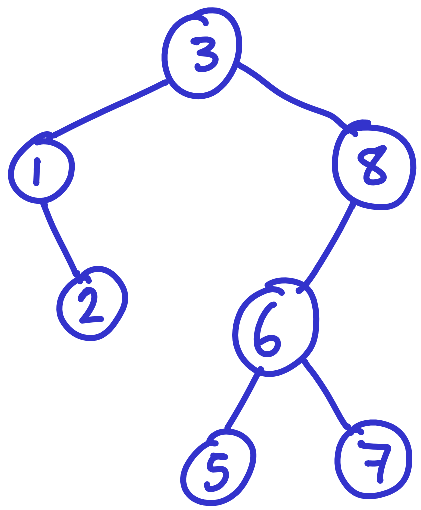
BINARY SEARCH TREES – BUILT RANDOMLY



Insert n elements into a BST

in the order that they're given.

Given array of elements : 3 1 8 2 6 7 5

3

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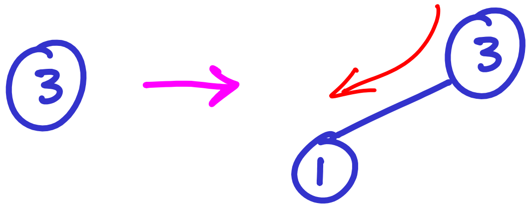

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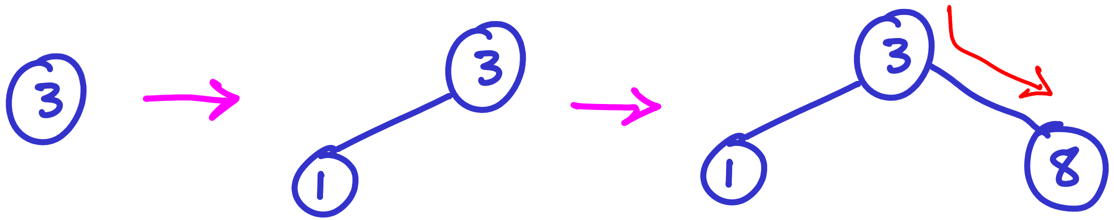

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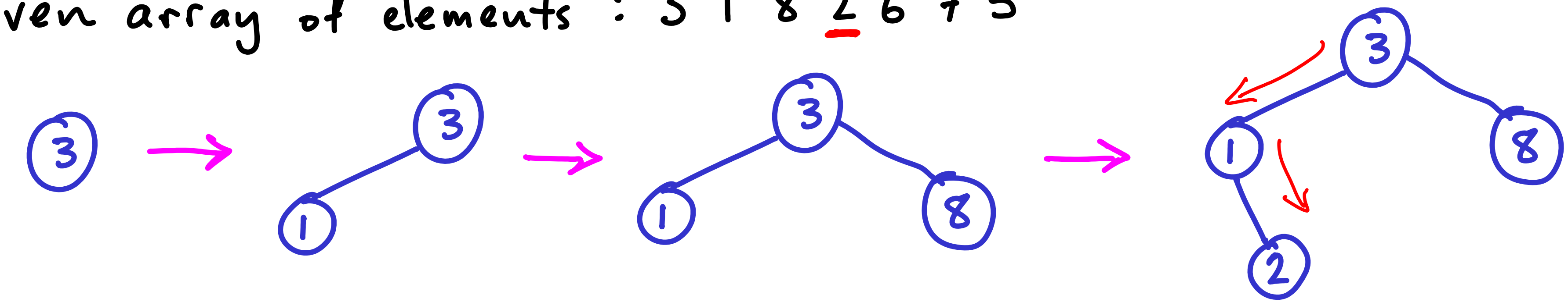

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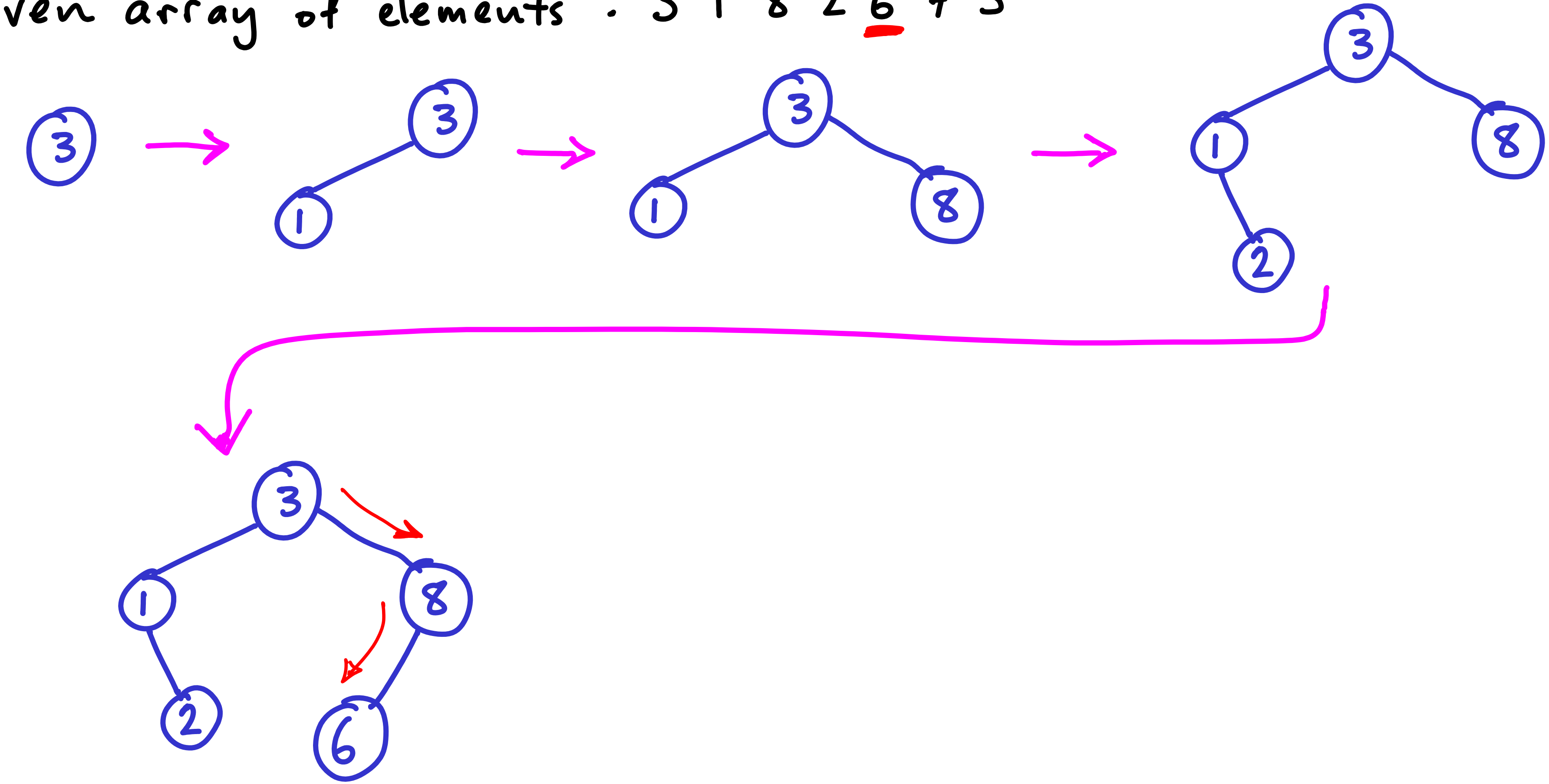

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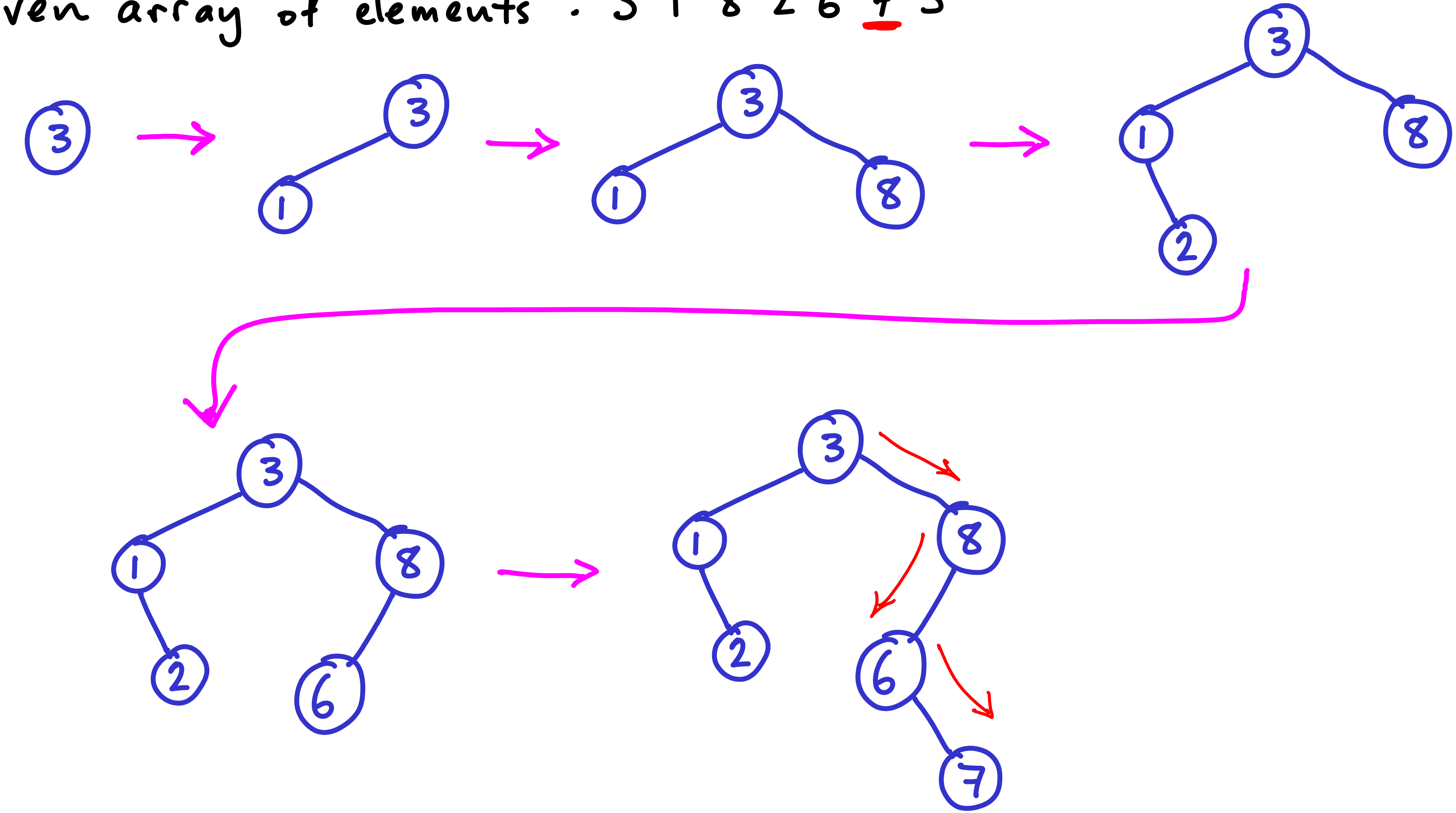

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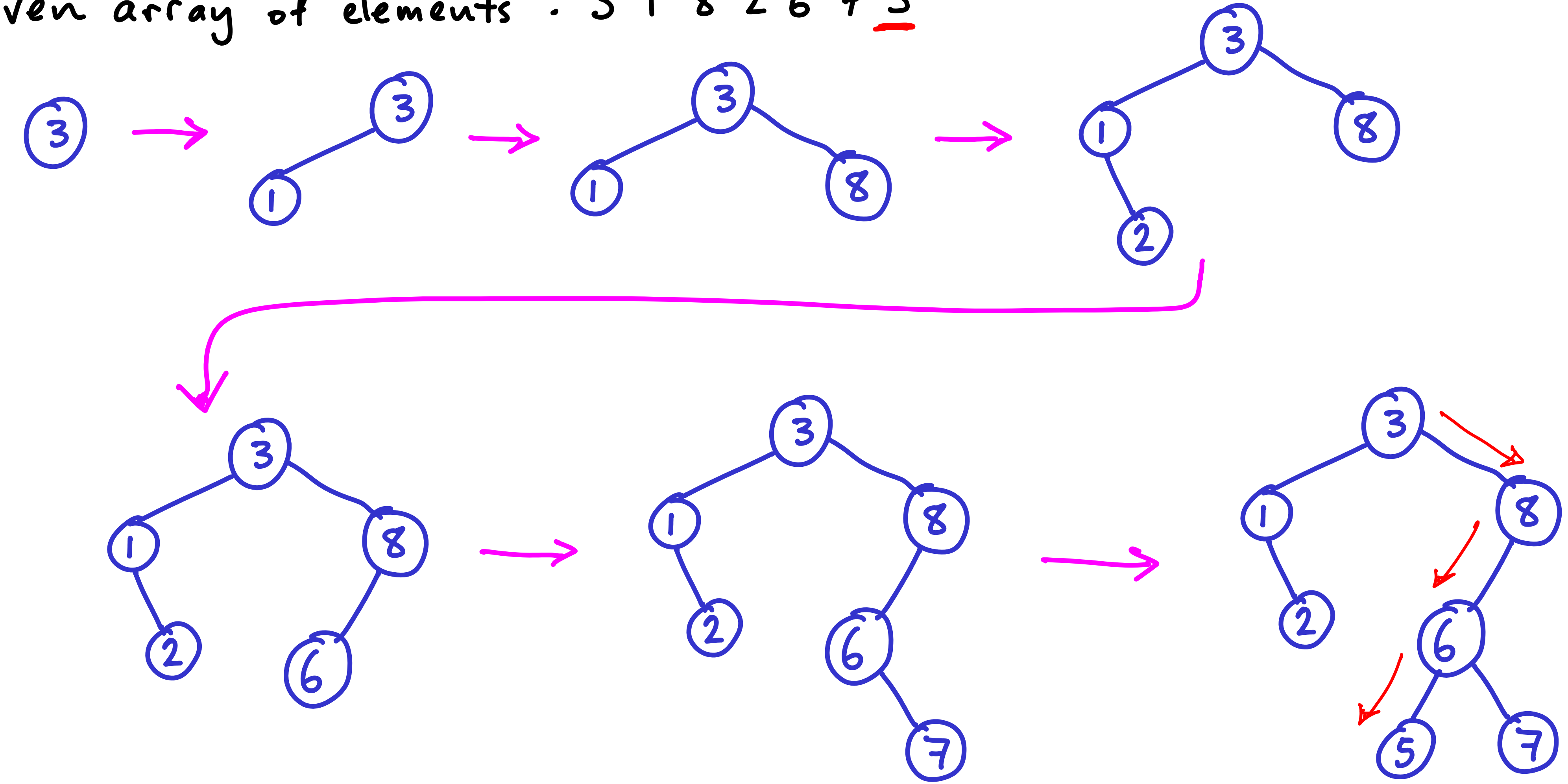

- What is the worst-case time complexity, and why?
- How unbalanced could the tree be ?

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↳ already sorted input, reverse-sorted, nearly sorted...

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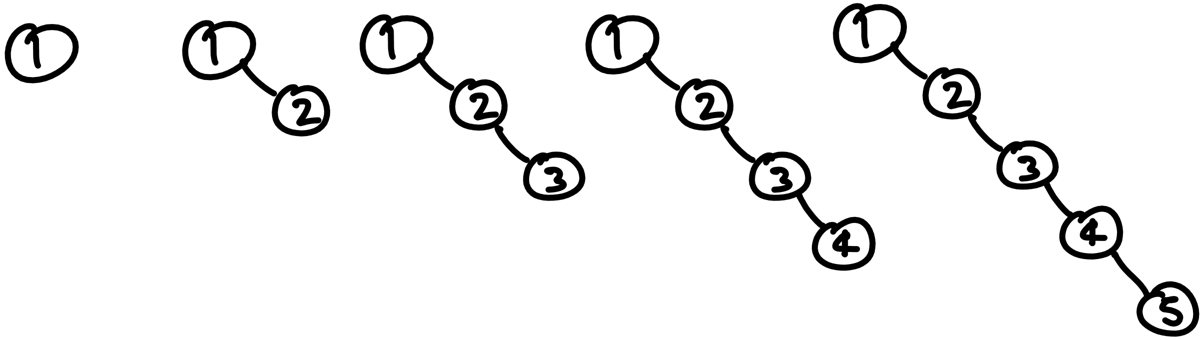

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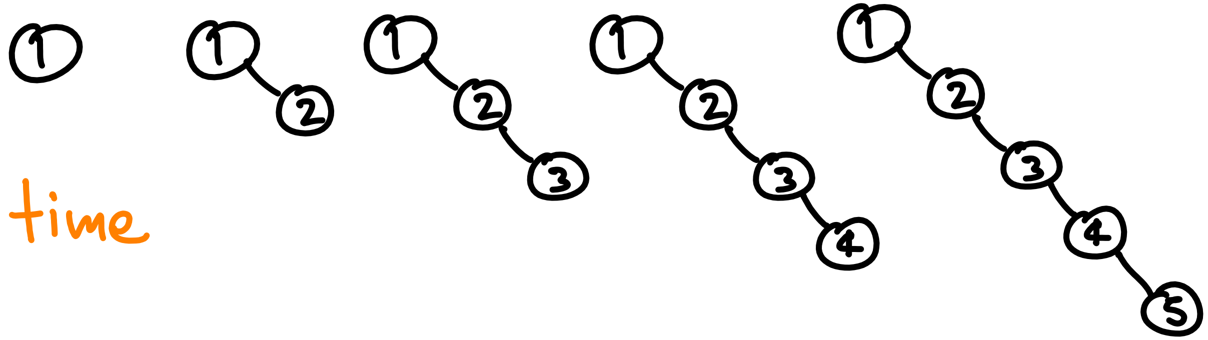

$O(n)$ depth, $O(n^2)$ time

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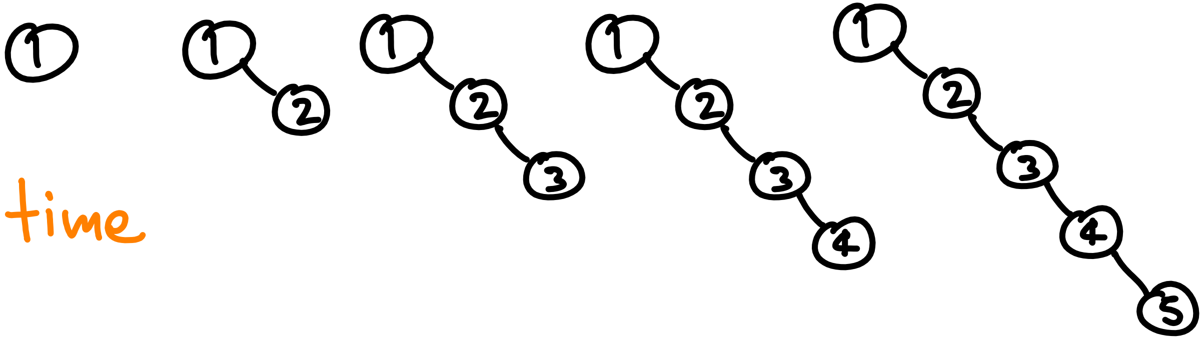

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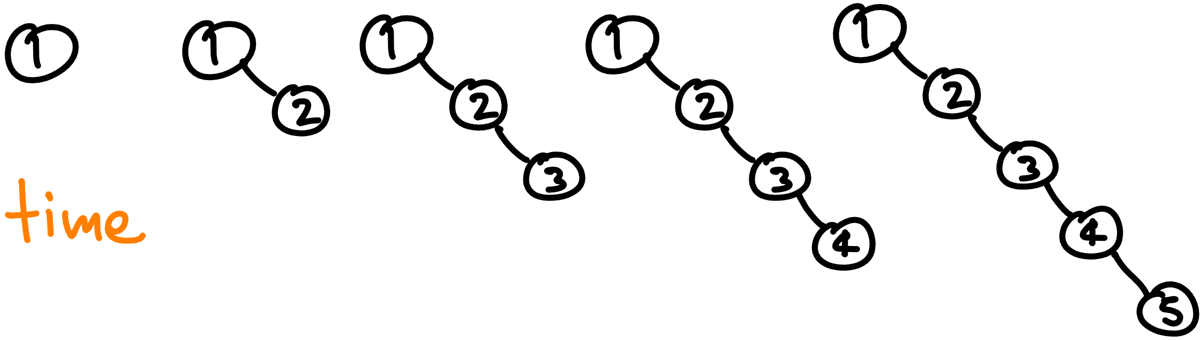

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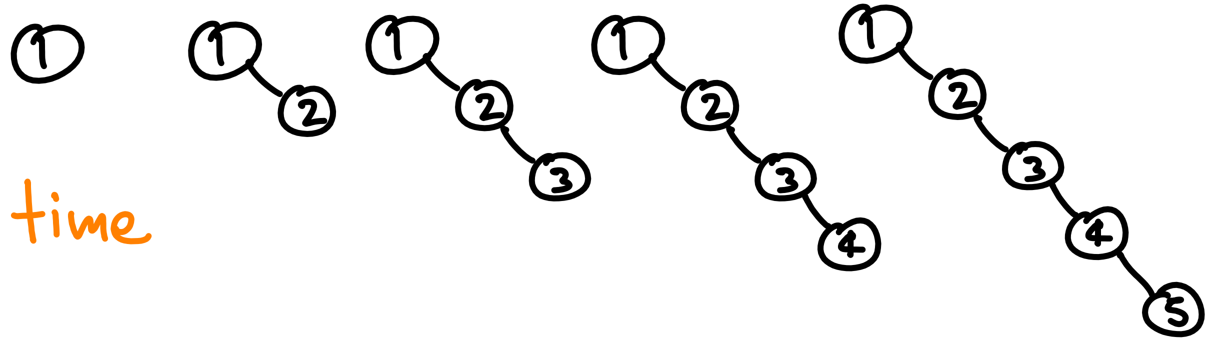

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- What would be ideal ? → balanced partition every time
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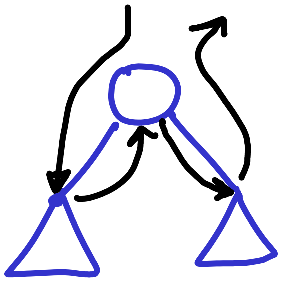
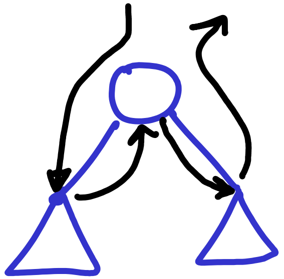
Hint: after constructing the BST, what does an in-order traversal give?



Constructing a BST essentially sorts data so the sorting lower bound applies



Constructing a BST essentially sorts data
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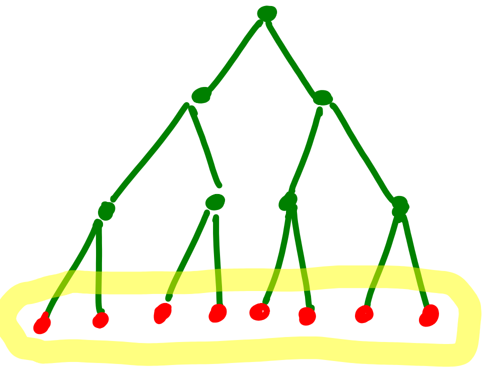
worst case $\Omega(n \log n)$

Constructing a BST essentially sorts data so the sorting lower bound applies } worst case $\Omega(n \log n)$

But we can make an even stronger claim

Constructing a BST essentially sorts data so the sorting lower bound applies $\Big\}$ worst case $\Omega(n\log n)$



In a balanced tree, $\sim\frac{n}{2}$ nodes have depth $\sim\log n$

so they take $\Omega(n\log n)$ time to insert.

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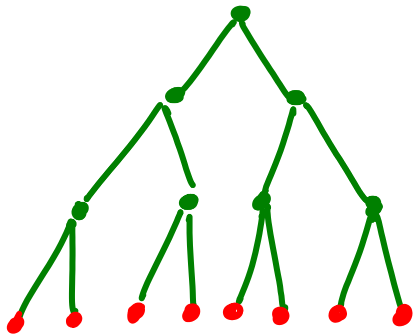

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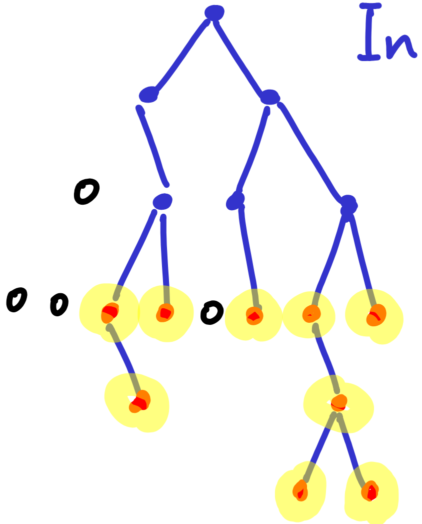

In any other tree we can find $\geq\frac{n}{2}$ nodes with depth $\geq\log n$

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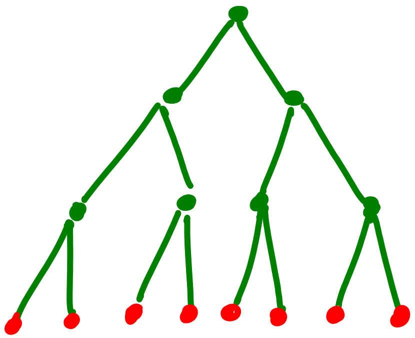

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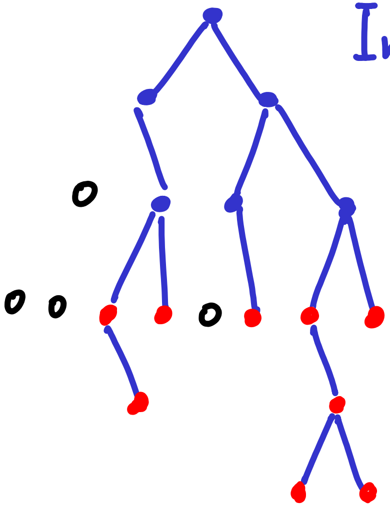

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$\hookrightarrow$ Every algorithm requires $\Omega(n\log n)$ time to construct any BST

Conclusion so far:

If lucky, $\Theta(n\log n)$ time

If unlucky, $\Theta(n^2)$ time

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Sounds like ... Quicksort

Stable quicksort

3 1 8 2 6 7 5

- use first elt to partition → 1 2 3 8 6 7 5

Stable quicksort

3 1 8 2 6 7 5

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- repeat on each side

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1 2 675 8

Stable quicksort

3 1 8 2 6 7 5

- use first elt to partition
- repeat on each side
- 3rd round

1 2 3 8 6 7 5

1 2 3 8 6 7 5

1 2 6 7 5 8

1 2 3 8 6 7 5

1 2 6 7 5 8

2 5 6 7

Stable quicksort

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- use first elt to partition → 1 2 3 8 6 7 5

- repeat on each side

  1 2 3 8 6 7 5

  1 2 6 7 5 8

- 3rd round

  1 2 3 8 6 7 5

  1 2 6 7 5 8

  2 5 6 7

- 4th round

  1 2 3 8 6 7 5

  1 2 6 7 5 8

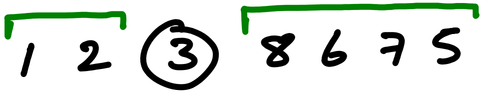
  2 5 6 7

  5 7

Stable quicksort

3 1 8 2 6 7 5

- use first elt to partition



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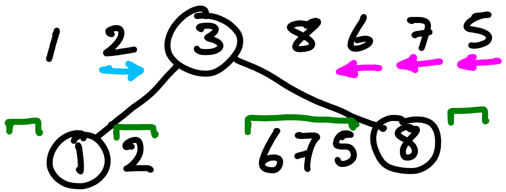

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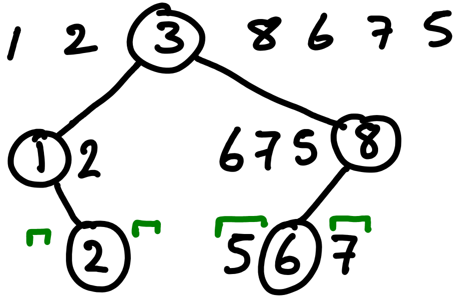
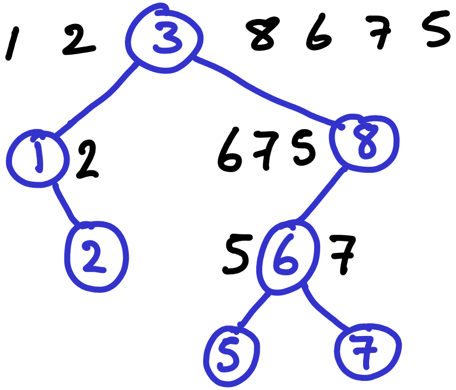


- 4th round

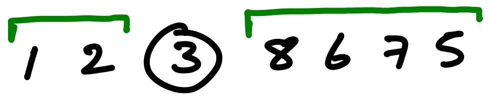


same tree as BST

Stable quicksort

3 1 8 2 6 7 5

- use first elt to partition →



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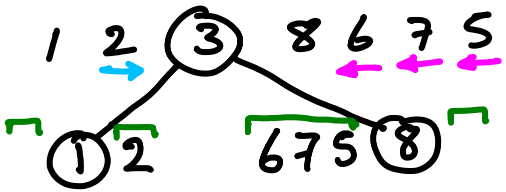

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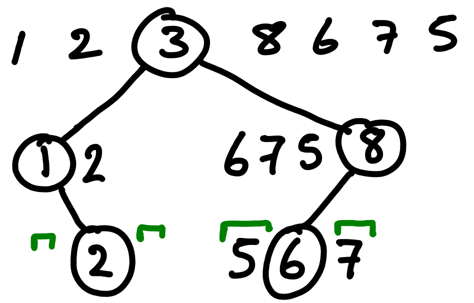

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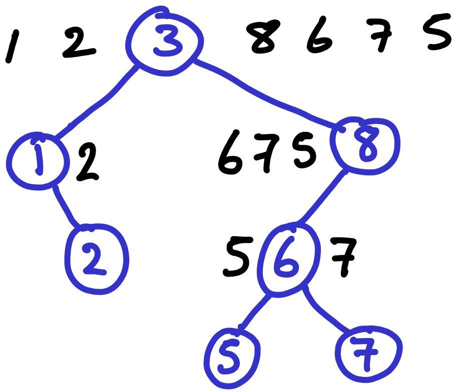
same tree as BST



quicksort round1: compare all elts to 3

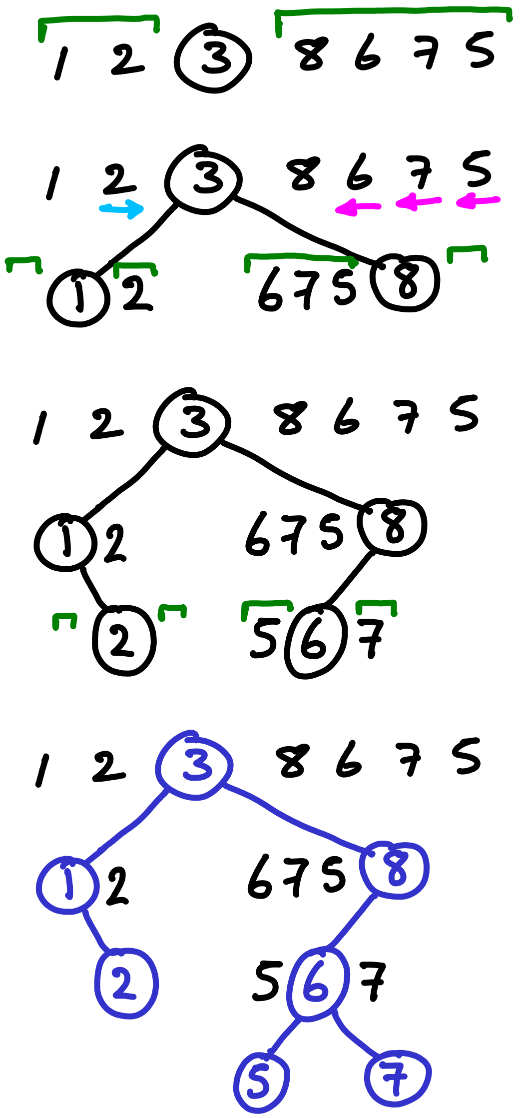
BST 3 = root; eventually all elts pass through.

Stable quicksort

③ 1 8 2 6 7 5

- use first elt to partition
- repeat on each side
- 3rd round
- 4th round

same tree as BST



quicksort round 1: compare all elts to ③

BST ③ = root; eventually all elts pass through.

quicksort: partitions into 2 groups
<③ & >③
each is independent

BST same

Stable quicksort

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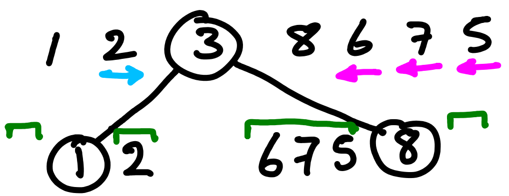

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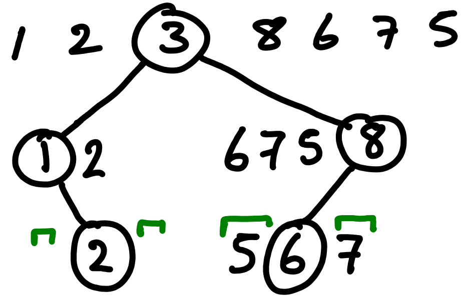

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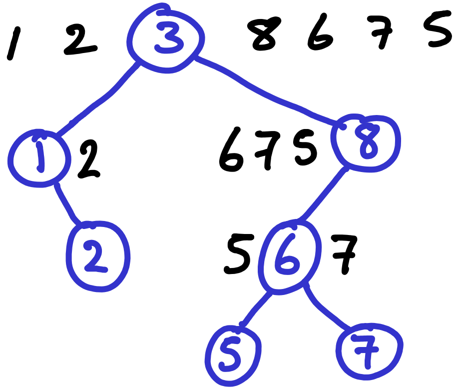

same tree as BST

quicksort round 1: compare all elts to 3

BST 3 = root; eventually all elts pass through.

quicksort: partitions into 2 groups <3 & >3 each is independent

BST same

exactly same comparisons but in different order

Conclusion:

The expected time complexity of building a BST on random data is the same as for Quicksort: $O(n \log n)$

Conclusion:

The expected time complexity of building a BST on random data is the same as for Quicksort: O(nlogn)

Unfortunately this doesn't imply anything useful about the expected depth of a random BST.

In particular it doesn't imply expected O(logn) depth.
There exist trees that have much greater depth, but that can be constructed in O(nlogn) time.