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1) produce correct output for all possible input
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4) be described clearly

etc

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- input type: e.g., are numbers distinct? integer/real/irrational/etc?
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We focus on:
- comparison-based algorithms
- constant time for basic ops (more later)

Insertion sort

- Start with a sorted prefix of size 1.
- Extend size of sorted prefix by 1
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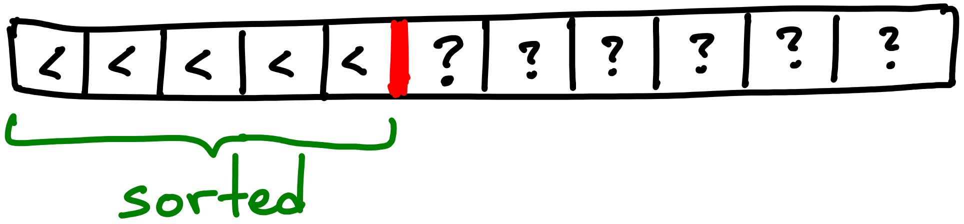
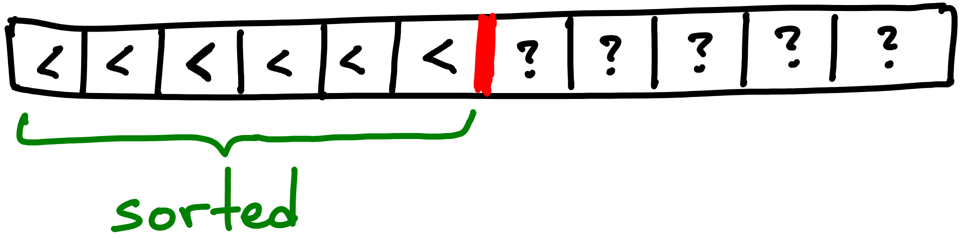
In general:

Before



After



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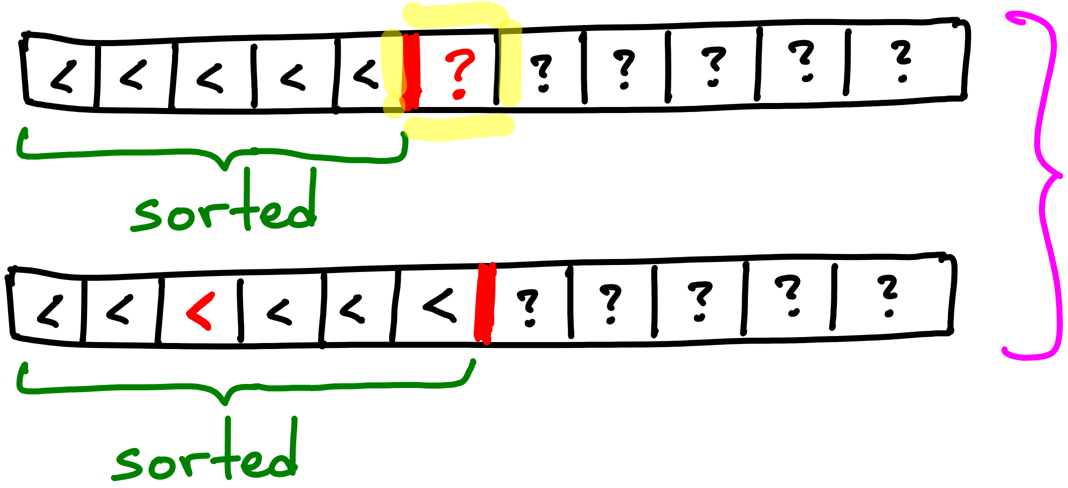
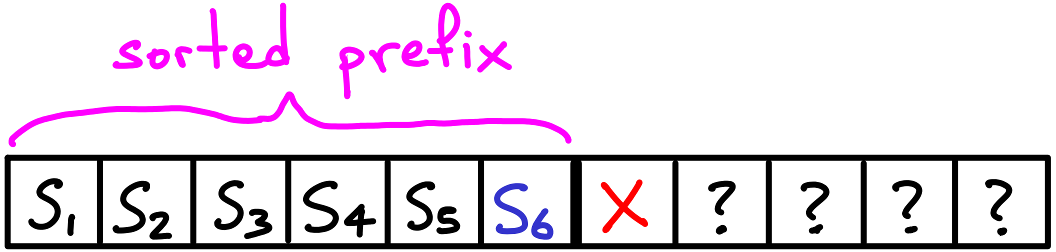
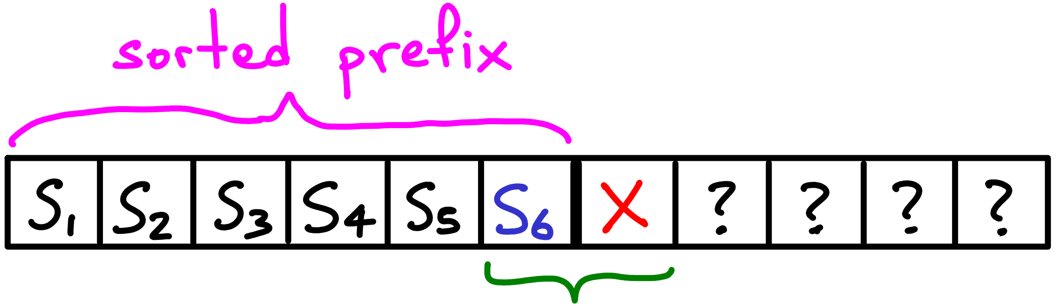


Use the element next to the prefix to extend the prefix...


sorted prefix
S1 S2 S3 S4 S5 S6 X ? ? ? ?



if $S_6 \leq x$ $\rightarrow$ trivial extension

$$\overbrace{\boxed{S_1}\boxed{S_2}\boxed{S_3}\boxed{S_4}\boxed{S_5}\boxed{S_6}}^{\text{sorted prefix}}\boxed{x}\boxed{?}\boxed{?}\boxed{?}\boxed{?}$$

if $S_6 \leq x$ $\rightarrow$ trivial extension

else $S_6 > x$

sorted prefix

S_1	S_2	S_3	S_4	S_5	S_6	X	?	?	?	?

if $S_6 \leqslant X$ $\rightarrow$ trivial extension

else $S_6 > X$ $\rightarrow$ swap

S_1	S_2	S_3	S_4	S_5	X	S_6	?	?	?	?

sorted prefix

S_1	S_2	S_3	S_4	S_5	S_6	X	?	?	?	?

if $S_6 \leq X$ → trivial extension

else $S_6 > X$ → swap & compare X to S_5

S_1	S_2	S_3	S_4	S_5	X	S_6	?	?	?	?

?

sorted prefix

S_1	S_2	S_3	S_4	S_5	S_6	X	?	?	?	?

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?

etc

If prefix size = j then we can insert X after at most j comparisons

$\leq j$ comparisons $\rightarrow$ increase the sorted prefix size from j to $j+1$

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$$= \frac{1}{2}n^2 - \frac{1}{2}n$$ = worst case #comparisons

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To actually implement, we need extra time & space

but just a constant amount per comparison e.g., $5\cdot\left(\frac{1}{2}n^2-\frac{1}{2}n\right)$

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- we don't focus on whether it takes 1 or 2 or 5 operations to compare, swap & iterate: $5 \cdot \left(\frac{1}{2}n^2 - \frac{1}{2}n\right)$

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This leads to Θ-notation

aka big-O notation