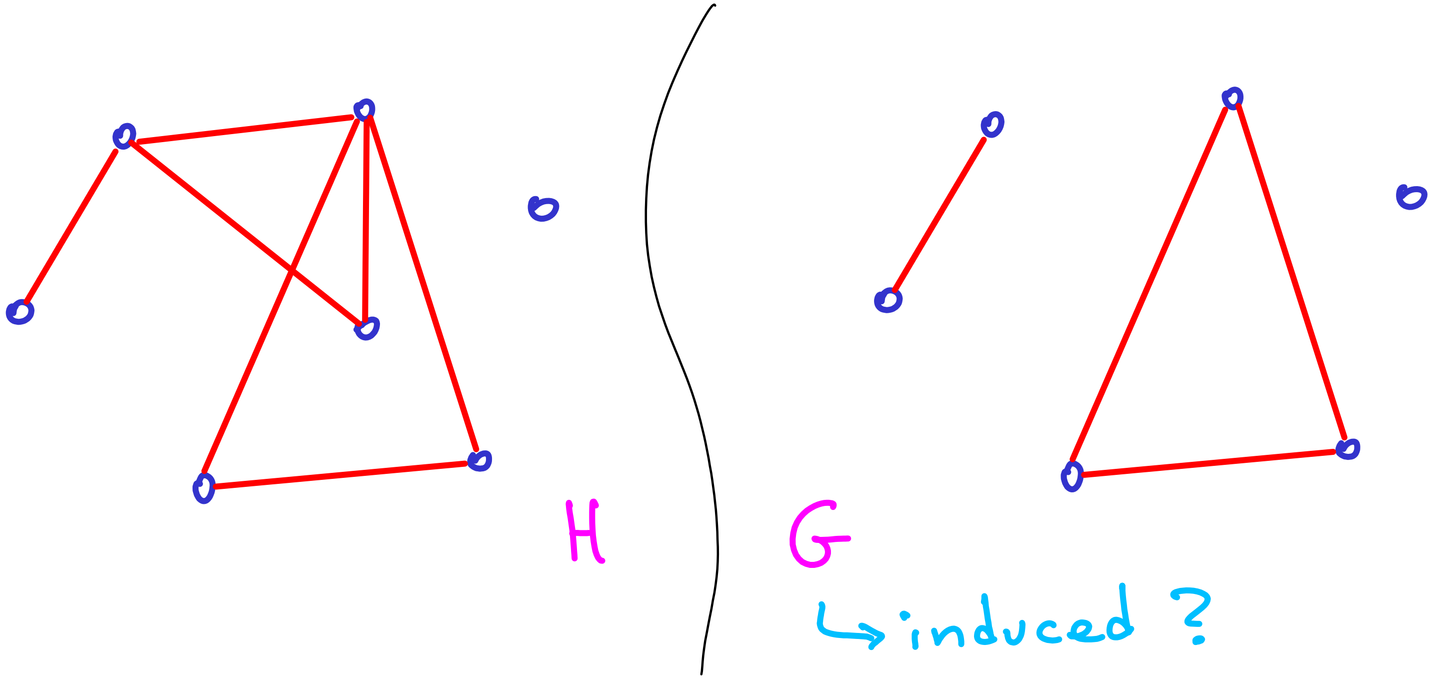

If you only remove edges as a result of removing vertices then G is an "induced" subgraph

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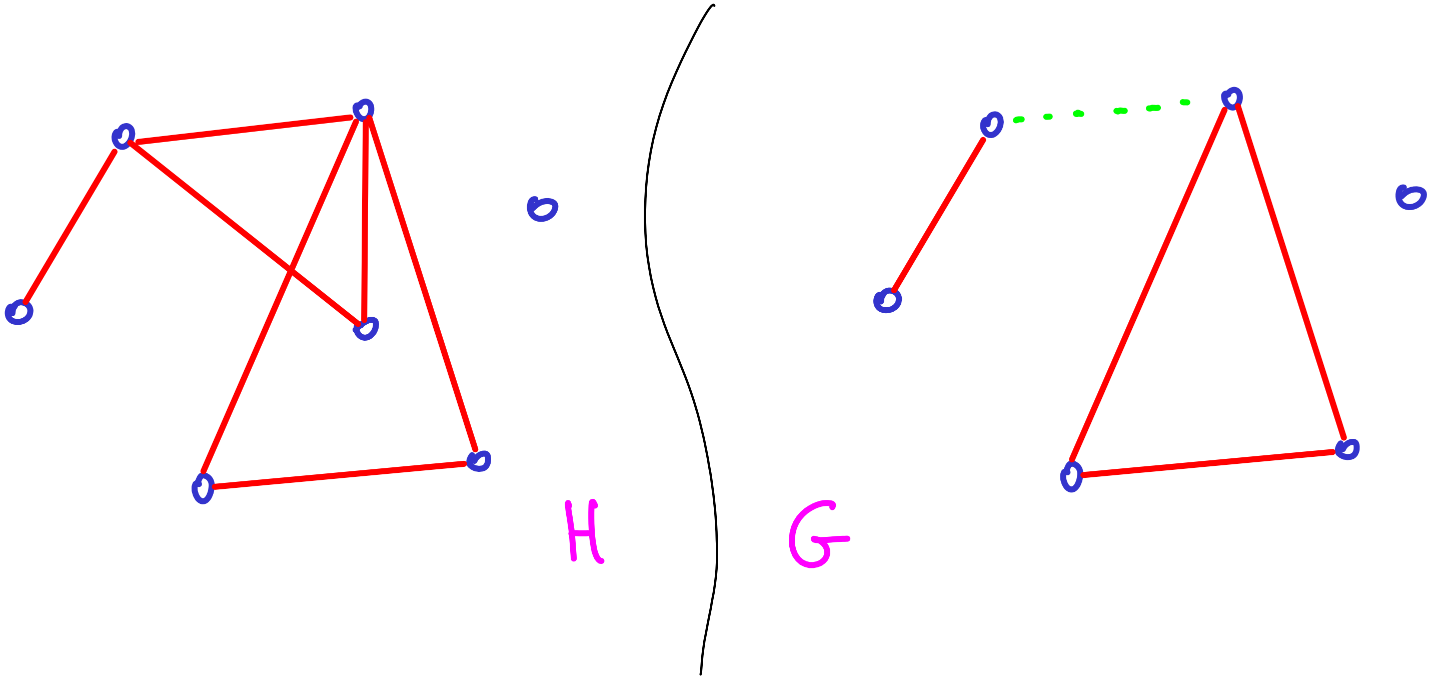

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