SELECTION

Given n values, find the k-th smallest

(Given n values → in an array or linked list)

(k-th smallest → rank = k)

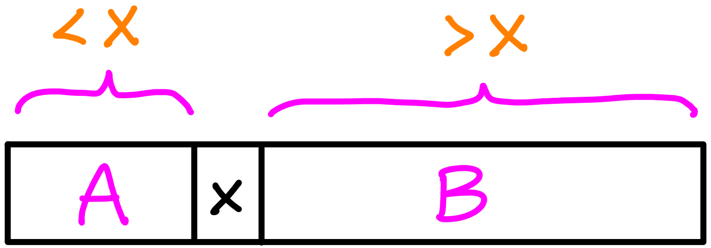
- easy if we sort first → $O(n \log n)$
- easy and fast if $k = O(1)$ or $k = n - O(1)$ → $O(n)$

harder as $k \to \frac{n}{2}$ (median)

SELECTION – Given n values, find the k-th smallest

Simple algorithm:

- pick a random input value: x
- use x as pivot, partition input → $\Theta(n)$
- if $\text{rank}(x) = k$, return x
  - if $k < \text{rank}(x)$, recurse on A → find value with rank k
  - if $\text{rank}(x) < k$, recurse on B → find value with rank $k - r$



$\text{size}(A) + 1 = \text{rank}(x) = r$

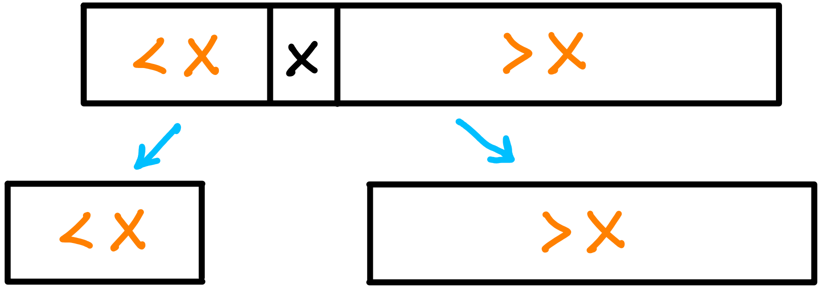
Worst case : $T(n) = \Theta(n) + T(n-1) = O(n^2)$

Deterministic SELECTION algorithm (1973)

↳ Blum, Floyd, Pratt, Rivest, Tarjan

- do something more clever: use some other x
- get O(n) time

- pick a ~~random~~ input value: x
- use x as pivot, partition input →

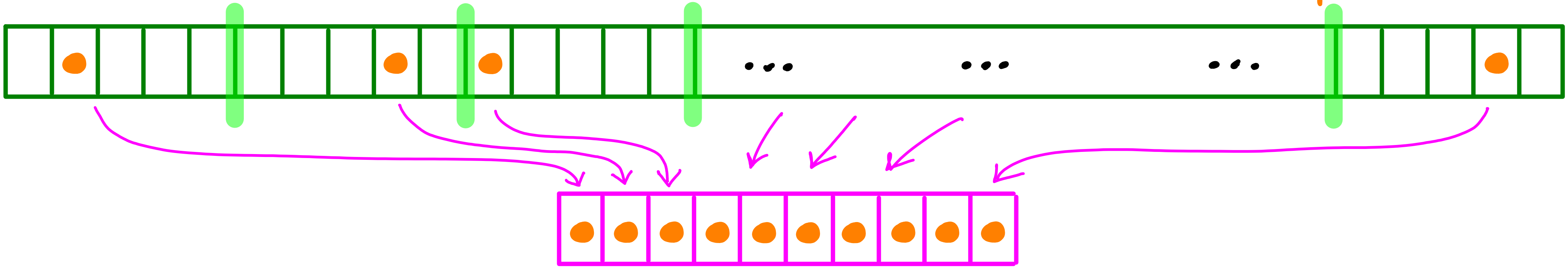


- recurse left or right if necessary

Deterministic SELECTION algorithm

For now, assume n = multiple of 5

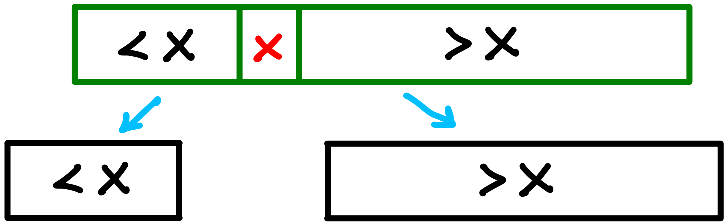
- for each block of size 5: find 3rd smallest value = representative



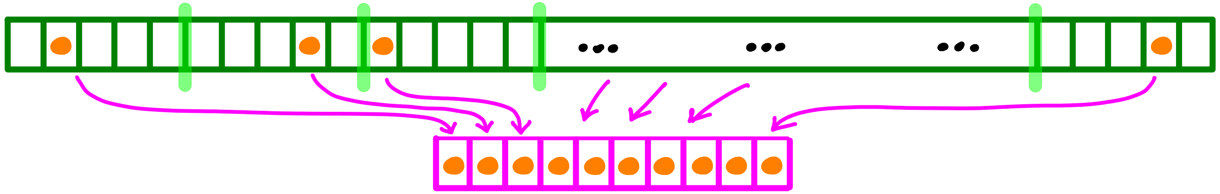
- copy representatives to new list
- find median of representatives → x

How? Use the algorithm (recurse)

- use x as pivot, partition input
- recurse left or right if necessary

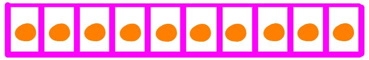


$T(n) = ?$

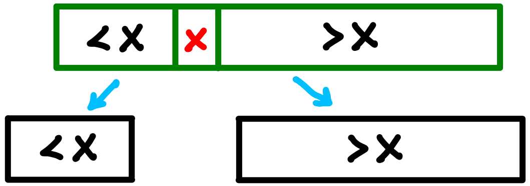


Time to run Selection on n elements

- for each block of size 5: find representative $= \frac{n}{5} \cdot \Theta(1) = \Theta(n)$
- copy representatives to new list $= \Theta(n)$



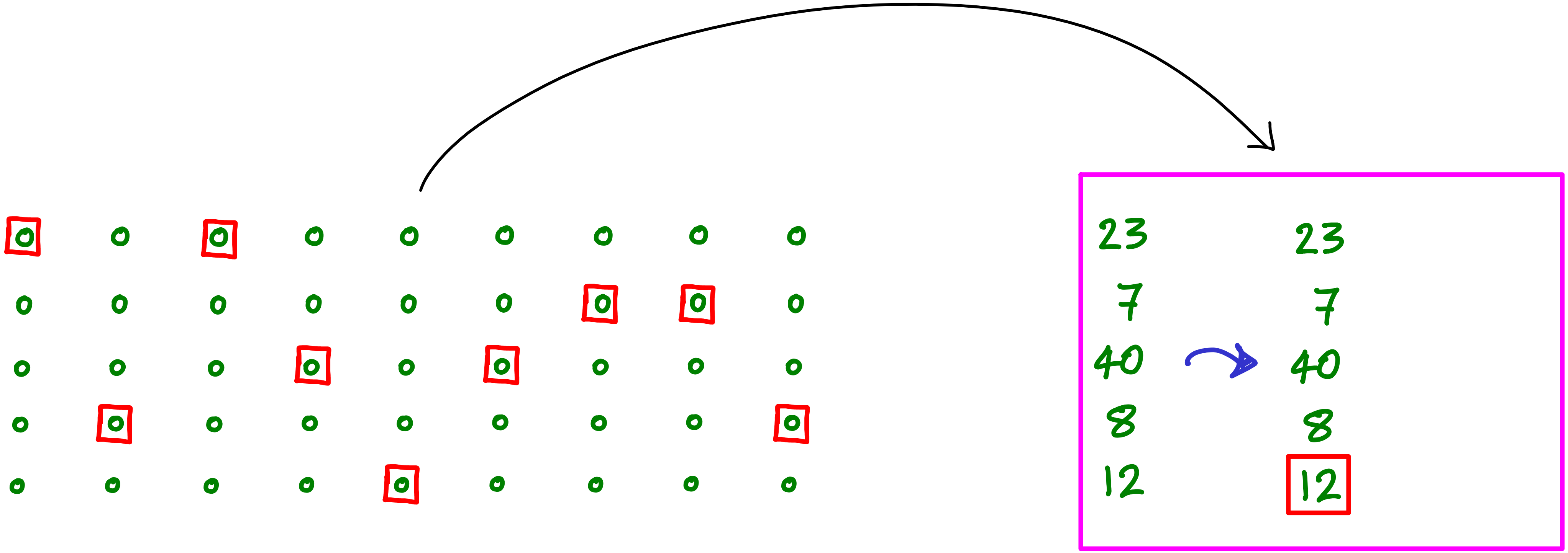
- find median of representatives $\rightarrow x$ (recurse) $= T(\frac{n}{5})$
- use x as pivot, partition input $= \Theta(n)$



- recurse left or right if necessary $= T(?)$

VISUALIZATION - not part of algorithm

- blocks of size 5
- find representatives



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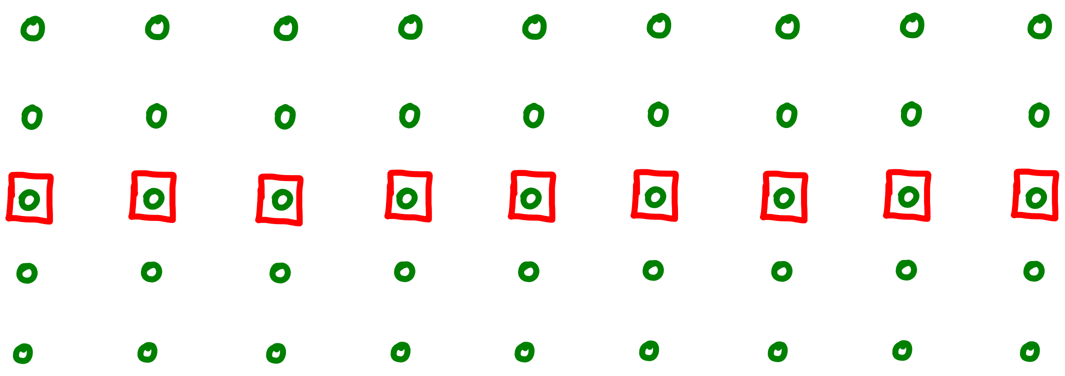

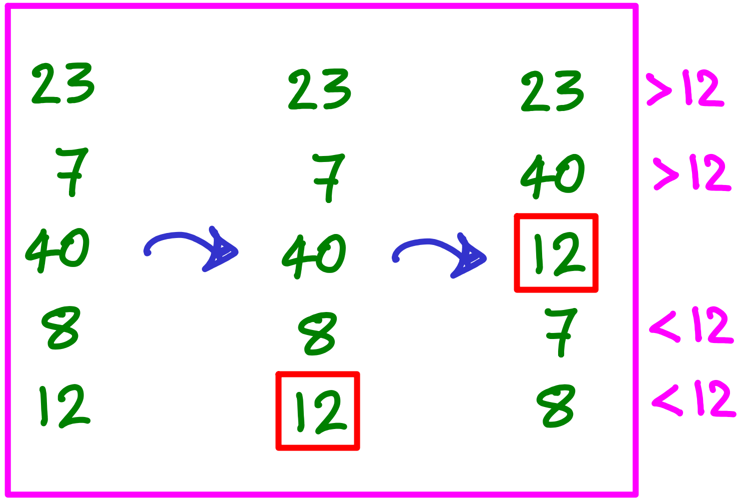

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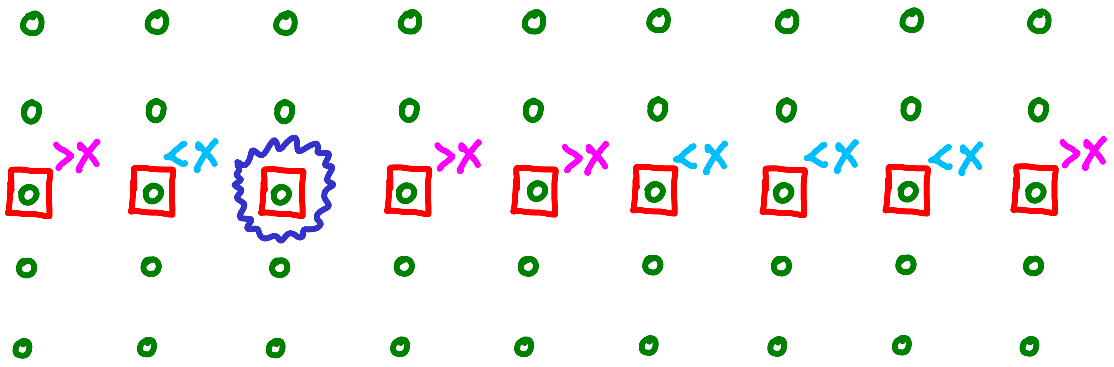

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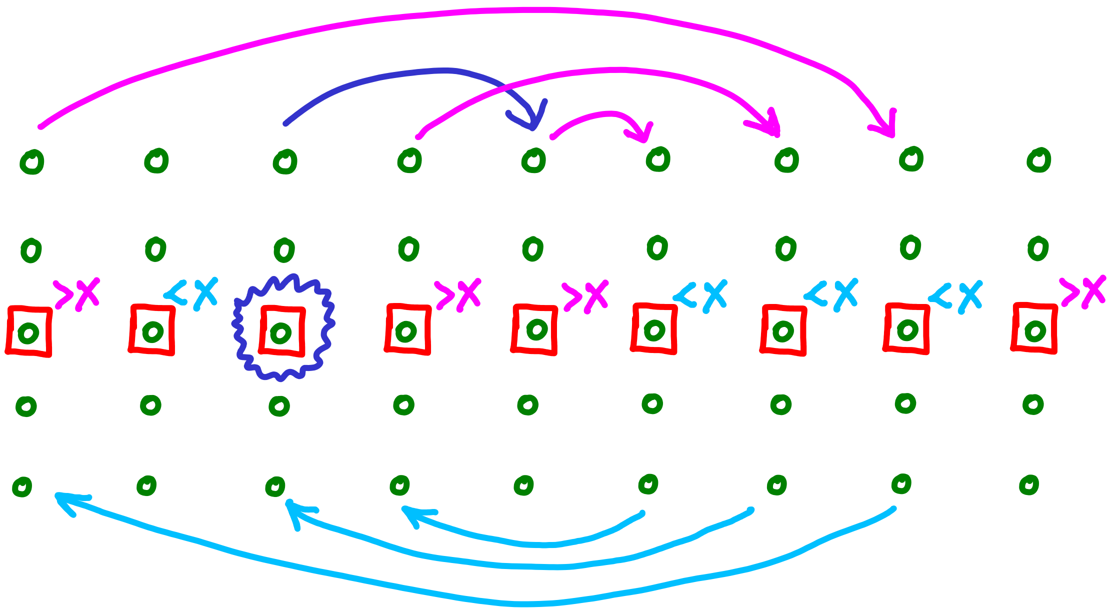

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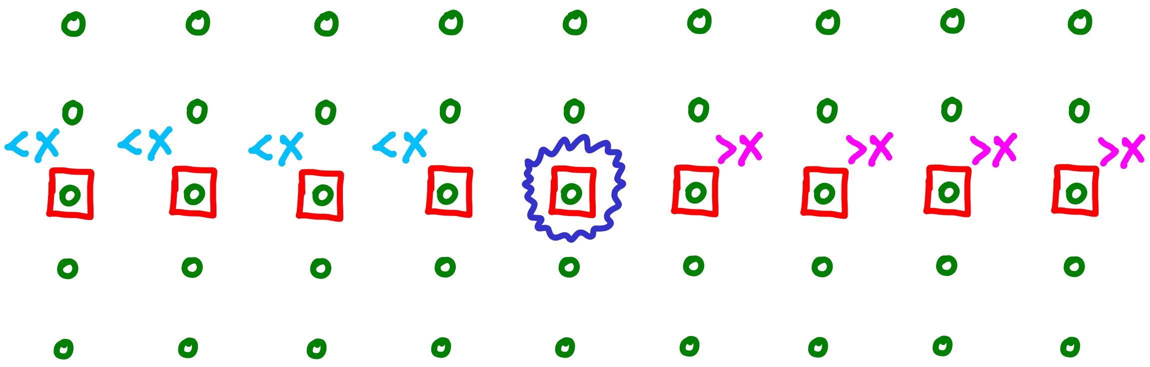
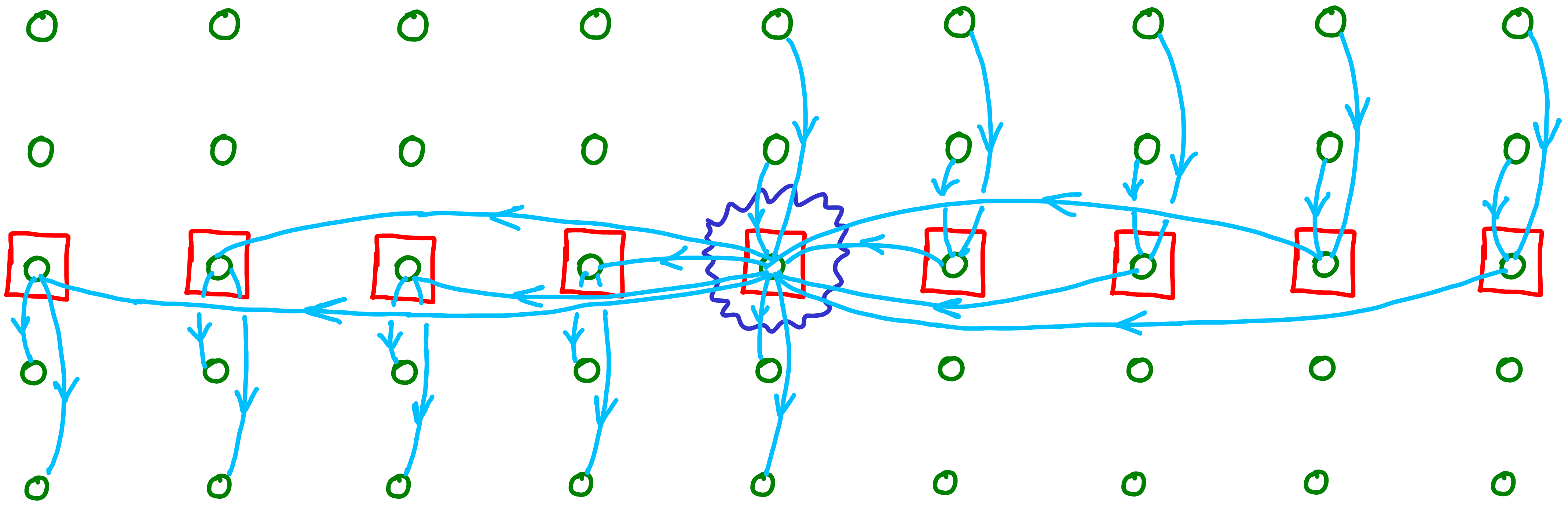
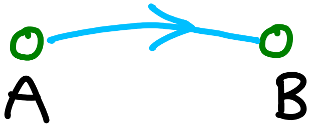
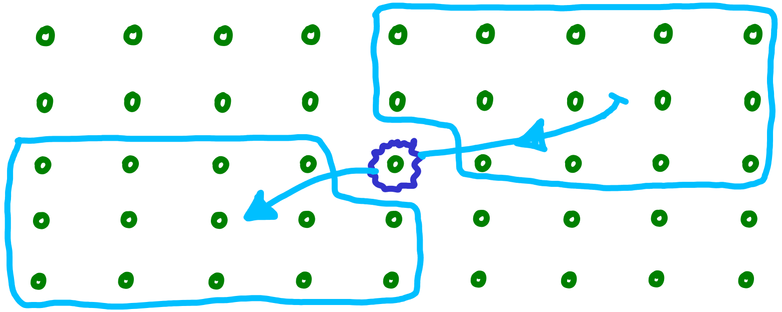
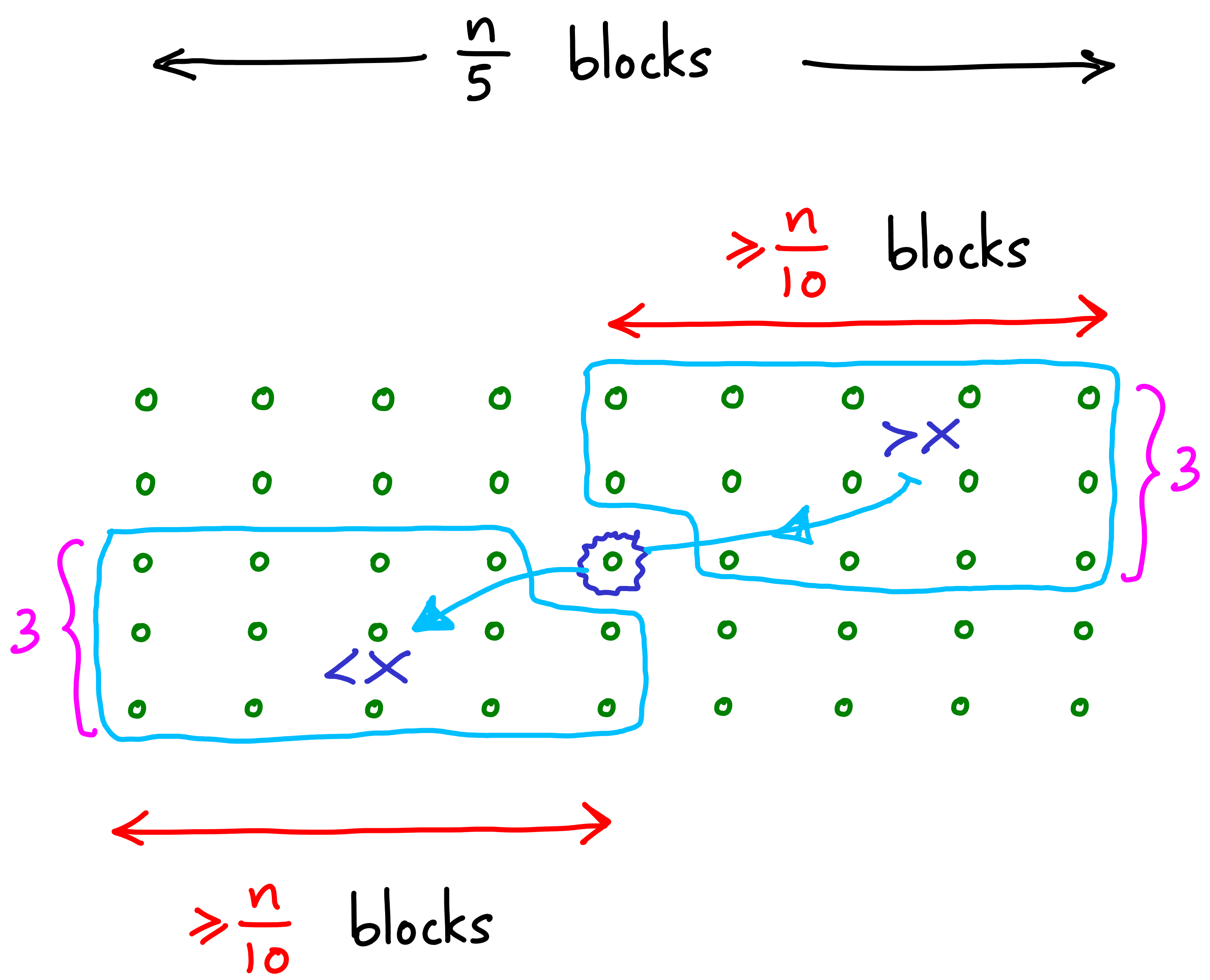




Let 

mean $A > B$





$\#\text{elements} > x$

$\geq 3 \cdot \frac{n}{10}$

$\#\text{elements} < x$

$\geq 3 \cdot \frac{n}{10}$

After partitioning, we recurse on at most $\frac{7n}{10}$ elements

- After partitioning, we recurse on at most $\frac{7n}{10}$ elements

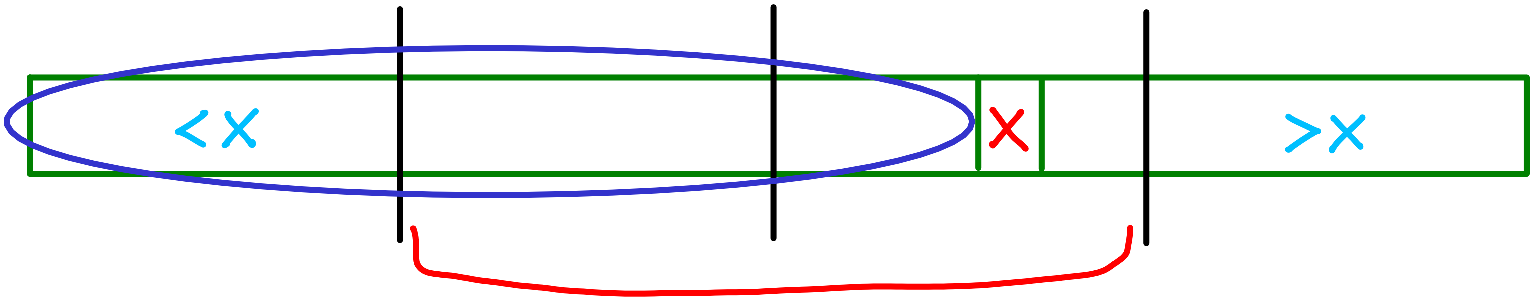
- If n is not a multiple of 5,

  we might need to recurse on a few more elements.

  (or spend extra $O(n)$ work dealing with this)

- For large enough n ($n > 50$), we recurse on at most $\frac{3n}{4}$ elements.

  ↳ x has rank between $\frac{n}{4}$ and $\frac{3n}{4}$



$$T(n) \leq \Theta(n) + T\left(\frac{n}{5}\right) + T\left(\frac{3n}{4}\right)$$

1) find representatives
2) copy to new list
3) find x
4) partition using x
5) recurse left or right

$$T(n) \le T\left(\frac{n}{5}\right) + T\left(\frac{3n}{4}\right) + dn$$

assume $T(k) \le ck$ for all $k < n$

$$T(n) \le c \cdot \frac{n}{5} + c \cdot \frac{3n}{4} + dn$$

$$= \frac{19}{20} cn + dn$$

$$= cn - \left(\frac{cn}{20} - dn\right)$$

$$\le cn \quad \text{...if } c = 20d$$