SORTING

Input: a list of numbers $8, \sqrt{2}, 5, 21, 1/3$

Output: a permutation that is in increasing order $1/3, \sqrt{2}, 5, 8, 21$

An algorithm should do the following:

1) produce correct output for all possible input
2) terminate quickly
3) not use a lot of space
4) be described clearly

etc

Issues that affect algorithmic design:

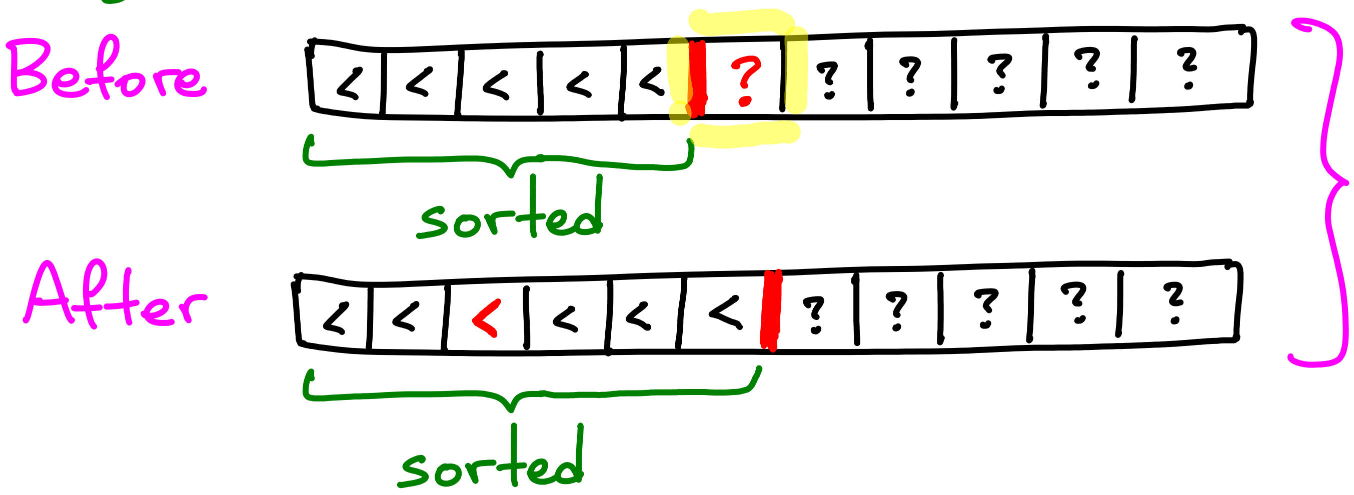
- input type: e.g., are numbers distinct? integer/real/irrational/etc? Do they have bounded size?
- allowed operations: compare, add, truncate, etc
- data structure: array? linked list? tree? etc
- model of computation: time/space complexity of operations

We focus on:
- comparison-based algorithms
- constant time for basic ops (more later)

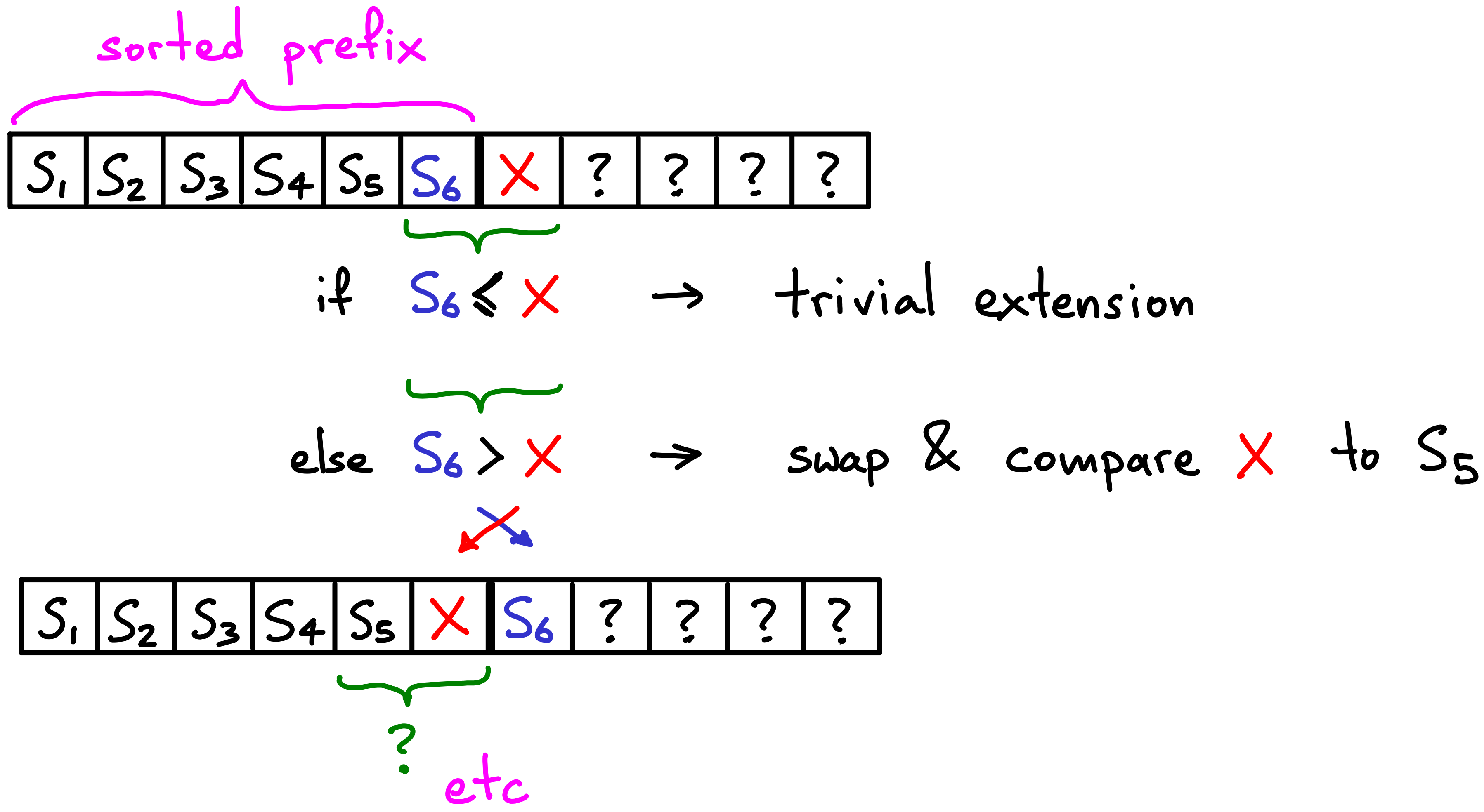
Insertion sort

- Start with a sorted prefix of size 1.
- Extend size of sorted prefix by 1
- Repeat

In general:



Use the element next to the prefix to extend the prefix...



If prefix size $= j$ then we can insert X after at most j comparisons

$\leq j$ comparisons $\rightarrow$ increase the sorted prefix size from j to $j+1$

Terminate when prefix size $= n$ (entire array)

comparisons $\leq \sum_{j=1}^{n-1} j = 1 + 2 + 3 + \cdots + (n-1) = \frac{n(n-1)}{2}$

$= \frac{1}{2}n^2 - \frac{1}{2}n$ = worst case #comparisons

To actually implement, we need extra time & space

but just a constant amount per comparison e.g., $5 \cdot \left(\frac{1}{2}n^2 - \frac{1}{2}n\right)$

At an introductory level:

- we don't focus on whether it takes 1 or 2 or 5 operations to compare, swap & iterate: $5 \cdot \left(\frac{1}{2}n^2 - \frac{1}{2}n\right)$
- we don't really care about non-leading terms: $5 \cdot \left(\frac{1}{2}n^2 - \frac{1}{2}n\right)$

When n is HUGE, both of the above are unimportant.

$$\left[\begin{array}{ll} \text{see} & 5 \cdot \left(\frac{1}{2}n^2 - \frac{1}{2}n\right) \\ \text{interpret} & n^2 \end{array}\right]$$

This leads to Θ-notation aka big-O notation