Suppose you want to prove that $A < C$

but it's complicated to compare directly.

Try to show that $A < B$, and $B < C$

We are specifically interested in comparing functions: $f(n)$ vs $g(n)$

$$f(n) \overset{?}{<} g(n)$$

$$f(n) \overset{?}{<} g(n)$$

Suppose $n \geq 1$

$g(n) = 12n^7$

$f(n) = n^7 + 3n^5 - 10n^4 + 8\log n$

$$f(n) \overset{?}{<} g(n)$$

Suppose $n \geq 1$

$$g(n) = 12n^7$$

$$f(n) = n^7 + 3n^5 \underline{-10n^4} + 8\log n$$

$$< n^7 + 3n^5 + 8\log n$$

$$f(n) \overset{?}{<} g(n)$$

Suppose $n \geq 1$

$g(n) = 12n^7$

$f(n) = n^7 + 3n^5 - 10n^4 + 8\log n$

$< n^7 + 3\underline{n^5} + 8\underline{\log n}$

$< n^7 + 3\underline{n^7} + 8\underline{n^7}$

$$f(n) \overset{?}{<} g(n)$$

Suppose $n \geqslant 1$

$$g(n) = 12n^7$$

$$f(n) = n^7 + 3n^5 - 10n^4 + 8\log n$$

$$< n^7 + 3n^5 + 8\log n$$

$$< n^7 + 3n^7 + 8n^7$$

$$= 12n^7 = g(n)$$

$f(n) \overset{?\checkmark}{<} g(n)$

$g(n)$ is an upper bound for $f(n)$

Suppose $n \geq 1$

$g(n) = 12n^7$

$$f(n) = n^7 + 3n^5 - 10n^4 + 8\log n$$

$$< n^7 + 3n^5 + 8\log n$$

$$< n^7 + 3n^7 + 8n^7$$

$$= 12n^7 = g(n)$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

Secondary goal: minimize $g(n)$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

$\left\{ \begin{array}{l} \text{Secondary goal:} \\ \text{minimize } g(n) \end{array} \right.$

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5}$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

{ Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{- n^4}_{<0} + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5}$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

{ Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{- n^4}_{<0} + \underbrace{8\log n^8}_{=64\log n} + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5}$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

{ Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{- n^4}_{<0} + \underbrace{\underbrace{8 \log n^8}_{=64 \log n}}_{<64n^7} + 3 \log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5}$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

{ Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{- n^4}_{<0} + \underbrace{\underbrace{8\log n^8}_{=64\log n}}_{<64n^7} + \underbrace{\underbrace{3\log^2 n}_{3n^2}}_{<3n^7} + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5}$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{-\,n^4}_{<0} + \underbrace{\underbrace{8\log n^8}_{=64\log n}}_{<64n^7} + \underbrace{\underbrace{3\log^2 n}_{3n^2}}_{<3n^7} + \underbrace{70^{\log_2 n}}_{=n^{\log_2 70}} + \frac{n^7}{n^{0.1}-0.5}$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

{ Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{- n^4}_{<0} + \underbrace{\underbrace{8\log n^8}_{=64\log n}}_{<64n^7} + \underbrace{\underbrace{3\log^2 n}_{3n^2}}_{<3n^7} + \underbrace{\underbrace{70^{\log_2 n}}_{=n^{\log_2 70}}}_{<n^{\log_2 128}} + \frac{n^7}{n^{0.1} - 0.5}$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{- n^4}_{<0} + \underbrace{\underbrace{8\log n^8}_{=64\log n}}_{<64n^7} + \underbrace{\underbrace{3\log^2 n}_{3n^2}}_{<3n^7} + \underbrace{\underbrace{\underbrace{70^{\log_2 n}}_{=n^{\log_2 70}}}_{<n^{\log_2 128}}}_{n^7} + \frac{n^7}{n^{0.1} - 0.5}$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{- n^4}_{<0} + \underbrace{\underbrace{8\log n^8}_{=64\log n}}_{<64n^7} + \underbrace{\underbrace{3\log^2 n}_{3n^2}}_{<3n^7} + \underbrace{\underbrace{\underbrace{70^{\log_2 n}}_{=n^{\log_2 70}}}_{<n^{\log_2 128}}}_{n^7} + \underbrace{\frac{n^7}{n^{0.1} - 0.5}}_{\leq \frac{n^7}{0.5}}$$

for all n

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{- n^4}_{<0} + \underbrace{\underbrace{8\log n^8}_{=64\log n}}_{<64n^7} + \underbrace{\underbrace{3\log^2 n}_{3n^2}}_{<3n^7} + \underbrace{\underbrace{\underbrace{70^{\log_2 n}}_{=n^{\log_2 70}}}_{<n^{\log_2 128}}}_{n^7} + \underbrace{\underbrace{\frac{n^7}{n^{0.1}-0.5}}_{\leq \frac{n^7}{0.5}}}_{2n^7}$$

for all n

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leqslant g(n)$

{ Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{- n^4}_{<0} + \underbrace{\underbrace{8\log n^8}_{=64\log n}}_{<64n^7} + \underbrace{\underbrace{3\log^2 n}_{3n^2}}_{<3n^7} + \underbrace{\underbrace{\underbrace{70^{\log_2 n}}_{=n^{\log_2 70}}}_{<n^{\log_2 128}}}_{n^7} + \underbrace{\underbrace{\frac{n^7}{n^{0.1}-0.5}}_{\leqslant \frac{n^7}{0.5} \text{ for all n}}}_{2n^7}$$

$$f(n) < n^7 + 3n^7 + 64n^7 + 3n^7 + n^7 + 2n^7$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leqslant g(n)$

$\left\{ \begin{array}{l} \text{Secondary goal:} \\ \quad \text{minimize } g(n) \end{array} \right.$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{-\,n^4}_{<0} + \underbrace{\underbrace{8\log n^8}_{=64\log n}}_{<64n^7} + \underbrace{\underbrace{3\log^2 n}_{3n^2}}_{<3n^7} + \underbrace{\underbrace{\underbrace{70^{\log_2 n}}_{=n^{\log_2 70}}}_{<n^{\log_2 128}}}_{n^7} + \underbrace{\underbrace{\frac{n^7}{n^{0.1}-0.5}}_{\leqslant \frac{n^7}{0.5}}}_{2n^7} \quad \text{for all } n$$

$$f(n) < n^7 + 3n^7 + 64n^7 + 3n^7 + n^7 + 2n^7 = 74n^7$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{- n^4}_{<0} + \underbrace{8 \log n^8}_{\underbrace{=64 \log n}_{<64n^7}} + \underbrace{3 \log^2 n}_{\underbrace{3n^2}_{<3n^7}} + \underbrace{70^{\log_2 n}}_{\underbrace{=n^{\log_2 70}}_{\underbrace{\leq n^{\log_2 128}}_{n^7}}} + \underbrace{\frac{n^7}{n^{0.1} - 0.5}}_{\cancel{\leq \frac{n^7}{0.5}}}$$

(alternative)

$$\underbrace{\frac{n^7}{n^{0.1} - 0.5}}_{< n^7}$$

if n > 57

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{-\,n^4}_{<0} + \underbrace{\underbrace{8\log n^8}_{=64\log n}}_{<64n^7} + \underbrace{\underbrace{3\log^2 n}_{3n^2}}_{<3n^7} + \underbrace{\underbrace{\underbrace{70^{\log_2 n}}_{=n^{\log_2 70}}}_{<n^{\log_2 128}}}_{n^7} + \underbrace{\underbrace{\frac{n^7}{n^{0.1}-0.5}}_{\cancel{\leq \frac{n^7}{0.5}}\ \text{(alternative)}}}_{<n^7 \text{ if } n>57}$$

$$f(n) < n^7 + 3n^7 + 64n^7 + 3n^7 + n^7 + n^7$$

EXAGGERATE & SIMPLIFY

Given: complicated $f(n)$

Wanted: simple $g(n)$, such that $f(n) \leq g(n)$

{ Secondary goal: minimize $g(n)$

$$f(n) = n^7 + \underbrace{3n^5}_{<3n^7} \underbrace{- n^4}_{<0} + \underbrace{\underbrace{8 \log n^8}_{=64 \log n}}_{<64n^7} + \underbrace{\underbrace{3 \log^2 n}_{3n^2}}_{<3n^7} + \underbrace{\underbrace{\underbrace{70^{\log_2 n}}_{=n^{\log_2 70}}}_{<n^{\log_2 128}}}_{n^7} + \underbrace{\frac{n^7}{n^{0.1} - 0.5}}_{\cancel{\leq \underbrace{\frac{n^7}{0.5}}_{<n^7}}}$$

(alternative)

if n > 57

$$f(n) < n^7 + 3n^7 + 64n^7 + 3n^7 + n^7 + n^7 = 73n^7$$

EXAGGERATE & SIMPLIFY

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5} \leq 73n^7$$

EXAGGERATE & SIMPLIFY

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5} \leq 73n^7$$

What if we want $\underbrace{f(n) \geq h(n)}_{\text{lower bound}}$ [h: simple] & try to maximize $h(n)$

EXAGGERATE & SIMPLIFY

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5} \leq 73n^7$$

UNDERESTIMATE & SIMPLIFY

What if we want $f(n) \geq h(n)$ [h: simple] & try to maximize $h(n)$

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5}$$

EXAGGERATE & SIMPLIFY

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5} \leq 73n^7$$

UNDERESTIMATE & SIMPLIFY

What if we want $f(n) \geq h(n)$ [h: simple] & try to maximize $h(n)$

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5}$$

$$> n^7 - n^4$$

EXAGGERATE & SIMPLIFY

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5} \leq 73n^7$$

UNDERESTIMATE & SIMPLIFY

What if we want $f(n) \geq h(n)$ [h: simple] & try to maximize $h(n)$

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5}$$

$$> n^7 - n^4 = \frac{1}{2}n^7 + \frac{1}{2}n^7 - n^4$$

EXAGGERATE & SIMPLIFY

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5} \leq 73n^7$$

UNDERESTIMATE & SIMPLIFY

What if we want $f(n) \geq h(n)$ [h: simple] & try to maximize $h(n)$

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5}$$

$$> n^7 - n^4 = \frac{1}{2}n^7 + \frac{1}{2}n^7 - n^4 = \frac{1}{2}n^7 + \left(\frac{1}{2}n^7 - n^4\right)$$

EXAGGERATE & SIMPLIFY

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5} \leq 73n^7$$

UNDERESTIMATE & SIMPLIFY

What if we want $f(n) \geq h(n)$ [h: simple] & try to maximize $h(n)$

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5}$$

$$> n^7 - n^4 = \frac{1}{2}n^7 + \frac{1}{2}n^7 - n^4 = \frac{1}{2}n^7 + \left(\frac{1}{2}n^7 - n^4\right) > \frac{1}{2}n^7 \quad \text{if } n \geq 2$$

EXAGGERATE & SIMPLIFY

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5} \leq 73n^7$$

if n > 57

UNDERESTIMATE & SIMPLIFY

What if we want $f(n) \geq h(n)$ [h: simple] & try to maximize $h(n)$

$$f(n) = n^7 + 3n^5 - n^4 + 8\log n^8 + 3\log^2 n + 70^{\log_2 n} + \frac{n^7}{n^{0.1} - 0.5}$$

$$> n^7 - n^4 = \frac{1}{2}n^7 + \frac{1}{2}n^7 - n^4 = \frac{1}{2}n^7 + \left(\frac{1}{2}n^7 - n^4\right) > \frac{1}{2}n^7 \quad \text{if } n \geq 2$$

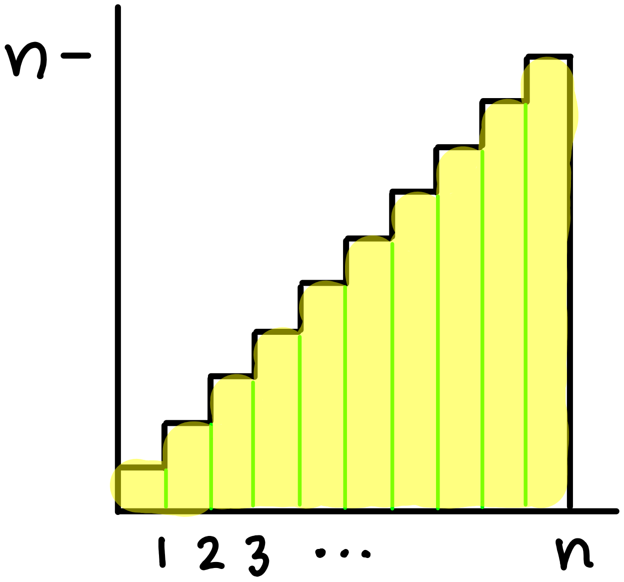
For both simplifications, sometimes you'll need a condition like n>...

EXAGGERATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x = 1 + 2 + 3 + \cdots + (n-2) + (n-1) + n$$

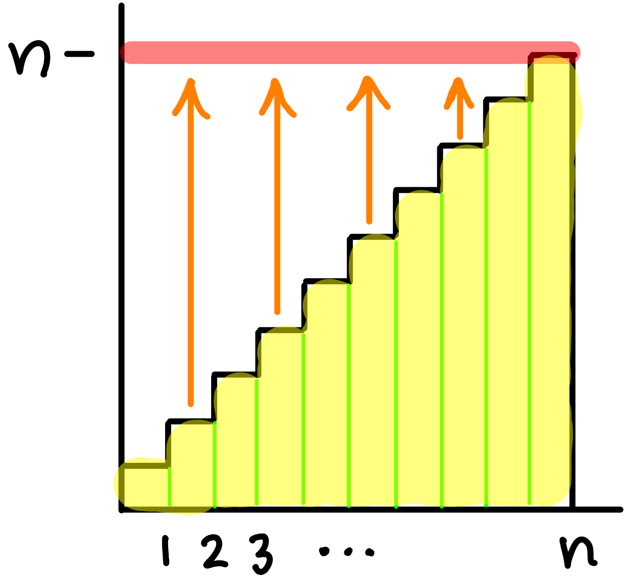
EXAGGERATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x = 1 + 2 + 3 + \cdots + (n-2) + (n-1) + n$$



EXAGGERATE & SIMPLIFY

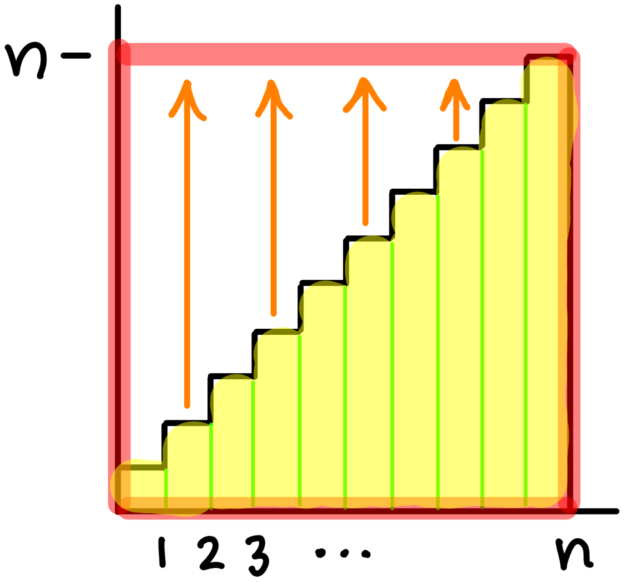
$$f(n) = \sum_{x=1}^{n} x = \underset{\overset{\downarrow}{n}}{1} + \underset{\overset{\downarrow}{n}}{2} + \underset{\overset{\downarrow}{n}}{3} + \cdots + \underset{\overset{\downarrow}{n}}{(n-2)} + \underset{\overset{\downarrow}{n}}{(n-1)} + n$$



EXAGGERATE & SIMPLIFY

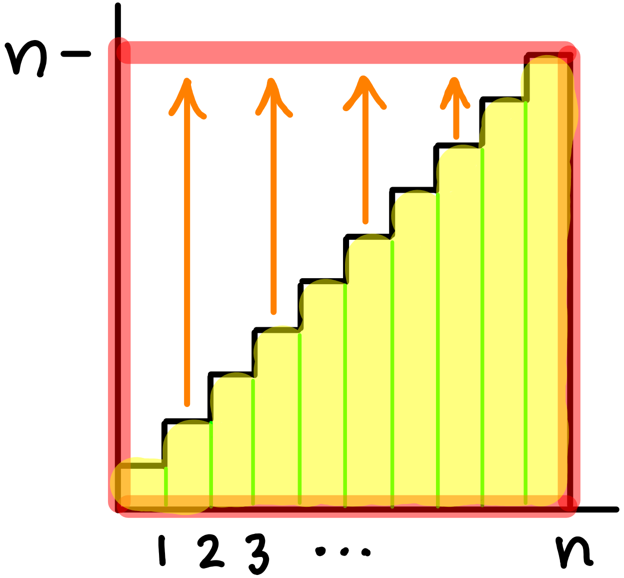
$$f(n) = \sum_{x=1}^{n} x = \underset{\substack{\downarrow \\ n}}{1} + \underset{\substack{\downarrow \\ n}}{2} + \underset{\substack{\downarrow \\ n}}{3} + \cdots + \underset{\substack{\downarrow \\ n}}{(n-2)} + \underset{\substack{\downarrow \\ n}}{(n-1)} + n$$

$$\leq \sum_{x=1}^{n} n = n^2$$



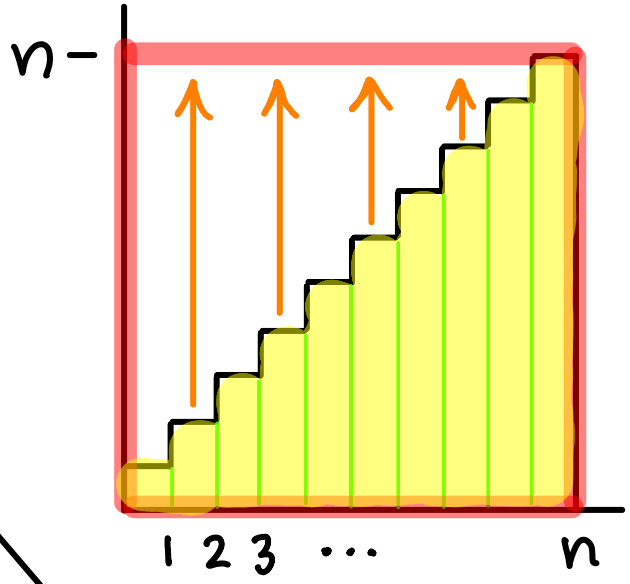
EXAGGERATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x \leqslant \sum_{x=1}^{n} n = n^2$$



EXAGGERATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x \leq \sum_{x=1}^{n} n = n^2$$



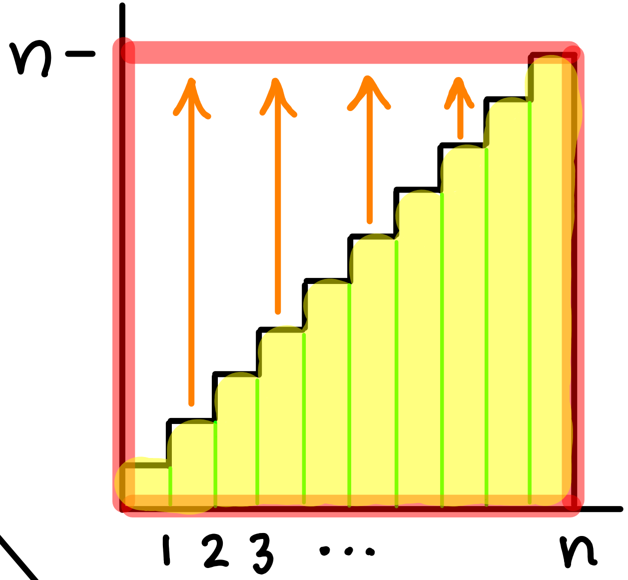
UNDERESTIMATE & SIMPLIFY

What if we want $f(n) \geq h(n)$ [h: simple]

& try to maximize $h(n)$

EXAGGERATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x \leq \sum_{x=1}^{n} n = n^2$$



UNDERESTIMATE & SIMPLIFY

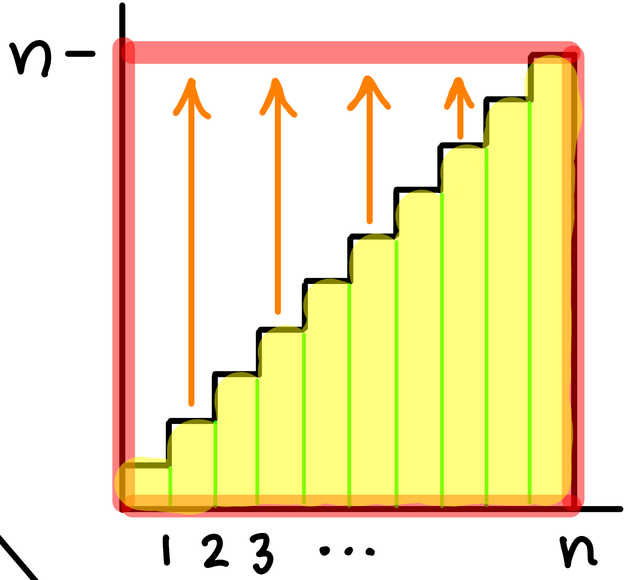
What if we want $f(n) \geq h(n)$ [h: simple]

& try to maximize $h(n)$

$$f(n) = \sum_{x=1}^{n} x \geq \sum_{x=1}^{n} 1$$

EXAGGERATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x \leq \sum_{x=1}^{n} n = n^2$$



UNDERESTIMATE & SIMPLIFY

What if we want $f(n) \geq h(n)$ [h: simple]

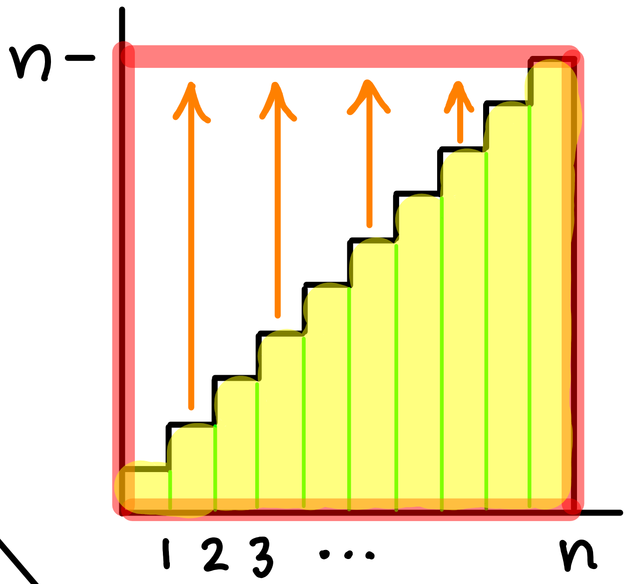
& try to maximize $h(n)$

$$f(n) = \sum_{x=1}^{n} x \geq \sum_{x=1}^{n} 1 = n$$

OK... Can we do better?

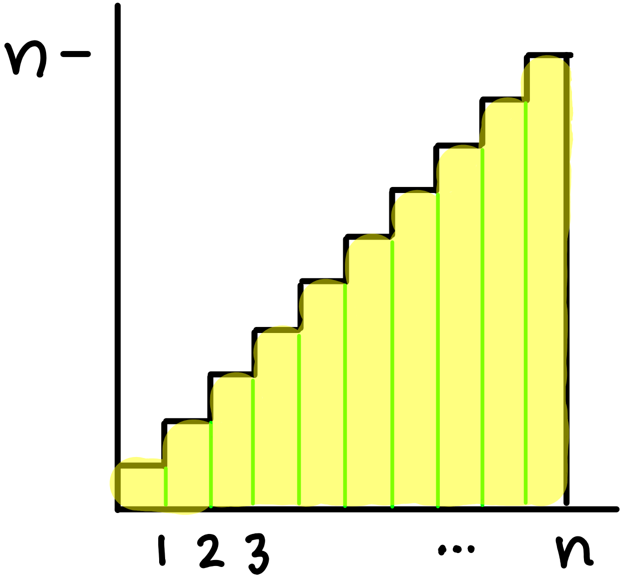
EXAGGERATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x \leq \sum_{x=1}^{n} n = n^2$$



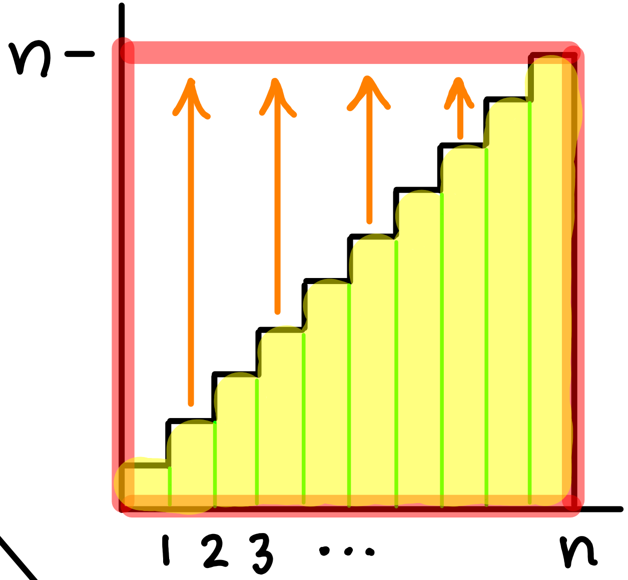
UNDERESTIMATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x = 1 + 2 + 3 + \cdots + (n-2) + (n-1) + n$$



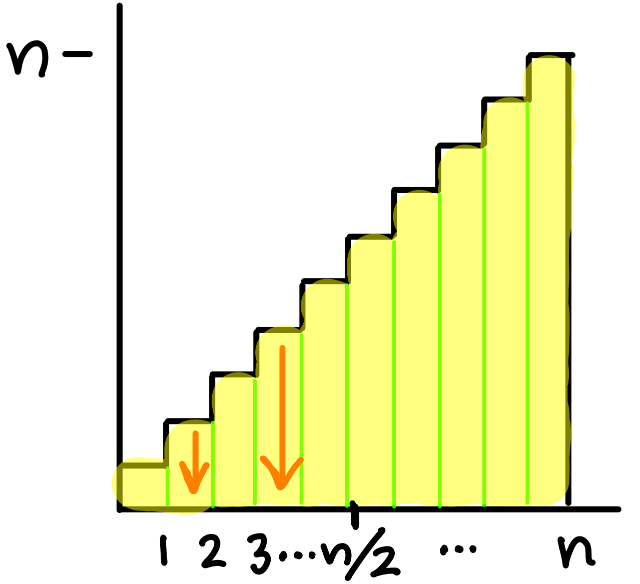
EXAGGERATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x \leqslant \sum_{x=1}^{n} n = n^2$$



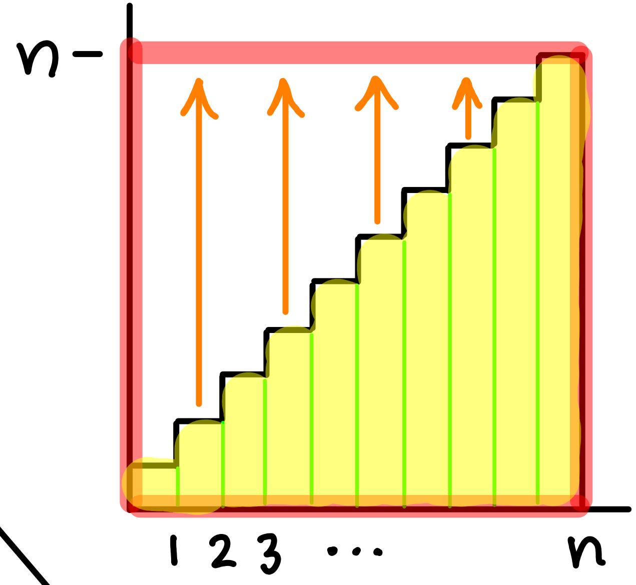
UNDERESTIMATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x = \underset{\substack{\downarrow \\ 0}}{1} + \underset{\substack{\downarrow \\ 0}}{2} + \underset{\substack{\downarrow \\ 0}}{3} + \cdots + (n-2) + (n-1) + n$$



EXAGGERATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x \leqslant \sum_{x=1}^{n} n = n^2$$

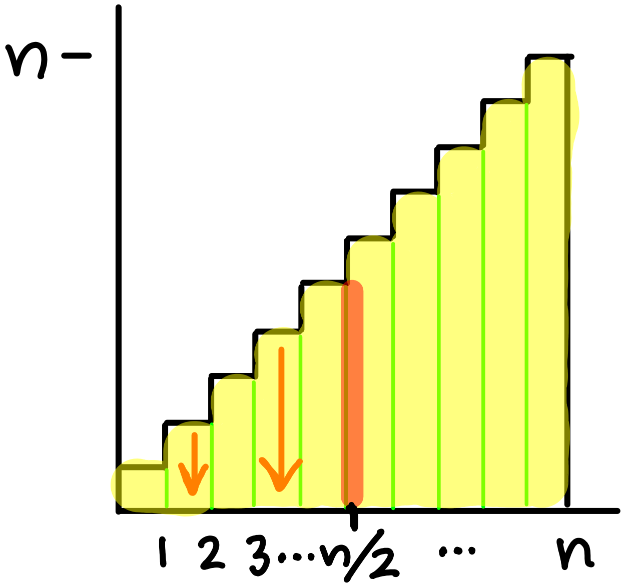


UNDERESTIMATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x = 1 + 2 + 3 + \cdots + (n-2) + (n-1) + n$$

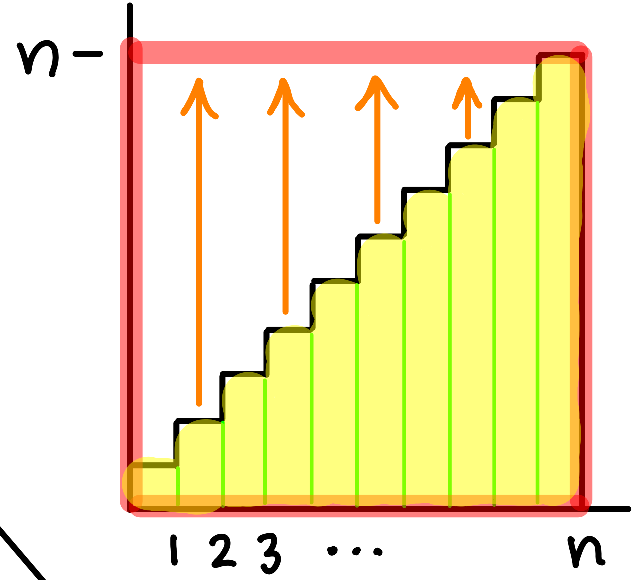
$$0 \quad 0 \quad 0$$

$$\geqslant \sum_{x=\lceil \frac{n}{2} \rceil}^{n} x$$



EXAGGERATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x \leqslant \sum_{x=1}^{n} n = n^2$$

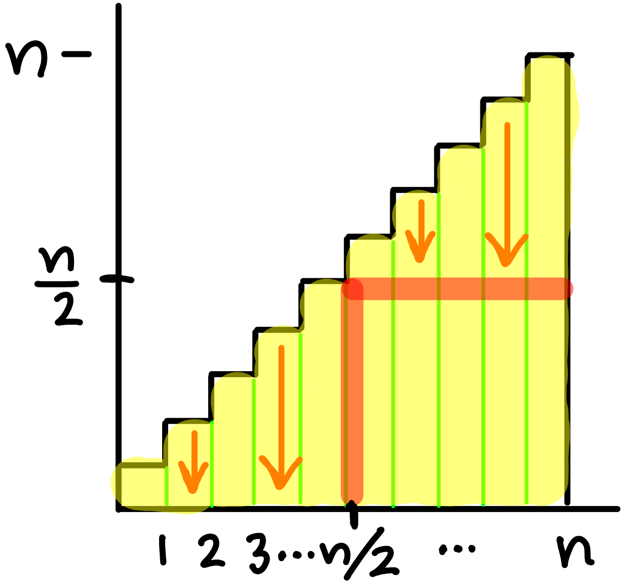


UNDERESTIMATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x = 1 + 2 + 3 + \cdots + (n-2) + (n-1) + n$$

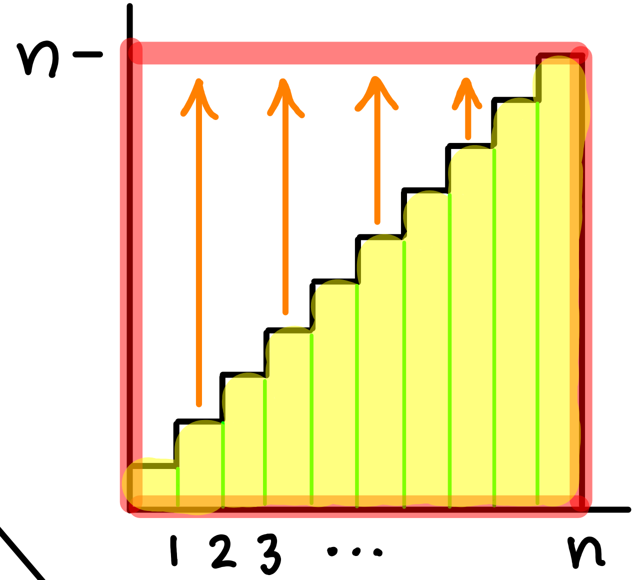
$$\begin{matrix} 1 & 2 & 3 & (n-2) & (n-1) & n \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0 & 0 & 0 & n/2 & n/2 & n/2 \end{matrix}$$

$$\geqslant \sum_{x=\lceil \frac{n}{2} \rceil}^{n} x$$



EXAGGERATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x \leq \sum_{x=1}^{n} n = n^2$$

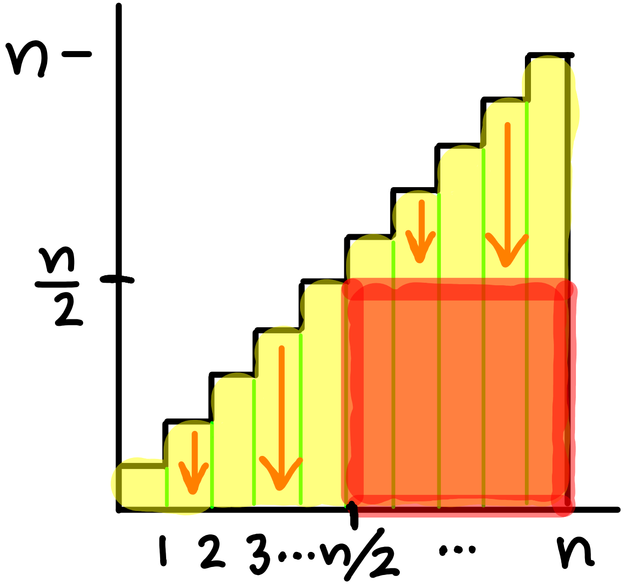


UNDERESTIMATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x = 1 + 2 + 3 + \cdots + (n-2) + (n-1) + n$$

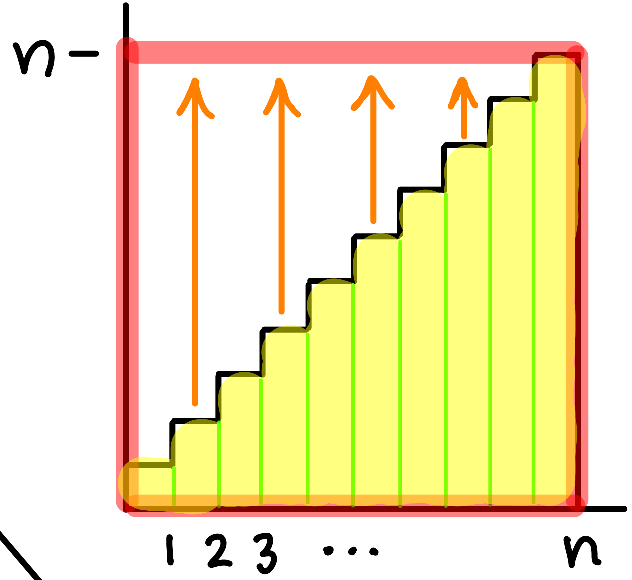
$0 \quad 0 \quad 0 \qquad n/2 \quad n/2 \quad n/2$

$$\geq \sum_{x=\lceil \frac{n}{2} \rceil}^{n} x \geq \sum_{x=\lceil \frac{n}{2} \rceil}^{n} \frac{n}{2}$$



EXAGGERATE & SIMPLIFY

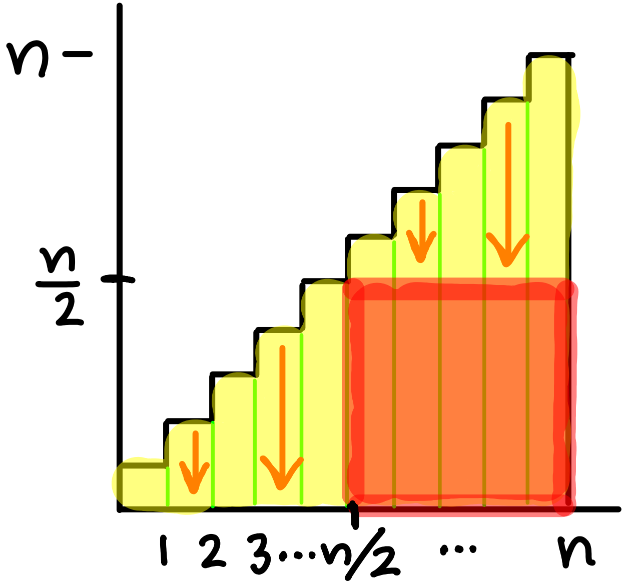
$$f(n) = \sum_{x=1}^{n} x \leq \sum_{x=1}^{n} n = n^2$$



UNDERESTIMATE & SIMPLIFY

$$f(n) = \sum_{x=1}^{n} x = \underset{\underset{0}{\downarrow}}{1} + \underset{\underset{0}{\downarrow}}{2} + \underset{\underset{0}{\downarrow}}{3} + \cdots + \underset{\underset{n/2}{\downarrow}}{(n-2)} + \underset{\underset{n/2}{\downarrow}}{(n-1)} + \underset{\underset{n/2}{\downarrow}}{n}$$

$$\geq \sum_{x=\lceil \frac{n}{2} \rceil}^{n} x \geq \sum_{x=\lceil \frac{n}{2} \rceil}^{n} \frac{n}{2} \geq \frac{n^2}{4}$$



$$\frac{n^2}{4} \leqslant \sum_{x=1}^{n} x \leqslant n^2$$

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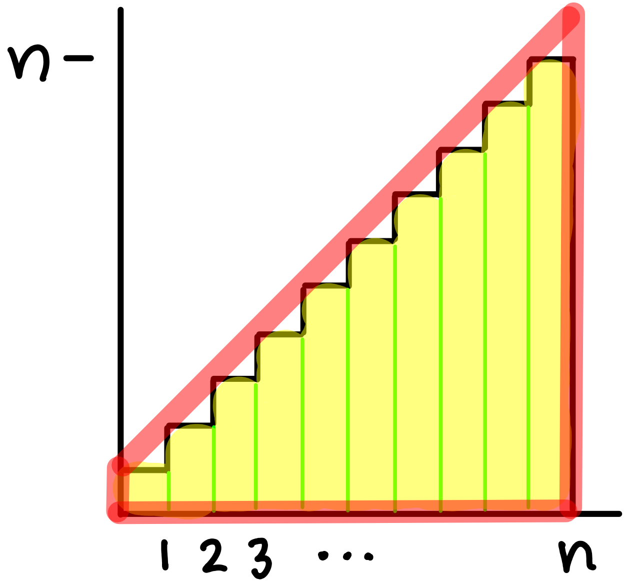
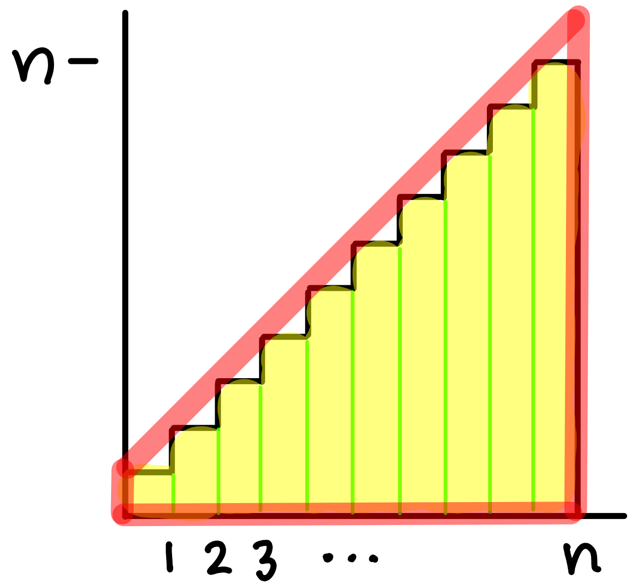
- We could have exaggerated less. →



$$\boxed{\frac{n^2}{4} \leq \sum_{x=1}^{n} x \leq n^2}$$

- We could have exaggerated less. $\rightarrow$



- Also, we can show $\sum_{x=1}^{n} x = \frac{n \cdot (n+1)}{2}$ e.g., by induction

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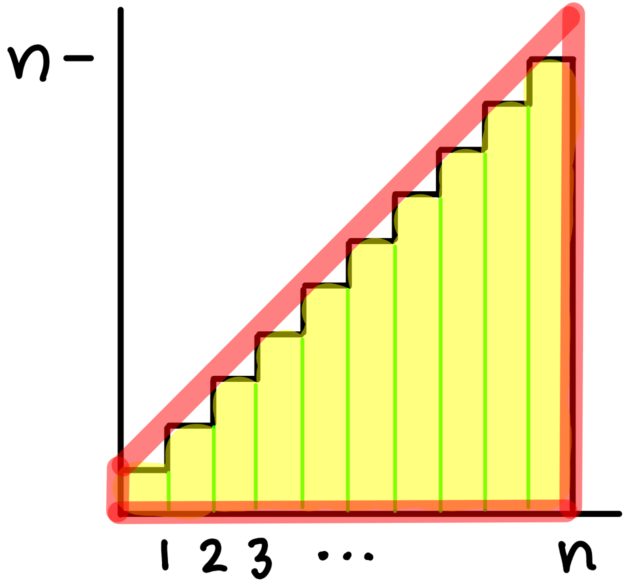

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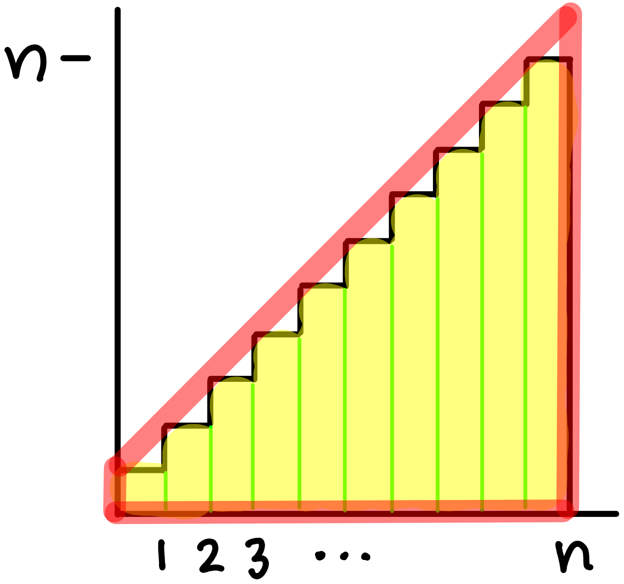
So what was the point?

$$\boxed{\frac{n^2}{4} \leqslant \sum_{x=1}^{n} x \leqslant n^2}$$

- We could have exaggerated less. →



- Also, we can show $\displaystyle\sum_{x=1}^{n} x = \frac{n\cdot(n+1)}{2}$ e.g., by induction

So what was the point? → Try $\displaystyle\sum_{x=1}^{n} x^9$